Professor Fearing EECS120/Problem Set 6 v 1.0 Spring 2014

Due at 12 pm, Wed. Mar. 19 in HW box under stairs (1st floor Cory)

1. (25 pts) DCT problem on web page

Download PS6-DCT.ipynb from the class web page. Answer the questions 1-5 in the python notebook, and attach a printout to your problem set.

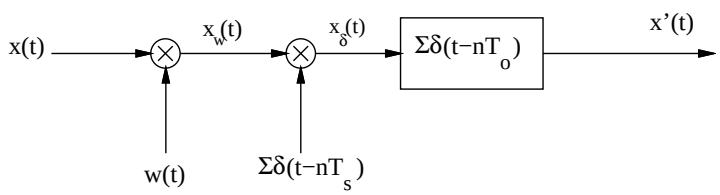
2. (25 pts) DFT zero padding

Consider the signal flow diagram shown in Figure 1. For each window $w(t)$, signal $x(t)$, and sampling combination below, sketch $x(t), x_w(t), x_\delta(t), x'(t)$ and their magnitude spectra. Also sketch magnitude and phase for $X[k]$ (derived from $X'(j\omega)$).

i. Let $w(t) = \Pi(t)$, $T_o = 8T_s$, $T_o = 1$ sec, $x(t) = \cos(5\pi t)$.

ii. Let $w(t) = \Pi(t)$, $T_o = 16T_s$, $T_o = 2sec$, $x(t) = \cos(5\pi t)$.

ii. Let $w(t) = \Pi(t - \frac{1}{2})$, $T_o = 16T_s$, $T_o = 2sec$, $x(t) = \cos(5\pi t)$.



DFT equivalent block diagram.

3. (30 pts)

This problem considers digital interpolation using upsampling and a digital low pass filter. Consider a 1 KHz cosine sampled at 8 KHz for 8 samples.

a) Determine $X[k]$ the DFT of $x[n]$. What is the spacing of the samples in the frequency domain? For example, what frequency does k=3 correspond to?

b) Create a new sequence $y[n]$ of length 16 by upsampling, and sketch $y[n]$. That is $y[n] = x[n/2]$ for n even and $y[n] = 0$ for n odd. Determine $Y[k]$ the 16 point DFT of $y[n]$. What is the new spacing of samples in the frequency domain?

c) We can now interpolate between even samples of $y[n]$ by filtering with the filter $H[k]$ so that $Z[k] = Y[k]H[k]$, where $H[k] = 1$ for k=0,1,2,3,12,13,14,15; else $H[k] = 0$. Determine and plot $z[n]$, the inverse DFT of $Z[k]$. What is the relationship between $z[n]$ and $x[n]$?

d) Consider the output of $x[k]$ at 8 K samples per second through a D/A converter with zero order hold, and $z[k]$ at 16 K samples per second through a D/A converter with zero order hold. Sketch both functions in time. Which signal will have less aliasing, and thus be easier to filter?

4. (20 pts) Laplace Transform

For each signal below calculate its bilateral Laplace Transform $X(s)$ and region of convergence, and determine whether the $j\omega$ axis is in the region of convergence.

i) $x(t) = \Pi(2t - 2)$ ii) $x(t) = \Pi(t/4)$ iii) $x(t) = e^{2t}cos(2\pi t)u(-t)$

iv) $x(t) = (e^{-2t} + e^{2t})u(t)$ v) $x(t) = e^{-5t}u(5t)$