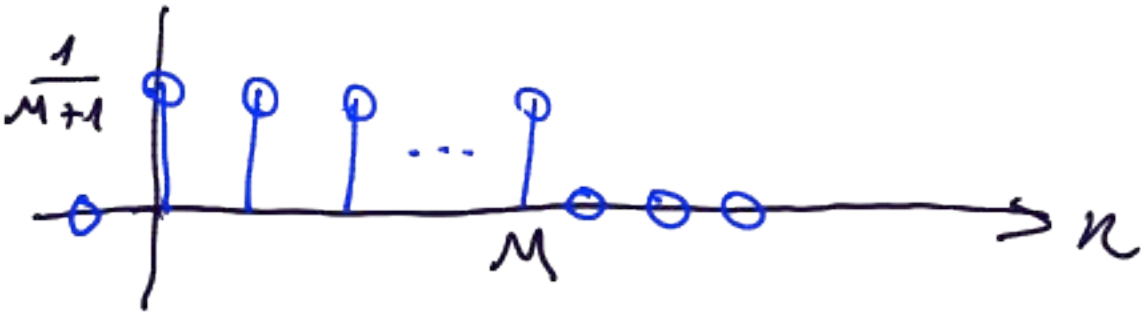
Lecture 11/30/11

Generalized linear-phase systems

$$H(e^{j\omega}) = \underbrace{A(e^{j\omega})}_{\text{Real, allow sign change}} e^{-j\alpha\omega + j\beta}$$

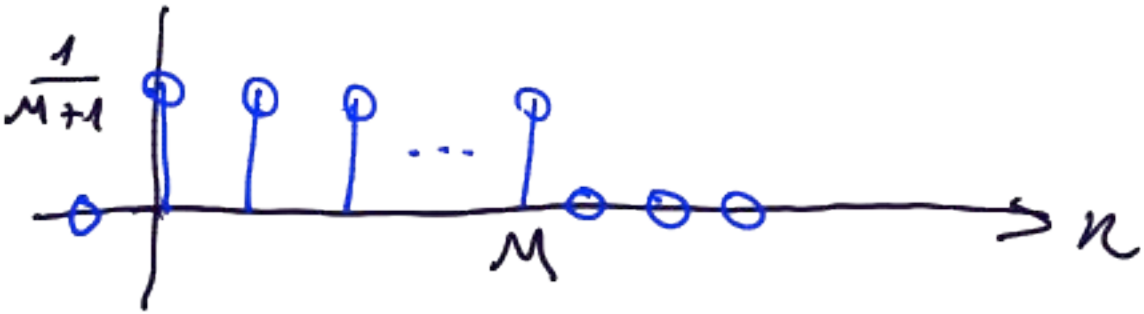
$$\text{grd}\left[H(e^{j\omega})\right] = \alpha \quad \left(\begin{array}{c}\text{except when} \\ A(e^{j\omega}) \text{ changes} \\ \text{sign}\end{array}\right)$$

Example (M+1)-point moving average



$$H(e^{j\omega}) = \underbrace{\boxed{}}_{A(e^{j\omega})} \boxed{}$$

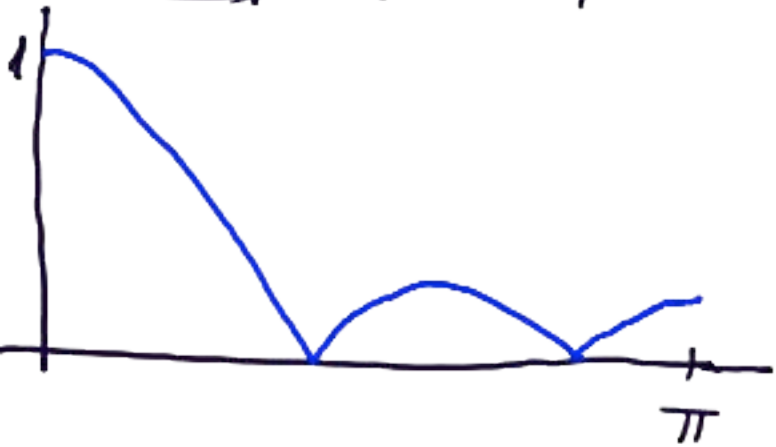
Example (M+1)-point moving average



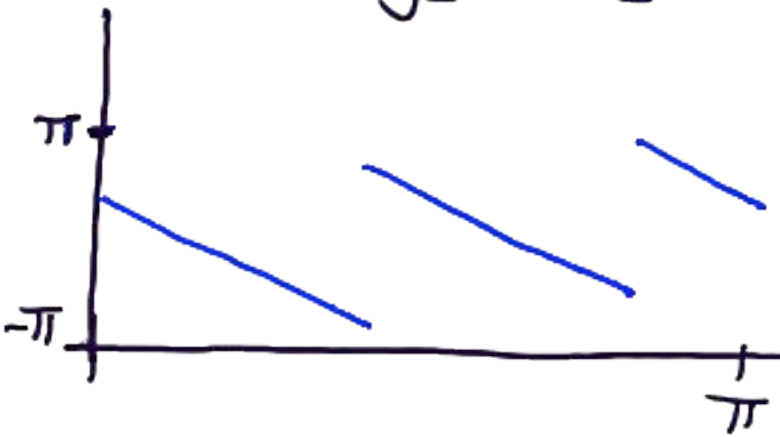
$$H(e^{j\omega}) = \underbrace{\boxed{\frac{1}{M+1}\,\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)}}}_{A(e^{j\omega})}\ \boxed{e^{-j\omega\underbrace{\frac{M}{2}}_{\alpha}}}$$

$\boxed{M=4}$

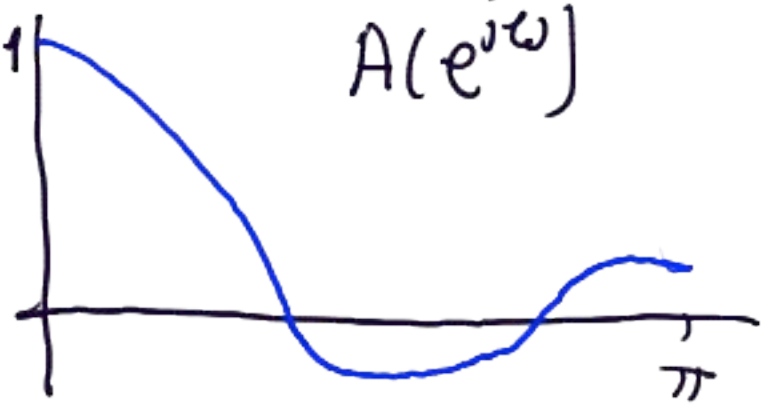
$|H(e^{j\omega})|$



$\arg[H(e^{j\omega})]$



$A(e^{j\omega})$

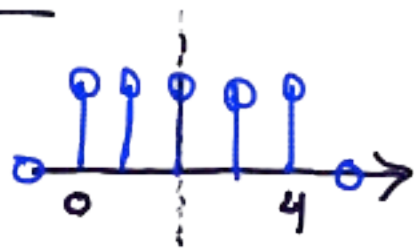


$grd[H(e^{j\omega})]$

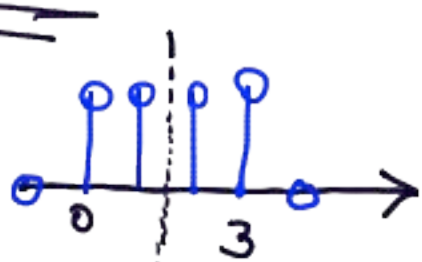
GLP for FIR → MUST have symmetry

$h[n] = h[M-n]$:

Type I (M even)



Type II (M odd)



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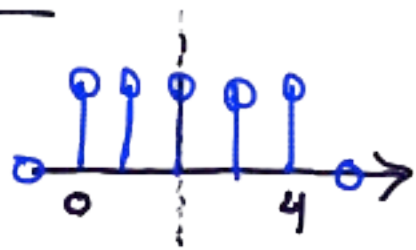
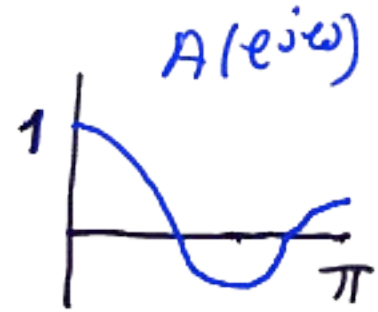
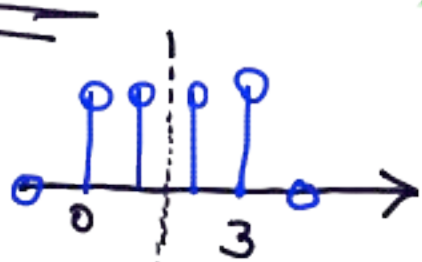
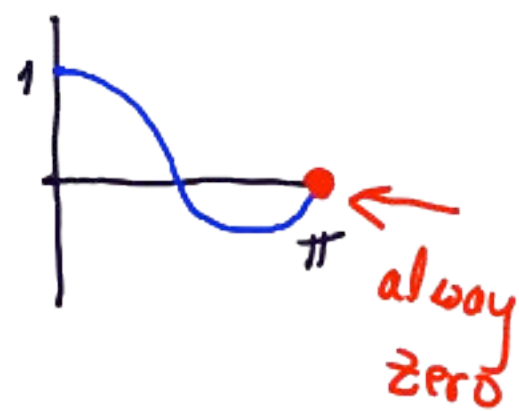




Type II (M odd)





GLP for FIR $\rightarrow$ MUST have symmetry

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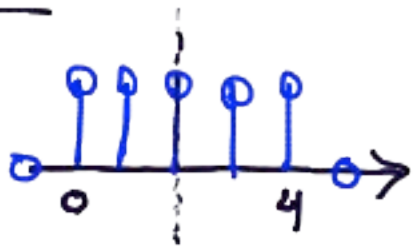
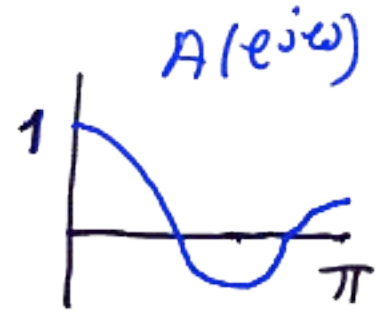
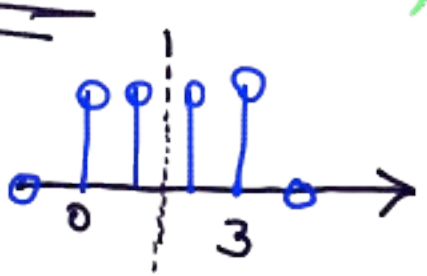
Type I (M even)

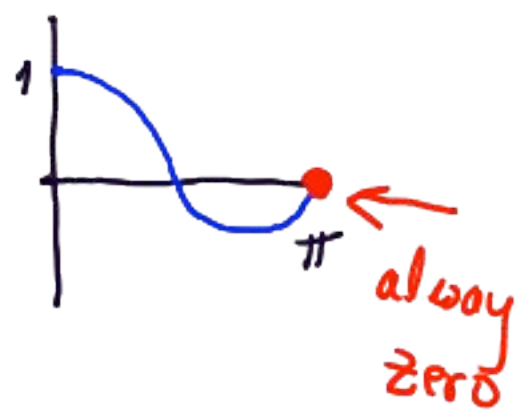




$$A(e^{j\omega}) = h\left[\frac{M}{2}\right] + 2\sum_{k=1}^{\frac{M}{2}} h\left[\frac{M}{2}-k\right]\cos(\omega k)$$

Type II (M odd)

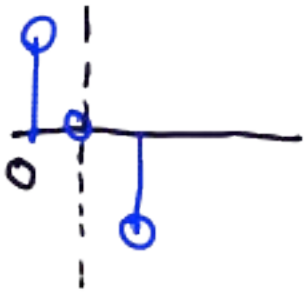


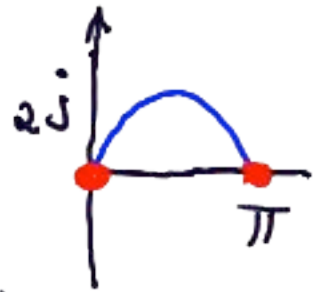


$A(e^{j\omega})$ = In the text

$$h[n] = -h[M-n]$$

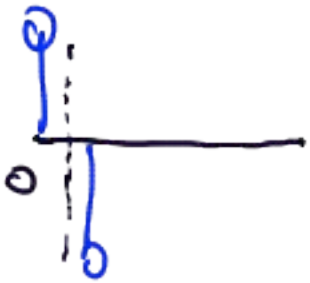
Type III (M even)

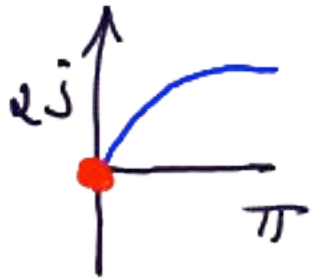




$$A(e^{j\omega}) = j\,2\sum_{k=1}^{\frac{M}{2}} h\left[\tfrac{M}{2}-k\right]\sin(\omega k)$$

Type IV (M odd)





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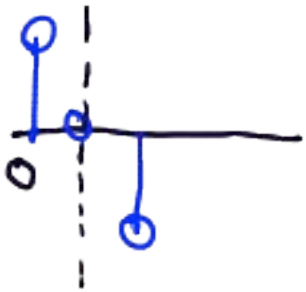
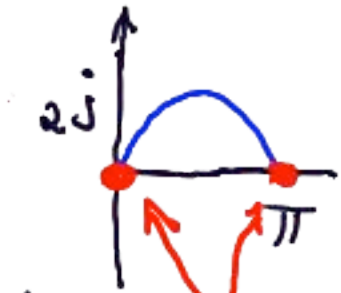
Type III (M even)

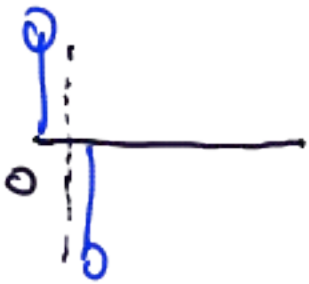


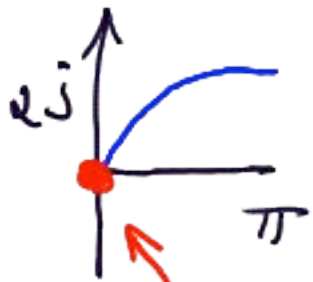


$$A(e^{j\omega}) = j\,2\sum_{k=1}^{\frac{M}{2}} h\left[\tfrac{M}{2}-k\right]\sin(\omega k)$$

always $=0$

Type IV (M odd)





always $=0$

$A(e^{j\omega}) =$ See text

Zeroes of GLP system

Type I, II: $h[n] = h[n-M]$

$$H(z) = \sum_{n=0}^{M} h[n] z^{-n} =$$

Zeroes of GLP system

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Zeroes of GLP system

Type I, II: $h[n] = h[n-M]$

$$H(z) = \sum_{n=0}^{M} h[n] z^{-n} =$$

$$= \sum_{n=0}^{M} h[M-n] z^{-n} = z^{-M} \sum_{n=0}^{M} h[M-n] z^{M-n}$$

Zeroes of GLP system

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$$H(z) = \sum_{n=0}^{M} h[n] z^{-n} =$$

$$= \sum_{n=0}^{M} h[M-n] z^{-n} = z^{-M} \sum_{n=0}^{M} h[\underbrace{M-n}_{\triangleq k}] z^{\underbrace{M-n}_{\triangleq k}}$$

$$= z^{-M} \sum_{k=0}^{M} h[k] z^{k}$$

Zeroes of GLP system

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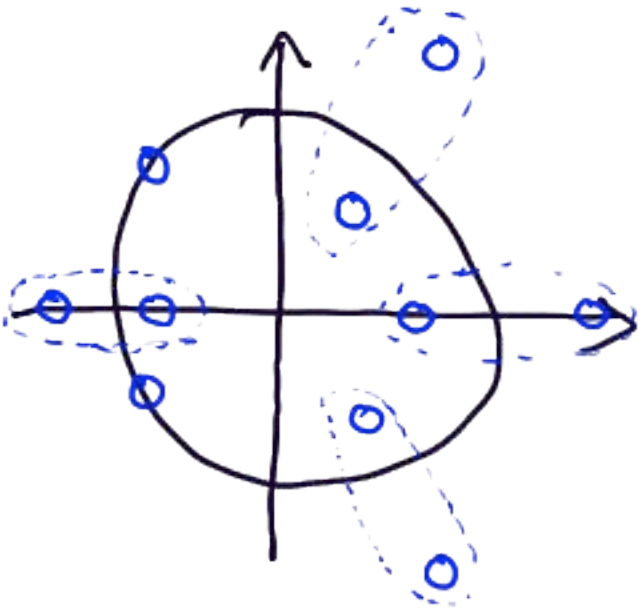
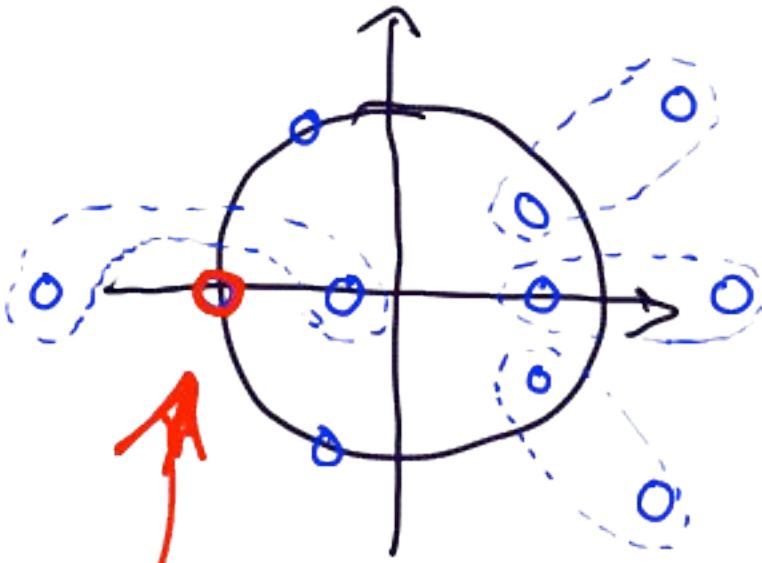
$$= z^{-M} \sum_{k=0}^{M} h[k] z^{k}$$

$$\Longrightarrow \boxed{H(z) = z^{-M} H(z^{-1})}$$

$H(z) = z^{-M} H(z^{-1})$ Type I, II





$H(-1) = 0$ Type II (Never high-pass)

→ FOR GLP, IF $a = re^{j\theta}$ is a zero

$\frac{1}{a^*}$ is also a zero

Zeroes of GLP system

Type I, II: $h[n] = h[n-M]$

$$H(z) = \sum_{n=0}^{M} h[n] z^{-n} =$$

$$= \sum_{n=0}^{M} h[M-n] z^{-n} = z^{-M} \sum_{n=0}^{M} h[\underbrace{M-n}_{\triangleq k}] z^{\underbrace{M-n}_{\triangleq k}}$$

$$= z^{-M} \sum_{k=0}^{M} h[k] z^{k}$$

$$\Longrightarrow \boxed{H(z) = z^{-M} H(z^{-1})}$$

for type II:

$$H(-1) = (-1)^{\overset{\text{odd}}{M}} H(-1) = -H(-1) \Rightarrow \boxed{H(-1) = 0}$$

similarly, can show for

type III, IV

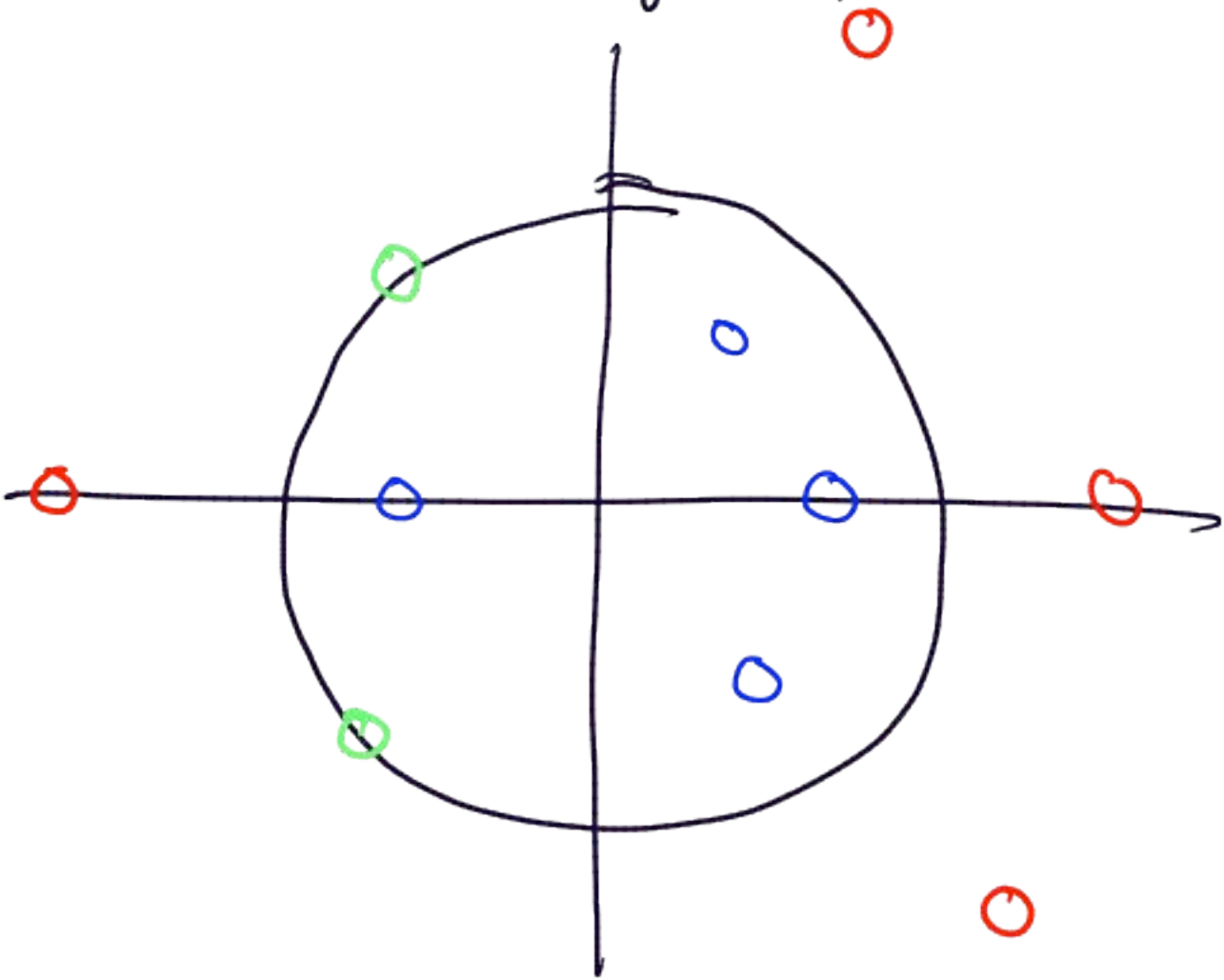
$$\boxed{H(z) = -z^{-M} H(z^{-1})}$$

$$\boxed{H(1) = 0} \rightarrow \text{Never low pass}$$

for type III

$$\boxed{H(-1) = 0} \quad \text{only band pass}$$

Relation of FIR GLP to min-phase systems



$$H(z) = H_{min}(z)\, H_{max}(z)\, H_{uc}(z)$$

minimum phase

maximum phase

There is much more to learn, ... but ....

This is

The END

(for now)

What's Next:

EE 121 → digital communication
EE 126 → Prob. & Random processes.
EE 127A → optimization

EE 145B → Image Proc. & tomography

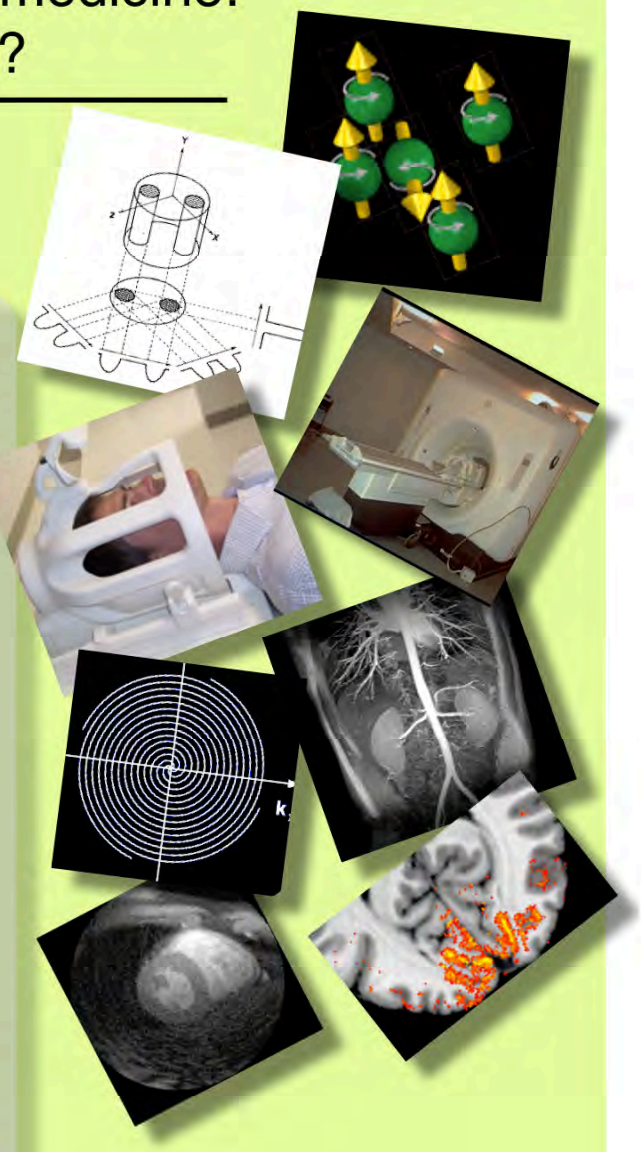
More advanced

EE 225A (EE123 + EE126) + +
EE 225B " ⇒ 2D +
CS 280 Vision
EE 224A, 226A, 227A, 229, CS 281A + B
EE 225E → Principles of MRI

Principles of Magnetic Resonance Imaging EEc225E / BIOEc265

MRI has revolutionized diagnostic medicine. Ever wondered how it really works?

Spring 2012
TT, 2-3:30 Cory 299
Prerequisites: Either EE 120 or BioEC165/El Eng C145B
Units: 3
Instructor: Michael (Miki) Lustig



Magnetic Resonance Imaging (MRI) is a non-invasive imaging modality. Unlike Computed Tomography (CT) that uses x-ray, MRI does not use any ionizing radiation and is considered safe. MRI provides a large number of flexible contrast parameters, which give excellent soft-tissue contrast.
The class will cover:

Fundamentals of MRI:
- Multi-dimensional Fourier Transforms and linear systems
- Nuclear Magnetic Resonance Physics
- Imaging Sequences
- Contrast Generation
- Image reconstruction
- MRI Hardware and Software
- Imaging tradeoffs and image artifacts

Advanced Topics:
- Rapid imaging
- Parallel Imaging
- Emerging research opportunities (High-field, dynamic imaging, functional imaging, hyperpolarization,compressed sensing)

Class includes hands-on Matlab labs for sequence design and reconstruction, tour to an MRI facility and guest lecture by a radiologist.

* This year, for the first time students will perform imaging experiments on a physical earth-field MRI system