Professor Fearing EECS120/Problem Set 9 v1.1 Spring 2014

Due at 12 pm, Wed. April 23 in HW box .

1. (20 pts) All-pass filter

A discrete time causal LTI system has transfer function:

$$H(z) = \frac{(1 + 0.3z^{-1})(1 - 4z^{-2})}{1 + 0.64z^{-2}}$$

a) Draw the pole-zero diagram for $H(z)$. Is the system stable?
b) Find the minimum phase system $H_{min}(z)$ and an all-pass system $H_{ap}(z)$ such that $H(z) = H_{min}(z)H_{ap}(z)$ and plot the respective pole-zero diagrams.

2. (30 pts) Digital filter design

A continuous time filter has impulse response: $h(t) = (e^{-t} - e^{-0.5t})u(t)$. For all parts assume $T = 0.25$.
a) Using impulse invariance, that is let $h_1[n] = h(nT)$, find $H_1(z)$.
b) Find $H(s)$ and the corresponding linear differential equation with input $x(t)$ and output $y(t)$.
c)Find $H_2(z)$ using the backward difference approximation to the derivative, $\frac{dy}{dt} \approx \frac{y[n]-y[n-1]}{T}$.
d) Find $H_3(z)$ using the bilinear transformation, $s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$.
e) Plot the pole-zero diagrams for $H_1(z), H_2(z), H_3(z)$. Plot magnitude responses for $H(j\omega)$ (the Fourier transform of $h(t)$), $H_1(e^{j\omega T}), H_2(e^{j\omega T})$ and $H_3(e^{j\omega T})$ versus frequency ω on the same graph. Explain the reasons for any differences between the plots. (You may sketch using the geometric arguments or plot using numpy, etc.)

3. (25 pts) FIR filter design

An FIR digital filter is to be designed which has the desired frequency response of $H(e^{j\Omega}) = 6$ for $0 \leq \Omega < \pi$ and $H(e^{j\Omega}) = 3$ for $-\pi < \Omega < 0$.
a) Find the unit sample response of the Finite Impulse Response filter $h[n]$ using 17 samples $n = -8, -7, ...7, 8$, and using $h[n] = 0$ for n outside this range.
b) Plot the amplitude and phase response of $H(e^{j\Omega})$. (You may use numpy or sketch using geometric arguments.)
c) The $h[n]$ found above is non-causal. Describe how the answer to b) changes if a causal version of $h[n]$ were used instead.

4. (25 pts) All-pass filter

For this problem use the iPython notebook `PythonPS9-Prob4-Question.ipynb` from the class web page. Turn in plots and printout of the parts you implemented. Consider the all-pass filter $H(z) = \frac{1-1.11111z^{-1}}{1-0.9z^{-1}}$.
a. Plot the magnitude and phase of $H(e^{j\Omega})$ and show that the magnitude response is all-pass.
b. Implement the LDE for $H(z)$ in iPython, and apply it to the given $x[n]$ (a sum of sinusoids). Briefly explain how and why the output is changed by the all-pass filter.
c. Implement the LDE for the all-pass filter on the *left* channel of the provided audio file. (Leave the *right* channel unmodified.) Play the sound back in mono and through stereo headphones. How has the phase change effected intelligibility? Does the speech sound distorted? You may need to scale your all-pass filter, as the values in the .wav file are limited to 16 but signed integers. Check the plot for distortion (e.g. clipping) of the waveforms.