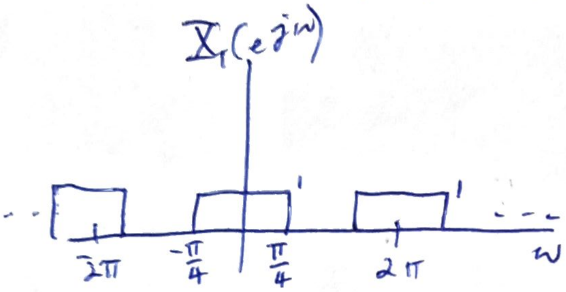
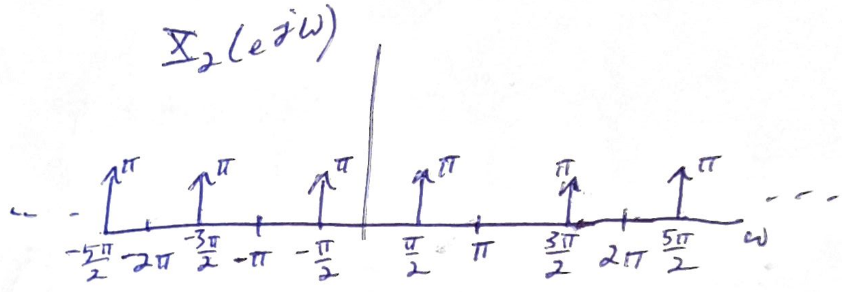
① Circular Convolution:

$$x_1[n] = \frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n} \qquad x_2[n] = \cos\left(\frac{\pi}{2}n\right)$$

Draw $X_1(e^{j\omega})$, $X_2(e^{j\omega})$





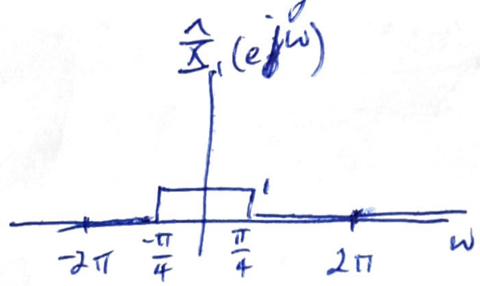
Write Multiplication rule for DTFT:

$$x_1[n] \cdot x_2[n] \longleftrightarrow \frac{1}{2\pi} \int_{\langle 2\pi \rangle} X_1(e^{j\theta}) X_2(e^{j(\omega-\theta)}) \, d\theta$$

Let's make this into our typical convolution form, so $\left(\int_{-\infty}^{\infty}\right)$.

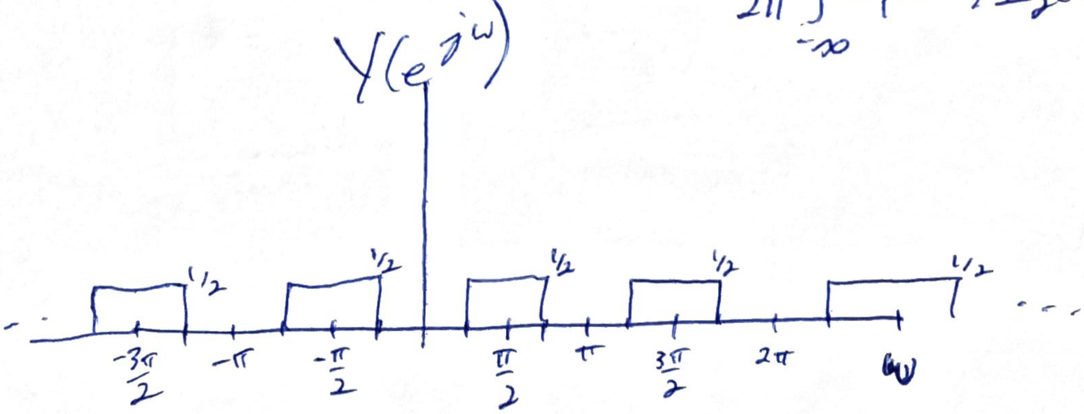
Re-write using $\hat{X}_1(e^{j\omega})$, aperiodic:

in the interval, $[-\pi, \pi]$, $\hat{X}_1(e^{j\omega}) = X_1(e^{j\omega})$



So: $$\frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(e^{j\theta}) X_2(e^{j(\omega-\theta)}) \, d\theta = \frac{1}{2\pi} \int_{-\pi}^{\pi} \hat{X}_1(e^{j\theta}) X_2(e^{j(\omega-\theta)}) \, d\theta$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{X}_1(e^{j\theta}) X_2(e^{j(\omega-\theta)}) \, d\theta$$



$\cancel{y[n] = x[n] + x[n-1]}$

$$Y(e^{j\omega}) - 1.2\,Y(e^{j\omega})e^{-j\omega} + 0.36\,Y(e^{j\omega})e^{-j2\omega} = X(e^{j\omega}) + X(e^{j\omega})e^{-j\omega}$$

$$Y(e^{j\omega})\left(1 - 1.2e^{-j\omega} + 0.36e^{-2j\omega}\right) = X(e^{j\omega})\left(1 + e^{-j\omega}\right)$$

$$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = H(e^{j\omega}) = \boxed{\frac{1+e^{-j\omega}}{1-1.2e^{-j\omega}+0.36e^{-2j\omega}}}$$

To find impulse response, $h[n]$, we would need to take IDTFT$\{H(e^{j\omega})\}$, but it is not in an easily computed form.

First, use $(z^2 - 1.2z + 0.36) = (z-0.6)^2$

$$\cancel{\frac{1+e^{-j\omega}}{1+}} \quad \cancel{\frac{1+z}{1-1.2z+0.36z^2} = \frac{A}{z-0.6} + \frac{B}{(z-0.6)^2}}$$

$$\left(1 - \frac{0.6}{z}\right)^2 = 1^2 - 1.2\frac{1}{z} + 0.36\frac{1}{z^2}$$

$$z = e^{j\omega}$$

$$(1-0.6z)^2 = 1^2 - 1.2z + 0.36z^2$$

$$\frac{1+z}{1-1.2z+0.36z^2} = \frac{1+z}{(1-0.6z)^2} = \frac{A}{1-0.6z} + \frac{B}{(1-0.6z)^2}$$

$$1+z = A(1-0.6z) + B$$

$$1+z = A - 0.6Az + B$$

$$A = -\frac{1}{0.6} = -\frac{5}{3}$$

$$A+B = -\frac{5}{3} + B = 1$$

$$B = \frac{8}{3}$$

$$H = -\frac{5}{3}\frac{1}{1-0.6z} + \frac{8}{3}\left(\frac{1}{(1-0.6z)^2}\right)$$

$$\boxed{h[n] = -\frac{5}{3}(0.6)^n u[n] + \frac{8}{3}(n+1)(0.6)^n u[n]}$$

DTFT practice:

a) $h[n] = \left(\frac{1}{2}\right)^n e^{j2n} u[n+2]$

This looks like:

$$a^n u[n] \cdot e^{j\omega_0 n}$$

but $\uparrow$ is $(n+2)$ in h, which corresponds to a time-shift.

the e^{j2n} will be a phase shift, or frequency shift.

Let $h[n] = h_1[n] e^{j2n} \xleftrightarrow{DTFT} H_1(e^{j(\omega-\omega_0)}) = H(e^{j\omega})$

so $h_1[n] = \left(\frac{1}{2}\right)^n u[n+2] = \left(\frac{1}{2}\right)^n u[n+2] \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{-2}$

$$h_1[n] = \left(\frac{1}{2}\right)^{n+2} u[n+2] \left(\frac{1}{2}\right)^{-2} = 4\left(\frac{1}{2}\right)^{n+2} u[n+2]$$

Let $h_2[n] = \left(\frac{1}{2}\right)^n u[n]$

so $h_1[n] = h_2[n+2]$

$h_2[n] \xleftrightarrow{DTFT} \frac{1}{1-\frac{1}{2}e^{-j\omega}} = H_2(e^{j\omega})$

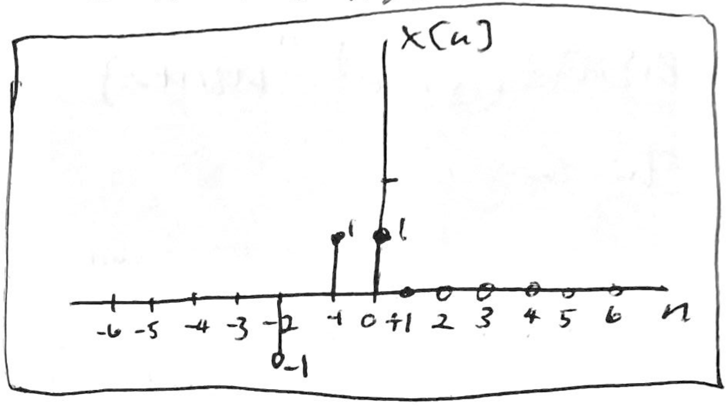
$h_1[n] \longleftrightarrow H_2(e^{j\omega}) e^{j2\omega} = H_1(e^{j\omega})$

$h[n] \longleftrightarrow H_1(e^{j(\omega-\omega_0)}) = H_2(e^{j(\omega-\omega_0)}) e^{j2(\omega-\omega_0)}$

$$= \boxed{\frac{e^{j2(\omega-\omega_0)}}{1-\frac{1}{2}e^{-j(\omega-\omega_0)}}} = H(e^{j\omega})$$

where $\omega_0 = 2$

③ b) 1st: Draw what we know: ① $x[n]=0 \quad n>0$:



Start w/ Property 3: $\mathrm{Im}\{X(e^{j\omega})\} = \sin(\omega) - \sin(2\omega)$

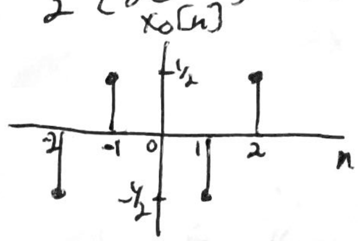
We know: $\underset{x_o[n]}{\mathrm{odd}\{x[n]\}} \xleftrightarrow{DTFT} j\,\mathrm{Im}\{X(e^{j\omega})\}$

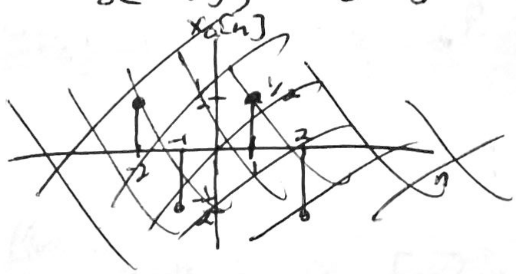
$$\mathrm{IDTFT}\{j\sin(\omega) - j\sin(2\omega)\} = x_o[n]$$

$$\mathrm{IDTFT}\left\{\cancel{j}\,\frac{e^{j\omega}-e^{-j\omega}}{2\cancel{j}} - \cancel{j}\,\frac{e^{j2\omega}-e^{-j2\omega}}{2\cancel{j}}\right\}$$

$$\frac{1}{2}\,\mathrm{IDTFT}\{\underset{n_0=-1}{e^{j\omega}} \underset{n_0=+1}{-e^{-j\omega}} \underset{n_0=-2}{-e^{j2\omega}} \underset{n_0=+2}{+e^{-j2\omega}}\}$$

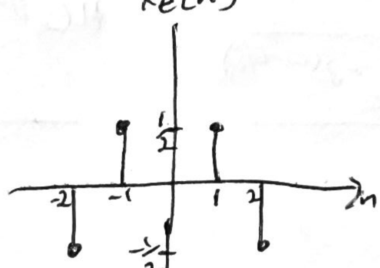
$$\frac{1}{2}\{\delta[n+1] - \delta[n-1] - \delta[n+2] + \delta[n-2]\} = x_o[n]$$





We know $x_o[n] + x_e[n] = x[n] = 0$ for $n>0$, so $x_e[1] = \frac{1}{2} = x_e[-1]$

$x_e[2] = -\frac{1}{2} = x_e[-2]$



We use Parseval's theorem to find $x_e[0]$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 3 = \sum_{n=-\infty}^{\infty} |x[n]|^2 = 3$$

$$3 = |x[-2]|^2 + |x[-1]|^2 + |x[0]|^2 = 2 + |x[0]|^2$$

so $\boxed{x[0] = 1}$ (we also know $x[0] > 0$)