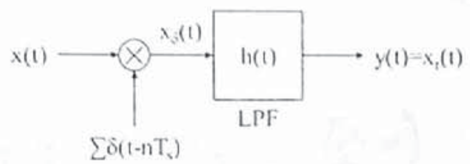
Solutions:

October 31, 2016 EE120 Fall 2016 GSI: Phil Sandborn

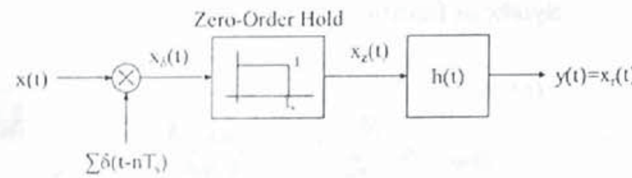
Discussion 9: Sampling and Laplace Transform Practice

1. Sampling with Zero-order-hold

Sinc-interpolation block diagram:



Zero-order-hold block diagram:

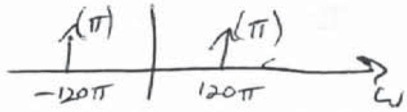


Consider $x(t) = \cos(2\pi 60t)$, $f_{sampling} = 360Hz$.

a) Sample the signal using Dirac deltas, show that we can recover the original signal using an ideal LPF (following sinc-interpolation block diagram).

b) Sample the signal using zero-order-hold, find h(t) such that we can recover the original signal.

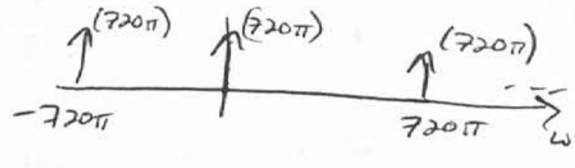
a) $x(t) = \cos(2\pi 60t) \longleftrightarrow X(j\omega) = \pi\delta(\omega-120\pi) + \pi\delta(\omega+120\pi)$



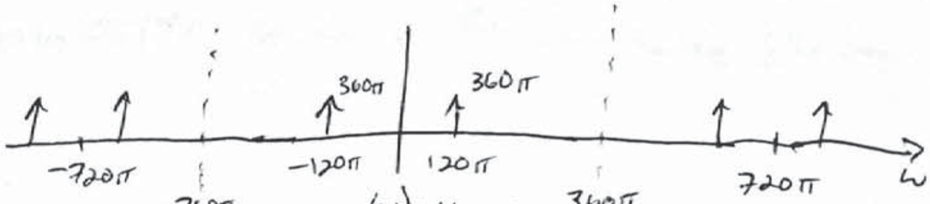
$T_s = \frac{1}{f_s} = \frac{1}{360}$

$\sum_n \delta(t-nT_s) \longleftrightarrow \frac{2\pi}{T_s}\sum_k \delta\left(\omega - \frac{2\pi k}{T_s}\right)$

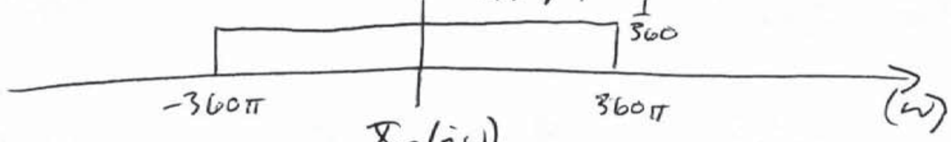
$= 720\pi \sum_k \delta(\omega - 720\pi k)$



$\frac{1}{2\pi} X(j\omega) * \frac{2\pi}{T_s}\sum_k \delta\left(\omega - \frac{2\pi k}{T_s}\right) = \frac{1}{T_s}\sum_k X\left(j\left(\omega - \frac{2\pi k}{T_s}\right)\right)$

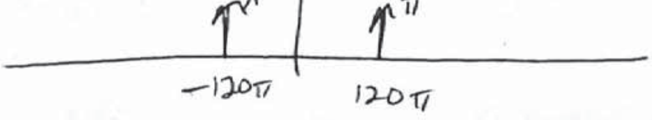


$H_{LPF}(j\omega)$

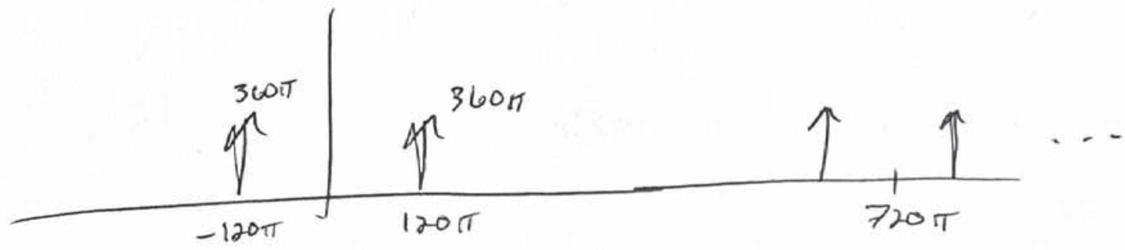


$X_r(j\omega)$

same as $X(j\omega)$ $\longleftarrow$



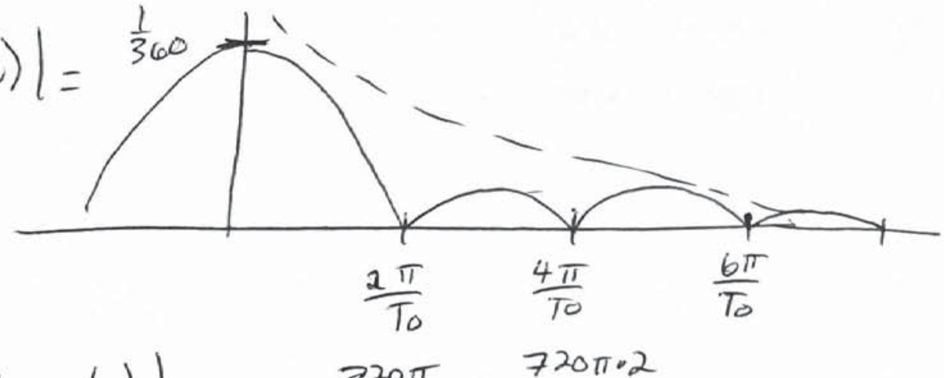
b) $X_\delta(j\omega)$ same as before?



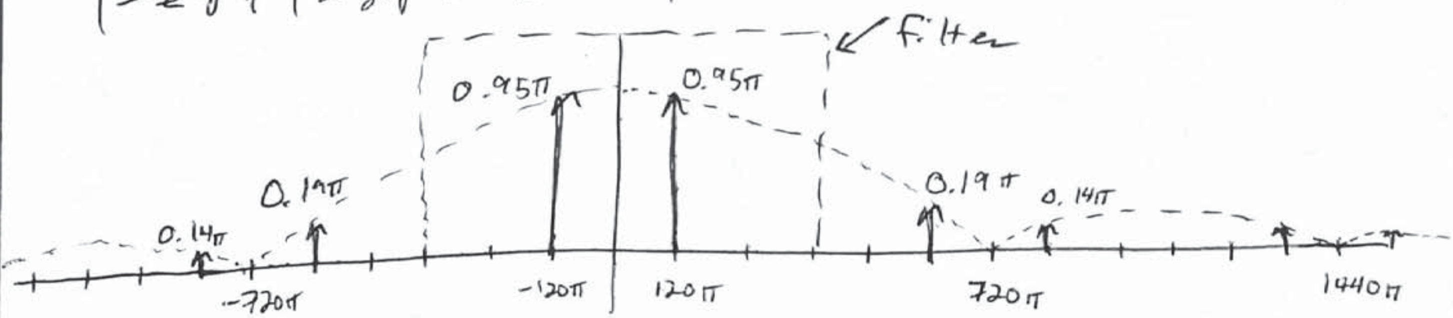
ZOH:

$$H_{ZOH}(\omega) = \frac{2\sin\left(\frac{T_0}{2}\omega\right)}{\omega} e^{-j\omega\frac{T_0}{2}} \qquad H_{ZOH}(0) = 2\frac{T_0}{2} = \frac{1}{360}$$

$|H_{ZOH}(\omega)| =$



$$|X_Z(j\omega)| = |X_\delta(j\omega)\cdot H_{ZOH}(\omega)|$$



system Reconstruction
~~Filter~~ has worse performance @ frequencies closer to |360π|

2. Laplace Transform Practice

Analysis Equation

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

Synthesis Equation

$$x(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}ds$$

a) Find $X(s) = \mathcal{L}\{e^{-\alpha t}u(t)\}$ using the analysis equation.
b) Find $X(s) = \mathcal{L}\{-e^{-\alpha t}u(-t)\}$ using the analysis equation.

a) $X(s) = \int_{-\infty}^{\infty} e^{-\alpha t}u(t)e^{-st}dt = \int_{0}^{\infty} e^{-(\alpha+s)t}dt = \left.\frac{e^{-(\alpha+s)t}}{-(\alpha+s)}\right|_0^{\infty}$ $\quad \alpha+\sigma>0$, $\sigma>-\alpha$

b) $X(s) = \int_{-\infty}^{\infty} -e^{-\alpha t}u(-t)e^{-st}dt = \int_{-\infty}^{0} -e^{-(\alpha+s)t}dt$ $\quad = 0 - \frac{1}{-(\alpha+s)} = \boxed{\frac{1}{s+\alpha}}$ if $\boxed{Re\{s\}=\sigma>-\alpha}$

$X(s) = \left.\frac{+e^{-(\alpha+s)t}}{+(\alpha+s)}\right|_{-\infty}^{0} = \frac{+1}{+(\alpha+s)} - 0 = \boxed{\frac{1}{s+\alpha}}$

$\alpha+\sigma<0$
$\sigma<-\alpha$ $\quad$ if $\boxed{Re\{s\}=\sigma<-\alpha}$

c) Find $x(t)$ if $X(s) = \frac{1}{(s+1)(s+2)}$, with region of convergence, $Re\{s\} > -1$.
d) Find $x(t)$ if $X(s) = \frac{1}{(s+1)(s+2)}$, with region of convergence, $-2 < Re\{s\} < -1$.

$X(s) = \frac{A}{s+1} + \frac{B}{s+2} \rightarrow A=1,\ B=-1 \qquad X(s) = \frac{1}{s+1} + \frac{-1}{s+2}$

$\frac{1}{s+1} \rightarrow e^{-1t}u(t)$ for $\sigma>-1$, $-e^{-t}u(-t)$ for $\sigma<-1$

$\frac{1}{s+2} \rightarrow e^{-2t}u(t)$ for $\sigma>-2$, $-e^{-2t}u(-t)$ for $\sigma<-2$

for (c), ROC: $Re\{s\}>-1 \rightarrow \sigma>-1$ means $\sigma>-2$, $\sigma>-1$

$$\boxed{x(t) = e^{-t}u(t) - e^{-2t}u(t)}$$

for (d): ROC: $-2<\sigma<-1 \rightarrow \frac{1}{s+2} \rightarrow e^{-2t}u(t)$, $\frac{1}{s+1} \rightarrow -e^{-t}u(-t)$

$$\boxed{x(t) = -e^{-t}u(-t) - e^{-2t}u(t)}$$