Circular Convolution:

\[ X(e^{j\omega}) = \sin \left( \frac{\pi}{4} \omega \right) \]
\[ x_2(e^{j\omega}) = \cos \left( \frac{\pi}{2} \omega \right) \]

Draw \( \hat{X}_1(e^{j\omega}) \), \( \hat{X}_2(e^{j\omega}) \)

Write Multiplication rule for DTFT:

\[ x_1(e^{j\omega}) \cdot x_2(e^{j\omega}) \leftrightarrow \frac{1}{2\pi} \int_{-\pi}^{\pi} \hat{X}_1(e^{j\theta}) \hat{X}_2(e^{j(\omega-\theta)}) d\theta \]

Let's make this into our typical convolution form, so.

Re-write using \( \hat{X}_1(e^{j\omega}) \), a periodic:

\[ \hat{X}_1(e^{j\omega}) \]

in the interval \( [-\pi, \pi] \), \( \hat{X}_1(e^{j\omega}) = \hat{X}_1(e^{j2\pi}) \)

So:

\[ \frac{1}{2\pi} \int_{-\pi}^{\pi} \hat{X}_1(e^{j\theta}) \hat{X}_2(e^{j(\omega-\theta)}) d\theta = \frac{1}{2\pi} \int_{-\pi}^{\pi} \hat{X}_1(e^{j\theta}) \hat{X}_2(e^{j(\omega-\theta)}) d\theta \]

\[ = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{X}_1(e^{j\theta}) \hat{X}_2(e^{j(\omega-\theta)}) d\theta \]
\[ Y(e^{j\omega}) = 1.2 Y(e^{j\omega}) e^{-j\omega} + 0.36 Y(e^{j\omega}) e^{-2j\omega} = X(e^{j\omega}) + X(e^{-j\omega}) e^{-j\omega} \]

\[ Y(e^{j\omega}) \left( 1 - 1.2 e^{-j\omega} + 0.36 e^{-2j\omega} \right) = X(e^{j\omega}) \left( 1 + e^{-j\omega} \right) \]

\[ \frac{Y(e^{j\omega})}{X(e^{j\omega})} = H(e^{j\omega}) = \frac{1 + e^{-j\omega}}{1 - 1.2 e^{-j\omega} + 0.36 e^{-2j\omega}} \]

To find impulse response, \( h[n] \), we would need to take IDTFT of \( H(e^{j\omega}) \), but it is not in an easily computed form.

First, use \((z^2 - 1.2z + 0.36z^2) = (z - 0.6)^2\)

\[ \frac{1 + z}{1 - 1.2z + 0.36z^2} = \frac{A}{z - 0.6} + \frac{B}{(z - 0.6)^2} \]

\[ \left(1 - \frac{0.6}{2}\right)^2 = 12 - 0.2 \cdot \frac{1}{2} + 0.36 \cdot \frac{1}{2} \]

\[ z = e^{j\omega} \]

\[ (-0.6)^2 = 12 - 1.2z + 0.36z^2 \]

\[ \frac{1 + z}{1 - 1.2z + 0.36z^2} = \frac{1 + z}{(1 - 0.6z)^2} = \frac{A}{1 - 0.6z} + \frac{B}{(1 - 0.6z)^2} \]

\[ H = -\frac{5}{3} \frac{1}{1 - 0.6z} + \frac{8}{3} \frac{1}{(1 - 0.6z)^2} \]

\[ H(n) = -\frac{5}{3} (0.6)^n u[n] + \frac{8}{3} (n+1)(0.6)^n u[n] \]

\[ 1 + z = A(1 - 0.6z) + B \]

\[ 1 + z = A - 0.6Az + B \]

\[ A = -\frac{1}{0.6} = -\frac{5}{3} \]

\[ A + B = -\frac{5}{3} + B = 1 \]

\[ B = \frac{8}{3} \]
DTFT practice:

a) \( h[n] = \left( \frac{1}{2} \right)^n e^{j2n} u[n+2] \)

This looks like:

\[ a^n u[n] \overset{\text{DFT}}{\rightarrow} H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h[k] e^{-j2\pi k}\omega \]

but \( u[n+2] \) is \((n+2)\) in \( h \) which corresponds to a time shift.

the \( e^{j2n} \) will be a phase shift, or frequency shift.

Let \( h[n] = h_1[n] e^{j2n} \)

\[ H_1(e^{j(\omega-\omega_0)}) = H(e^{j\omega}) \]

so \( h_1[n] = \left( \frac{1}{2} \right)^n u[n+2] = \left( \frac{1}{2} \right)^n u[n+2] \left( \frac{1}{2} \right)^2 = \left( \frac{1}{2} \right)^{n+2} u[n+2] \)

Let \( h_2[n] = \left( \frac{1}{2} \right)^n u[n] \)

so \( h_1[n] = h_2[n+2] \)

\[ h_2[n] \overset{\text{DFT}}{\rightarrow} \frac{1}{1 - \frac{1}{2} e^{-j\omega}} = H_2(e^{j\omega}) \]

\( h_1[n] \overset{\text{DFT}}{\rightarrow} H_2(e^{j\omega}) e^{j2\omega} = H_1(e^{j\omega}) \)

\( h[n] \overset{\text{DFT}}{\rightarrow} H_1(e^{j(\omega-\omega_0)}) = H_2(e^{j(\omega-\omega_0)}) e^{j2(\omega-\omega_0)} \)

\[ = \frac{e^{j2(\omega-\omega_0)}}{1 - \frac{1}{2} e^{j(\omega-\omega_0)}} = H(e^{j\omega}) \]

where \( \omega_0 = 2 \).
Start with property 3: \( \text{Im} \{ X(e^{j\omega}) \} = \sin(\omega) - \sin(2\omega) \)

We know:

\( \text{odd}\{ x[n] \} \leftrightarrow -\text{Im} \{ X(e^{j\omega}) \} \)

\( \text{IDFT} \{ \sin(\omega) - j\sin(2\omega) \} = \sum x_0[n] \)

\( \text{IDFT} \{ \frac{e^{j\omega} - e^{-j\omega}}{2j} - \frac{e^{j2\omega} - e^{-j2\omega}}{2j} \} \)

\( \frac{1}{2} \text{IDFT} \{ e^{j\omega} - e^{-j\omega} - e^{j2\omega} + e^{-j2\omega} \} \)

\( n_0=1 \quad n_0=1 \quad n_0=2 \quad n_0=2 \)

\( \frac{1}{2} \{ s[n+1] - s[n-1] - s[n+2] + s[n-2] \} = x_0[n] \)

We know:

\( x_0[n] + x_e[n] = x[n] = 0 \) for \( n > 0 \), so \( x_e[1] = \frac{1}{2} = x_e[-1] \)

\( x_e[2] = -\frac{1}{2} = x_e[-2] \)

We use Parseval's theorem to find \( x_e[0] \)

\( \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = \sum_{n=-\infty}^{\infty} |x[n]|^2 = 3 \)

\( 3 = (x[-2])^2 + (x[-1])^2 + (x[0])^2 = 2 + |x[0]|^2 \)

So \( |x[0]| = 1 \) (we also know \( x[0] > 0 \))