EECS 126: Probability and Random Processes

Discussion 10

Note: Please work on the problems before the discussion session.

Problem 37. We are given that $\mathbf{E}[X] = 1$, $\mathbf{E}[Y] = 2$, $\mathbf{E}[X^2] = 5$, $\mathbf{E}[Y^2] = 8$, and $\mathbf{E}[XY] = 1$. Find the linear least squares estimator of Y given X.

Problem 34. Let $X = Y - Z$ where Y and Z are nonnegative random variables such that $YZ = 0$.

(a) Show that $\text{cov}(Y, Z) \leq 0$.

(b) Show that $\text{var}(X) \geq \text{var}(Y) + \text{var}(Z)$.

(c) Use the result of part (b) to show that

$$\text{var}(X) \geq \text{var}\big(\max\{0, X\}\big) + \text{var}\big(\max\{0, -X\}\big).$$

Problem 35. Consider two random variables X and Y. Assume for simplicity that they both have zero mean.

(a) Show that X and $\mathbf{E}[X \,|\, Y]$ are positively correlated.

(b) Show that the correlation coefficient of Y and $\mathbf{E}[X \,|\, Y]$ has the same sign as the correlation coefficient of X and Y.