EE120 GSI: Ming

- LTI system
- Fourier Series
- Filtering.

properties $x(t) \to \boxed{h(t)} \to y(t)$

Causality: $h(t)=0$ for $t<0$

BIBO: $h(t)$ absolutely integ.

TI: if $x(t) \Rightarrow y(t)$, then $x(t-t_0) \Rightarrow y(t-t_0)$

linearity: if $H\{x_1(t)\} \to y_1(t)$, $H(ax_1(t)+bx_2(t)) \to ay_1(t)+by_2(t)$

1. (a) False. $y(t) = t\,x(t)$ (Sp 04, P1)

$t\,x(t-t_0) \neq (t-t_0)\,x(t-t_0)$

(b). True. for $|x(t)|<M$, $M>0$

$1 \leq (1+x^2(t)) < M^2+1$

since $-1 \leq \cos t \leq 1$.

$\frac{1}{M^2+1} \leq (1+x^2(t))^{\cos t} < M^2+1 \Rightarrow y(t)$ bounded

(c). False. Assume $x[n]$ periodic with $N=2\pi m$ for $m, N \in \mathbb{Z}_+$

then $x[n+N] = \cos(n+N) = \cos(n+2m\pi) = \cos(n)$

However, $\pi = \frac{N}{2m}$ is not possible, since π is irrational.

(d). True. $x(t) = \cos(t) + \cos(2t) = \frac{1}{2}e^{jt} + \frac{1}{2}e^{-jt} + \frac{1}{2}e^{j2t} + \frac{1}{2}e^{-j2t}$

Recall that complex exponentials are eigenfunctions of LTI

$e^{j\omega t} \to \boxed{LTI} \to \lambda e^{j\omega t}$ $\lambda = H(j\omega)$

However $y(t) = \underline{\frac{1}{2}} + \frac{1}{4}(e^{jt} + e^{-jt} + e^{j2t} + e^{-j2t} + \underline{e^{j3t} + e^{-j3t}})$

problem 2 (Sp 99, P4)

(a). $x(t) = \sin^2(t) = \frac{1-\cos 2t}{2} = \frac{1}{2} - \frac{1}{4}(e^{j2t} + e^{-j2t})$

$= \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$, $\omega_0 = \frac{2\pi}{T_0} = 2$.

$a_0 = \frac{1}{2}$, $a_1 = a_{-1} = -\frac{1}{4}$, $a_k = 0$ for all other k

(b). $y(t) = x(t) * h_1(t)$

$\mathcal{F}\{h_1(t)\} = H(j\omega) = \frac{1}{1+j\omega} = \frac{1-j\omega}{1+\omega^2}$

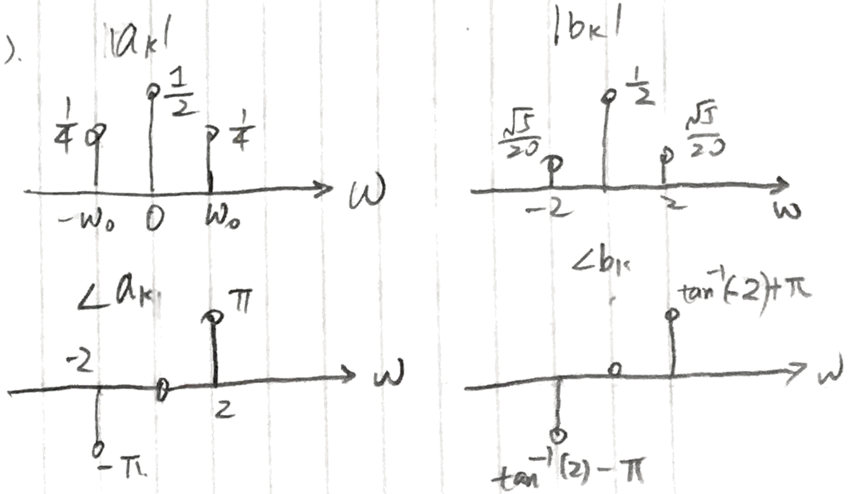
$y(t) = \sum_{k=-\infty}^{\infty} b_k \cdot e^{jk2t}$, $b_k = H(j\omega)\,a_k$; $\omega = k\omega_0$

$b_0 = \frac{1}{2}$, $b_1 = -\frac{1}{4}\frac{1-2j}{1+2^2} = -\frac{1}{20}(1-2j)$

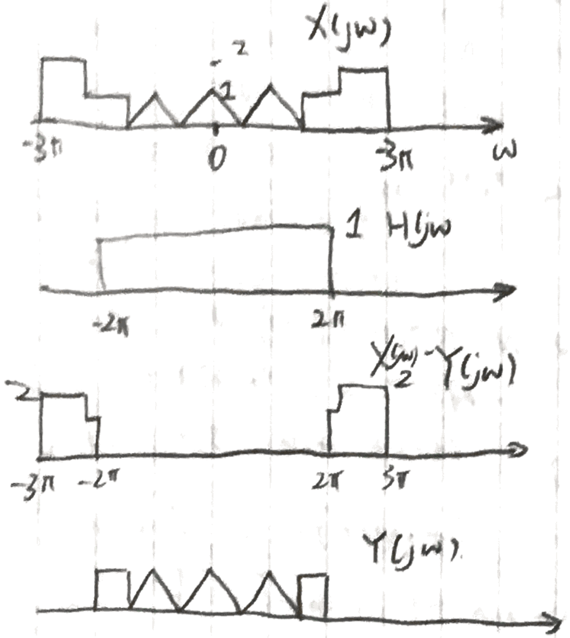
$b_{-1} = -\frac{1}{4}\frac{1+2j}{1+2^2} = -\frac{1}{20}(1+2j)$

$b_k = 0$ for all other k

(c).



(SP04, p5)



$$\frac{\sin at}{\pi t} \xrightarrow{\mathcal{F}} \begin{cases} 1 & |\omega| < a \\ 0 & |\omega| > a \end{cases}$$

$$h(t) = \frac{2 \sin \pi 2t}{\pi 2t} = \frac{\sin 2\pi t}{\pi t}$$

$$\int_{-\infty}^{\infty} |x(t) - y(t)|^2 dt \overset{\text{parseval's}}{=} \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega) - Y(j\omega)|^2 d\omega$$

$$= \frac{1}{2\pi} \left(2 \cdot 4 \cdot \frac{3\pi}{4} + 2 \cdot 1 \cdot \frac{\pi}{4} \right)$$

$$= \frac{13}{4}$$