EECS 126: Probability and Random Processes

Problem Set 1 Due on September 13th, 2005 in class

Note: Please submit a photocopy of your work. If you collaborate on the assignment, please list the names of students in your study group.

Problem 2. Let A and B be two sets. Under what conditions is the set $A \cap (A \cup B)^c$ empty?

Problem 3. Let A and B be two sets.

(a) Show that $(A^c \cap B^c)^c = A \cup B$ and $(A^c \cup B^c)^c = A \cap B$.

(b) Consider rolling a six-sided die once. Let A be the set of outcomes where an odd number comes up. Let B be the set of outcomes where a 1 or a 2 comes up. Calculate the sets on both sides of the equalities in part (a), and verify that the equalities hold.

Problem 6. We roll a four-sided die once and then we roll it as many times as is necessary to obtain a different face than the one obtained in the first roll. Let the outcome of the experiment be (r_1, r_2) where r_1 and r_2 are the results of the first and the last rolls, respectively. Assume that all possible outcomes have equal probability. Find the probability that:

(a) r_1 is even.

(b) Both r_1 and r_2 are even.

(c) $r_1 + r_2 < 5$.

Problem 8. You enter a special kind of chess tournament, whereby you play one game with each of three opponents, but you get to choose the order in which you play your opponents. You win the tournament if you win two games in a row. You know your probability of a win against each of the three opponents. What is your probability of winning the tournament, assuming that you choose the optimal order of playing the opponents?

Problem 29. Let A and B be events such that $A \subset B$. Can A and B be independent?

Problem 31. Suppose that A, B, and C are independent. Use the definition of independence to show that A and $B \cup C$ are independent.

Problem 25. A particular jury consists of 7 jurors. Each juror has a 0.2 chance of making the wrong decision, independently of the others. If the jury reaches a decision by majority rule, what is the probability that it will reach a wrong decision?

Problem 26. Three persons roll a fair n-sided die once. Let A_{ij} be the event that person i and person j roll the same face. Show that the events A_{12}, A_{13}, and A_{23} are pairwise independent but are not independent.

Problem 14. We roll two fair 6-sided dice. Each one of the 36 possible outcomes is assumed to be equally likely.

(a) Find the probability that doubles were rolled.

(b) Given that the roll resulted in a sum of 4 or less, find the conditional probability that doubles were rolled.

(c) Find the probability that at least one die is a 6.

(d) Given that the two dice land on different numbers, find the conditional probability that at least one die is a 6.

Problem 19. A magnetic tape storing information in binary form has been corrupted, so it can only be read with bit errors. The probability that you correctly detect a 0 is 0.9, while the probability that you correctly detect a 1 is 0.85. Each digit is a 1 or a 0 with equal probability. Given that you read a 1, what is the probability that this is a correct reading?

Problem 18. Alice and Bob have $2n+1$ coins, each with probability of a head equal to p. Bob tosses $n+1$ coins, while Alice tosses the remaining n coins. Show that the probability that after all the coins have been tossed, Bob will have gotten more heads than Alice is $1/2$.

Problem 35. Consider three independent rolls of a fair six-sided die.

(a) What is the probability that the sum of the three rolls is 11?

(b) What is the probability that the sum of the three rolls is 12?

(c) In the seventeenth century, Galileo explained the experimental observation that a sum of 10 is more frequent than a sum of 9, even though both 10 and 9 can be obtained in six distinct ways. Can you retrace Galileo's thinking?

Problem 37. Count the number of distinguishable ways in which you can arrange the letters in the words:

(a) children

(b) bookkeeper