EE120 Discussion 8, GSI: Ming

Problem 1

For each of the following statements, if you believe it is true, give a justification. If you believe it is false, give a counterexample.
(a) A linear causal continuous-time system is always time-invariant.
(b) The system with (real-valued) input x(t) and output given by

$$y(t) = (1+(x(t))^2)^{(\cos(t))}$$

is stable.
(c) The discrete-time signal x[n] = cos(n) is a periodic signal.
(d) For an otherwise completely unknown system, it is known that when the input is given by

$$x(t) = \cos(t) + \cos(2t),$$

the output is

$$y(t) = .5(1+\cos(t)+\cos(2t)+\cos(3t)).$$

This system cannot be a linear time-invariant (LTI) system.

Problem 2 (Fourier Series)

$$x(t) = \sin^2(t) \longrightarrow \boxed{h_1(t) = e^{-t}\,u(t)} \longrightarrow y(t)$$

a. Find the Fourier series expansion of $x(t)$.

b. Find the Fourier series expansion of $y(t)$.

c. Sketch the 2-sided amplitude and phase spectrum of $x(t)$ and $y(t)$. Label salient features.

Problem 3 (Filtering)

The signal $x(t)$ with spectrum $X(j\omega)$ as shown in Figure 2 is passed through a linear time-invariant (LTI) system with impulse response

$$h(t) = 2\text{sinc}(2t),$$

where, as defined in class,

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}.$$

Denote the output of the system by $y(t)$. Calculate the error between $x(t)$ and $y(t)$, given by

$$\int_{-\infty}^{\infty} |x(t) - y(t)|^2\, dt.$$

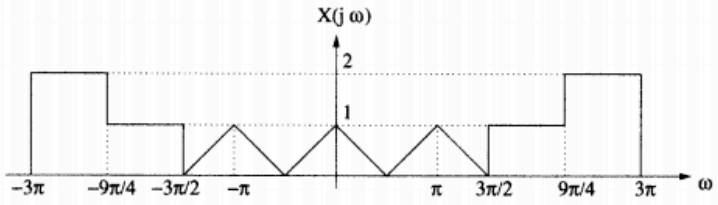

Figure. The spectrum of the signal $x(t)$.