**Due at 12 pm, Wed. Mar. 12 in HW box under stairs (1st floor Cory)**

1. (16 pts) Sampling and Reconstruction

A signal $x(t) = \cos(2\pi t)$ is sampled at 6 Hz:

$$\tilde{x}(t) = x(t) \cdot \sum_{n=-\infty}^{n=\infty} \delta(t - \frac{n}{6}) \tag{1}$$

a) Sketch $X(j\omega)$ and $\tilde{X}(j\omega)$.

b) Find an ideal reconstruction filter $R(j\omega)$ such that $\hat{X}(j\omega) = R(j\omega)\tilde{X}(j\omega)$ and show that the reconstructed signal $\hat{x}(t)$ is identical to $x(t)$.

c) Show in the time domain, (using time domain sketches), that $r(t) * \tilde{x}(t)$ reconstructs the original signal.

2. (24 pts) DFT/Making Sampled Signals Periodic

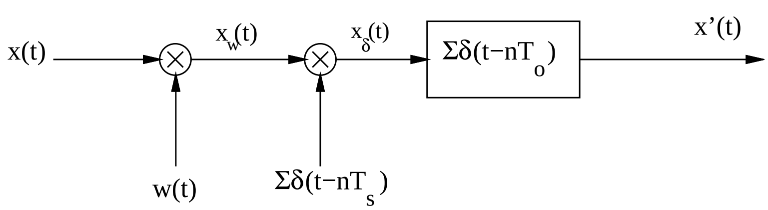
Consider the signal flow diagram shown in Figure 1. For each window $w(t)$, signal $x(t)$, and sampling combination below, do the following:

a. Sketch $x(t), x_w(t), x_\delta(t), x'(t)$ and their magnitude spectra.

b. What is the relationship between $X'(j\omega)$ and $X[k]$ (the DFT of $x[n]$)? Sketch X[k].

i. Let $w(t) = 1$, $T_o = 8T_s$, $T_o = 2$ sec, $x(t) = \Pi(t) * \Pi(t)$.

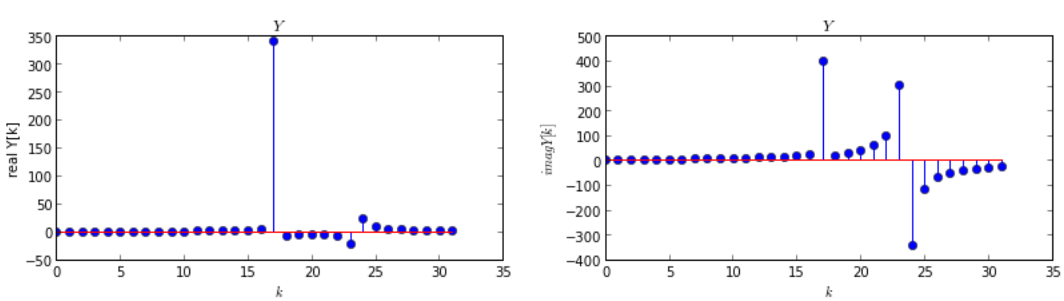
ii. Let $w(t) = \Pi(t)$, $T_o = 10T_s$, $T_o = 1sec$, $x(t) = \cos(4\pi t)$.




DFT equivalent block diagram.

3. (16 pts)

The DFT of the signal $x(t) = \cos(2\pi 23.5t) + \cos(2\pi 17t)$ is calculated, with $T_o = 1$ sec, and $N = 1024$, as shown below for samples $X[0]...X[31]$. Using reasoning as in problem 2 above, explain why the DFT of $x(t)$ differs substantially from $X(j\omega)$.



4. (12 pts)
Consider $x[n] = e^{j\omega_o n}$ for $n = 0, 1, ..., N-1$ and 0 otherwise.
a) Find the DTFT, $X(e^{j\omega})$.
b) Find the N-point DFT, $X[k]$.
c) Find the N-point DFT if $\omega_o = 2\pi k_o/N$ for some integer $k_o$.

5. (20 pts) DFT and `numpy`
Download `PS5-Prob5-Beethoven.ipynb` and `beethoven.wav` from the class web page.
a) Using the DFT, construct an "ideal" low pass filter with a 400 Hz cutoff, by writing a few lines of code in the Python notebook at the indicated location. Plot the resulting time signal from sample 30000 to sample 32000.
b) Listening to the saved .wav file, choose a better LPF cutoff (and write code for this new LPF) which would sound reasonable through a cell phone quality channel, and plot the resulting low-pass filtered signal from sample 30000 to sample 32000. State which cutoff frequency you used.

6. (12 pts) Superheterodyne receiver
Consider a superheterodyne receiver with intermediate frequency $\omega_{IF}$. (The purpose of the intermediate frequency is to increase gain and improve selectivity). Two AM broadcast stations A and B are transmitting simultaneously. Station 1 transmits $(1 + x_1(t))cos\omega_A t$ and Station 2 transmits $(1 + x_2(t))cos\omega_B t$. $x_1(t)$ and $x_2(t)$ have spectra (Fourier Transforms) $X_1(j\omega)$ and $X_2(j\omega)$ respectively, as shown in the figure below. (Let $\omega_{IF} = 2\pi(5 \times 10^5)$, $\omega_A = 2\pi \times 10^6$, $\omega_B = 2\pi(2 \times 10^6)$, and $\omega_C = \omega_A + \omega_{IF}$ ). Assume that the bandpass filter has a bandwidth of $\pm 2\pi 10$ kHz centered about $\omega_{IF}$.
For the portion of the receiver shown below, sketch the spectra $Y_1(j\omega)$, $Y_2(j\omega), Y_3(j\omega)$. Would the detector be able to separate station A and B? Briefly explain.