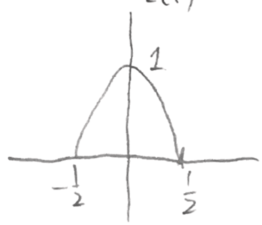
1.

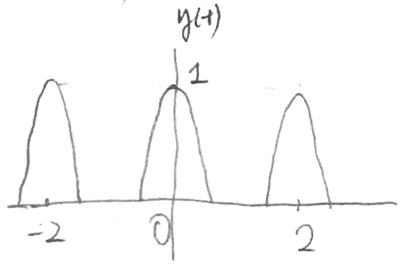
(a).

$z(t) = x(t)w(t) = \cos(\pi t)\,\Pi(t)$

$y(t) = \cos(\pi t)\,\Pi(t) * \sum_{n=-\infty}^{\infty} \delta(t-2n)$

$= \sum_{n=-\infty}^{\infty} \cos(\pi(t-2n))\,\Pi(t-2n)$




$Z(\omega) = \frac{1}{2\pi}\, \mathcal{F}\{\Pi(t)\} * \mathcal{F}\{\cos \pi t\}$

$= \frac{1}{2\pi}\, \frac{2\sin\frac{\omega}{2}}{\omega} * (\pi\delta(\omega-\pi) + \pi\delta(\omega+\pi))$

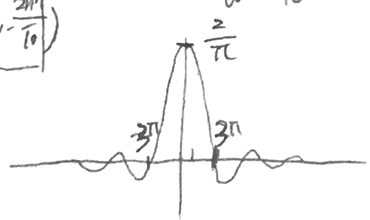
$\boxed{\Pi\left(\frac{t}{T}\right) \sim \frac{2\sin\frac{\omega T}{2}}{\omega}}$

$= \frac{\sin\frac{\omega-\pi}{2}}{\omega-\pi} + \frac{\sin\frac{\omega+\pi}{2}}{\omega+\pi}$

$= -\frac{\cos\frac{\omega}{2}}{\omega-\pi} + \frac{\cos\frac{\omega}{2}}{\omega+\pi}$

$\boxed{\sum_{n=-\infty}^{\infty} \delta(t-nT_0) \sim \frac{2\pi}{T_0}\sum_{n=-\infty}^{\infty} \delta\left(\omega-\frac{2n\pi}{T_0}\right)}$

$= \cos\frac{\omega}{2}\left(\frac{-2\pi}{\omega^2-\pi^2}\right) \Rightarrow \boxed{\frac{\pi\sin\frac{\omega}{2}}{2\omega}}$



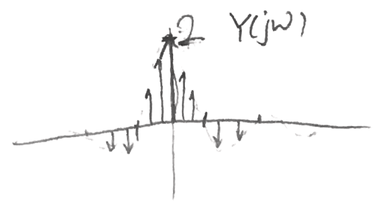

even?

$Y(j\omega) = \mathcal{F}\{z(t) * h(t)\}$

$= Z(j\omega)\,H(j\omega)$

$= \frac{2\pi}{\pi^2-\omega^2}\cos\frac{\omega}{2}\,\frac{2\pi}{2}\sum \delta(\omega-n\pi)$

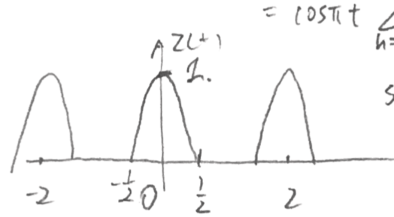
$= \pi\sum_{n=-\infty}^{\infty} \frac{2\pi}{\pi^2-(n\pi)^2}\cos\frac{n\pi}{2}$




$\omega = n\frac{2\pi}{T_0} = n\pi.$

(b). $z(t) = x(t)w(t) = \cos\pi t\left[\Pi(t) * \sum_{n=-\infty}^{\infty}\delta(t-2n)\right]$

$= \cos\pi t\sum_{n=-\infty}^{\infty}\Pi(t-2n)$




same as $y(t)$!

$y(t) = z(t) * \delta(t) = z(t)$.

Note: solutions for (c) and (d) can be found in PS#4, SP14 (c) and (d).

2 Duality

a). $x_1(t) \overset{\mathcal{F}}{\sim} X_1(j\omega)$

$x_1(t) = u(t)e^{-\alpha t}\cos \omega_0 t$

$\mathcal{F}\{X_1(t)\} = 2\pi x_1(-\omega) = 2\pi\, u(-\omega)e^{\alpha\omega}\cos \omega_0\omega$

b). $x_2(t) \overset{\mathcal{F}}{\sim} X_2(j\omega)$

$X_2(j\omega) = \frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)] + \frac{j\omega}{\omega_0^2-\omega^2}$

$\mathcal{F}^{-1}\{x_2(\omega)\} = \mathcal{F}^{-1}\{\frac{1}{2\pi}\mathcal{F}\{X_2(-t)\}\}$

$= \frac{1}{2\pi} X_2(-t)$

$= \frac{1}{4}[\delta(-t-\omega_0)+\delta(-t+\omega_0)] - \frac{jt}{\omega_0^2-t^2}$

$\boxed{\text{Time-reversal}: \mathcal{F}\{x(-t)\} = X(-\omega)}$

c) $u(t) = \frac{1}{2}(1+\mathrm{Sgn}(t))$

$\delta(t) \sim 1, \therefore \frac{1}{2} \sim \pi\delta(\omega)$

$\frac{2}{jt} \sim -2\pi\,\mathrm{Sgn}(\omega)$, $\mathrm{Sgn}(t) = \frac{2}{j\omega}$

$u(t) \sim \pi\delta(\omega) + \frac{1}{j\omega}$