Assignment 2
Due February 7th

1. Self-grade Homework 1.
2. Read Chapter 3 Oppenheim and Schafer, 3rd ed.
4. The Z-transform of a right-sided sequence is given by:

\[ X(z) = \frac{z^{-2}}{1 - 2.3z^{-1} + 1.6z^{-2} - 0.3z^{-3}}. \]

Find \( x[n] \) by doing partial fraction expansion with the help of Scipy’s `scipy.signal.residue` function (You need to import scipy).
5. The signal \( x[n] \) has the spectrum

\[ X(e^{j\omega}) \]

The signal \( z[n] \) is given by

\[ z[n] = x[n]y[n]. \]

Draw the DTFT \( Z(e^{j\Omega}) \) if \( y[n] \) is:
   (a) \( y[n] = \cos(\pi n) \)
   (b) \( y[n] = \cos(\pi n/2) \)
   (c) \( y[n] = \cos(\pi n/4) + \cos(3\pi n/4) \)
   (d) \( y[n] = \cos(\pi n + \pi/2) \)

   a) Consider \( x[n] \) a sequence of length \( L \) between \( 0 \leq n < L \), whos DTFT is \( X(e^{j\omega}) \). Let \( y[n] \) be a sequence whoes DTFT is \( Y(e^{j\omega}) = |X(e^{j\omega})|^2 \). What can you ALWAYS say about \( y[n] \) (circle all that apply and briefly explain)?

<table>
<thead>
<tr>
<th>( y[n \geq L] = 0 )</th>
<th>( y[n &lt; 0] = 0 )</th>
<th>Conjugate symmetric</th>
<th>Real</th>
<th>Even length</th>
<th>Odd length</th>
</tr>
</thead>
</table>

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b) We would like to compute $y[n]$ from part (a) by using the DFT. We compute the following:

$$\tilde{Y}[k] = \mathcal{DFT}\{x[n]\} = \sum_{n=0}^{L-1} x[n]W_N^{kn}$$

Then compute

$$\tilde{y}[n] = \mathcal{IDFT}\{|\tilde{Y}[k]|^2\}$$

Finally we set:

$$y[n] = \tilde{y}[m[n]]$$

What are the appropriate $N$ and $m[n]$ that would result in the right $y[n]$? ($m[n]$ is index mapping function, for example $x[m[n]]$ where $m[n] = n - 1 \mod L$ circularly shifts an L-length sequence $x[n]$ by one index to the right)

7. From Midterm I fall’11: A stable linear time invariant system has an impulse response

$$H(z) = \frac{3(1 - z^{-1})}{(1 - \frac{1}{2}z^{-1})(1 - 2z^{-1})}.$$  

a) Find the impulse response $h[n]$ of this system.

b) Another system has an impulse response $g[n]$ which is given by

$$g[n] = j^n h[n]$$

where $h[n]$ is the impulse response you found in part (a). Plot the poles and zeros of $G(z)$, which is the z-transform of $g[n]$. Indicate the region of convergence for $G(z)$.

8. Adapted from Midterm I fall’10:

The following values from the 8-point DFT of a length-8, real-valued sequence $x[n]$ are known:


(a) Evaluate $x[0]$.

(b) Find the 8-point DFT of the circular convolution:

$$x[n] \ast \delta[n - 1],$$

where $\delta[n]$ is the unit impulse.

(c) Consider a length-4 sequence $w[n]$ whose 4-point DFT is given by

$$W[k] = X[2k], \quad k = 0, 1, 2, 3.$$  

Find an expression for $w[n]$ in terms of $x[n]$. What’s going on here?

9. From Midterm I spring’15:

The FFTW package provides functions for efficiently computing DFTs. We will later see the main idea behind the implementation, which is famously known as the FFT.

The input to the complex-valued DFT function in FFTW expects as an input an array in which the real and imaginary components are interleaved. You would like to compute the $\text{DFT}_N$ of a $N$-length complex sequence $x[n] = x_r[n] + jx_i[n]$. You prepare an input array $h[n]$, $0 \leq n < 2N$ such that $h[2n] = x_r[n]$ and $h[2n + 1] = x_i[n]$. Unfortunately instead of calling the complex-valued DFT function, you accidentally call the real-valued function which treats your input array as a $2N$-length real-valued array and returns a $2N$-length complex array $H[k]$ corresponding to the $\text{DFT}_{2N}$ of $h[n]$. The question is about computing $X[k]$, the $\text{DFT}_N$ of $x[n]$ from $H[k]$ with minimal computation.
(a) Find an expression for the first half of \( H[k] \), \( 0 \leq k < N \) in terms of \( X_r[k] \) and \( X_i[k] \) the \( DFT_N \) of \( x_r[n] \) and \( x_i[n] \) respectively.

(b) Find an expression for the second half, \( H[k+N] \), \( 0 \leq k < N \) in terms of \( X_r[k] \) and \( X_i[k] \) the \( DFT_N \) of \( x_r[n] \) and \( x_i[n] \) respectively.

(c) Find an expression to \( X[k] \) in terms of \( H[k] \) with minimum number of multiplications/additions that can not be precomputed. How many additions/multiplications are required?

10. *From Midterm I fall’11:* Consider this discrete-time system

\[
\begin{align*}
x[n] & \quad H(e^{j\omega}) \quad y[n]
\end{align*}
\]

The frequency response \( H(e^{j\omega}) \) is shown below

\[
\begin{array}{c|c}
\omega & H(e^{j\omega}) \\
\hline
-\pi & 0 \\
0 & j \\
\pi & -j \\
\end{array}
\]

This is a very useful system, called a Hilbert-filter, and is often used in communication. Over the interval \(-\pi < \omega < \pi\) the frequency response is given by

\[
H(e^{j\omega}) = \begin{cases} 
  j & -\pi < \omega < 0 \\
  -j & 0 < \omega < \pi \\
  0 & \omega = 0
\end{cases}
\]

a) What is the symmetry of the impulse response of this system \( h[n] \)? Is it even, odd, Hermitian, or none of the above? Is it real, imaginary, or complex?

b) Assume the input to this system is \( x[n] = \cos(\omega_0 n) \)

where \(|\omega_0| < \pi\). Find the output \( y[n] \).

c) We apply a general signal \( x[n] \) to two such systems in series

\[
\begin{align*}
x[n] & \quad H(e^{j\omega}) \quad y[n]
\end{align*}
\]

Find \( y[n] \).

d) Consider the samples of a speech signal \( x[n] \) with the following magnitude spectrum \(|X(e^{j\omega})|\):
Design and draw a system diagram that produces a baseband (around DC) Upper-Sideband signal from $x[n]$. That is, it should look like the above image, except with the lower sideband removed.