EECS 126: Probability and Random Processes

Problem Set 11 Due on November 29^{th}, 2005 in class

Note: Please submit a photocopy of your work. If you collaborate on the assignment, please list the names of students in your study group.

Problem 1 *Finite State Markov Chain*

Bob goes to Las Vegas. He does not want to lose a lot of money so decides to gamble with only \$3 and to stop playing if he loses these \$3 dollars or reaches \$5. He approaches to a roulette wheel, which contains 18 red, 18 black and 2 green holes. He decides to always bet \$1 on red on the roulette wheel.

a. *Formulate the Markov Chain corresponding to the amount of dollars that Bob has and classify the states.*

b. *Find the transition probability matrix of the Markov Chain. Determine its eigenvalues and eigenvectors. Express the row vector containing probability that Bob has \$i at the end of n-th game for* $i = 0, 1, 2, 3, 4, 5$ *in terms of the eigenvalues and eigenvectors.*

   Use Matlab as you see fit.

c. *Find the steady state probability that Bob has \$i at the end of n-th game for* $i = 0, 1, 2, 3, 4, 5$.

d. *Repeat part (b) and (c) for the case when Bob starts with \$2 and stops playing if he loses all his money or has \$5.*

Problem 2 *Markov Chain*

Let X_n *be a Bernoulli i.i.d. process with parameter* $\frac{1}{2}$.

a. *Is* X_n *a Markov Chain? Why or why not?*

b. *Another sequence* Y_n *is defined by* $Y_n = X_n + X_{n-1}$. *Is* Y_n *a Markov process? Why or why not?*

c. *Another sequence* Z_n *is defined by* $Z_n = (X_n, X_{n-1})$. *Is* Z_n *a Markov process? Why or why not?*

Problem 3 *Synthesis of Markov Chains*

You are given a description of a two-state discrete-time Markov chain with a specified transition matrix A and a specified distribution for X_0.

a. *Assume you are given access to a process of iid continuous uniform random variables V_i that are uniformly distributed on $(0, 1)$. Give an algorithm that simulates the Markov chain using the V_k random variables as the underlying probability space. Use one V_k for every time step.*

b. *Assume that you are given access to a Bernoulli($\frac{1}{2}$) process B_k. Give an algorithm that simulates the Markov chain using the random bits coming from B_k. Now, feel free to treat the B_k as a resevoir of randomness that can be used as you see fit. You can use a variable number of bits per time step.*

c. *For the algorithm you gave in part (a), how many bits do you need to examine on average for every state transition?*

d. *Suppose instead that you were given access to successive samples of $\{X_t\}$. Give an algorithm that will use these to simulate a Bernoulli($\frac{1}{2}$) process B_k. What do you require from the Markov chain for this to be guaranteed to work?*