Due at 4 pm, Fri. Nov. 4 in HW box under stairs (1st floor Cory)

Note: \( \Pi(t) = u(t + \frac{1}{2}) - u(t - \frac{1}{2}) \), and \( \text{comb}(t) = \sum_{n=-\infty}^{\infty} \delta(t-n) \).

1. (30 pts) Lec15 OW Ch. 7, Up/Down sample handout

A sampled signal \( x[n] = \frac{2 \sin(\pi n/2)}{\pi n} \) is upsampled by 3, low-pass filtered, and then down sampled by 2 to obtain a new signal \( x_b[n] \) which is equivalent to the original signal sampled at 1.5 times faster sample rate. Following the handout and noting heights and widths, sketch the pairs:

\[ x[n] \leftrightarrow X(e^{j\Omega}), \quad x_p[n] \leftrightarrow X_p(e^{j\Omega}), \quad x_u[n] \leftrightarrow X_u(e^{j\Omega}), \quad x_d[n] \leftrightarrow X_d(e^{j\Omega}), \quad \text{and} \quad x_b[n] \leftrightarrow X_b(e^{j\Omega}) \]

Also specify height of \( H(e^{j\Omega}) \) (note: should be \( \Omega \) in LPF block).

Fig. 1.

2. (20 pts) Sampling and Reconstruction (OW 7.1)

In many cases of conversion from discrete to continuous time, we considered an ideal low-pass filter for reconstruction. In practice, a digital-to-analog converter, which can be modelled as a zero-order hold, is followed in many cases of conversion from discrete to continuous time, we considered an ideal low-pass filter for reconstruction. In practice, a digital-to-analog converter, which can be modelled as a zero-order hold, is followed by a low-complexity analog filter. Consider the system shown in Fig. 2, where \( x(t) = \cos(400\pi t), T_{s1} = \frac{1}{600} \) sec, \( T = T_s \), and \( h(t) = 400\pi e^{-400\pi t} u(t) \).

a. Sketch \( x(t), x_s(t), x_z(t), x_r(t) \) (for \( 0 \leq t \leq 6T_{s1} \)) and associated magnitude spectra. \( x_r(t) \) will be an approximate sketch.

b. Numerically estimate the fraction of the power in \( x_r(t) \) which is not at the original signal frequency of 200 Hz.

c. With upsampling, a discrete time signal can be be generated with a higher effective sampling rate (if the original signal is bandlimited to less than half the original sampling frequency). Consider now \( T_{s2} = \frac{1}{10} T_{s1} \), and \( T = T_{s2} \). Sketch the new \( x_{r2}(t) \) and \( |X_{r2}(j\omega)| \) and numerically estimate the fraction of the power in \( x_r(t) \) which is not at the original signal frequency of 200 Hz.

3. (20 pts) DFT H.O., Up/down sample H.O.

This problem considers up and down sampling with DFT instead of DTFT. Consider a 1 kHz cosine sampled at 8 kHz for 32 samples. (Assume \( x[0] = 1 \)).

a) Determine \( X[k] \) the DFT of \( x[n] \). What is the spacing of the samples in the frequency domain? For example, what frequency does \( k=3 \) correspond to?

b) Create a new sequence \( v[n] \) of length 16 by downsampling \( x[n] \), and sketch \( v[n] \). That is \( v[n] = x[2n] \). Determine and sketch \( V[k] \) the 16 point DFT of \( v[n] \). What is the new spacing of samples in the frequency domain?

c) Create a new sequence \( y[n] \) of length 64 by upsampling \( v[n] \). That is \( y[n] = v[n/4] \) for \( n \) a multiple of 4 and \( y[n] = 0 \) for \( n \) otherwise. Determine \( Y[k] \) the 64 point DFT of \( y[n] \). What is the new spacing of samples in the frequency domain?

d) A filter \( H[k] \) can be used to interpolate between values of \( y[n] \) to obtain \( z[n] \), that is \( Z[k] = H[k]Y[k] \). Sketch \( H[k] \), \( Z[k] \) and \( z[n] \) the inverse DFT of \( Z[k] \). What is the equivalent spacing of samples in time domain?
4. (10 pts) Region of convergence, pole/zero diagram (Lec 16,17 OW 9.2)
For each part below, \( y(t) \) is the output for an LTI system with impulse response \( h(t) \) and input \( x(t) \). Show the pole and zero locations, and the region of convergence in the \( \sigma - j\omega \) plane for each \( Y(s) \), using the Laplace transform integral.

i. \( x(t) = te^{-2t}u(t), h(t) = e^{+2t}u(t) \)

ii. \( x(t) = e^{-6t}u(t), h(t) = \sin(4\pi t)u(t) \).

5. (20 pts) Laplace Transform Lec 17,18 OW 9.3, 9.5-9.7
For each part below, use Laplace transforms and LTI properties (tables ok) to find the output \( y(t) \) for an LTI system with impulse response \( h(t) \) and input \( x(t) \).

i. \( x(t) = e^{-t}u(t), h(t) = u(t) \).

ii. \( x(t) = e^{-2t}u(t), h(t) = e^{-3t}u(t-1) \).

iii. \( x(t) = u(t), h(t) = \cos(8\pi t)u(t) \).

iv. \( x(t) = e^{-2t}u(t) + tu(t), h(t) = \delta(t - 1) + \dot{\delta}(t - 0.5) \).

v. \( x(t) = \cos(2\pi t)u(t), h(t) = tu(t) \).