EECS 126: Probability and Random Processes

Discussion 1

Note: Please work on the problems before the discussion session.

Problem 1. We are given that $P(A) = 0.55$, $P(B^c) = 0.35$, and $P(A \cup B) = 0.75$. Determine $P(B)$ and $P(A \cap B)$.

Problem 4. Let A and B be two sets with a finite number of elements. Show that the number of elements in $A \cap B$ plus the number of elements in $A \cup B$ is equal to the number of elements in A plus the number of elements in B.

Problem 11. Show the following generalizations of the formula

$$\mathbf{P}(A \cup B \cup C) = \mathbf{P}(A) + \mathbf{P}(A^c \cap B) + \mathbf{P}(A^c \cap B^c \cap C).$$

(a) Let A, B, C, and D be events. Then

$$\mathbf{P}(A \cup B \cup C \cup D) = \mathbf{P}(A) + \mathbf{P}(A^c \cap B) + \mathbf{P}(A^c \cap B^c \cap C) + \mathbf{P}(A^c \cap B^c \cap C^c \cap D).$$

(b) Let $A_1, A_2, \ldots, A_n$ be events. Then

$$\mathbf{P}\left(\cup_{k=1}^n A_k\right) = \mathbf{P}(A_1) + \mathbf{P}(A_1^c \cap A_2) + \mathbf{P}(A_1^c \cap A_2^c \cap A_3) + \cdots + \mathbf{P}(A_1^c \cap \cdots \cap A_{n-1}^c \cap A_n).$$

Problem 30. We are told that events A and B are independent. In addition, events A and C are independent. Is it true that A is independent of $B \cup C$? Provide a proof or counterexample to support your answer.

Problem 21. A peculiar six-sided die has uneven faces. In particular, the faces showing 1 or 6 are 1×1.5 inches, the faces showing 2 or 5 are 1×0.4 inches, and the faces showing 3 or 4 are 0.4×1.5 inches. Assume that the probability of a particular face coming up is proportional to its area. We independently roll the die twice. What is the probability that we get doubles?