EECS 126: Probability and Random Processes

Discussion 1

Note: Please work on the problems before the discussion session.

**Problem 1.** We are given that \( P(A) = 0.55 \), \( P(B^c) = 0.35 \), and \( P(A \cup B) = 0.75 \). Determine \( P(B) \) and \( P(A \cap B) \).

**Problem 4.** Let \( A \) and \( B \) be two sets with a finite number of elements. Show that the number of elements in \( A \cap B \) plus the number of elements in \( A \cup B \) is equal to the number of elements in \( A \) plus the number of elements in \( B \).

**Problem 11.** Show the following generalizations of the formula

\[
P(A \cup B \cup C) = P(A) + P(A^c \cap B) + P(A^c \cap B^c \cap C).
\]

(a) Let \( A, B, C, \) and \( D \) be events. Then

\[
P(A \cup B \cup C \cup D) = P(A) + P(A^c \cap B) + P(A^c \cap B^c \cap C) + P(A^c \cap B^c \cap C^c \cap D).
\]

(b) Let \( A_1, A_2, \ldots, A_n \) be events. Then

\[
P \left( \bigcup_{k=1}^{n} A_k \right) = P(A_1) + P(A_1^c \cap A_2) + P(A_1^c \cap A_2^c \cap A_3) + \cdots + P(A_1^c \cap \cdots \cap A_{n-1}^c \cap A_n).
\]

**Problem 30.** We are told that events \( A \) and \( B \) are independent. In addition, events \( A \) and \( C \) are independent. Is it true that \( A \) is independent of \( B \cup C \)? Provide a proof or counterexample to support your answer.

**Problem 21.** A peculiar six-sided die has uneven faces. In particular, the faces showing 1 or 6 are \( 1 \times 1.5 \) inches, the faces showing 2 or 5 are \( 1 \times 0.4 \) inches, and the faces showing 3 or 4 are \( 0.4 \times 1.5 \) inches. Assume that the probability of a particular face coming up is proportional to its area. We independently roll the die twice. What is the probability that we get doubles?