EE 120  
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HW due Friday (9/2)

- Ice-breaking
- DFT basics
- Phasor analysis
  - time domain
  - freq domain

\[ X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi nk}{N}} \]

\[ x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j \frac{2\pi nk}{N}} \]

Why DFT?
- Signal processing: low-pass filter
- Communication: AM
- Image compression

(a) \[ x[n] = \cos \left( \frac{2\pi k}{N} n \right) \]

\[ x[n] = \frac{1}{2} e^{-j \frac{2\pi k}{N} n} + \frac{1}{2} e^{j \frac{2\pi k}{N} n} \]

\[ X[k] = \frac{1}{2} \sum_{n=0}^{N-1} X[k] e^{-j \frac{2\pi nk}{N}} + \frac{1}{2} \sum_{n=0}^{N-1} X[k] e^{j \frac{2\pi nk}{N}} \]

\[ X[k] = \delta_{k,0} + \delta_{k,N} \]

(b) Signal superposition

\[ \frac{1}{N} \sum_{n=0}^{N-1} (x[n] + \beta y[n]) e^{-j \frac{2\pi nk}{N}} \]

\[ = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi nk}{N}} + \frac{1}{N} \sum_{n=0}^{N-1} \beta y[n] e^{-j \frac{2\pi nk}{N}} \]

\[ = \alpha X[k] + \beta Y[k] \]

Note: \( X[0] \) periodic

(c) \[ \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi nk}{N}} \]

\[ = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi nk}{N}} + \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi nk}{N}} \]

\[ = X[k] e^{-j \frac{2\pi Mk}{N}} \]

(d) \[ \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{j \frac{2\pi nk}{N}} \]

\[ = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{j \frac{2\pi nk}{N}} + \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{j \frac{2\pi nk}{N}} \]

\[ = X[k] e^{j \frac{2\pi Mk}{N}} \]

(e) \[ \frac{1}{N} \sum_{n=0}^{N-1} x[n] y[n] \]

\[ = \frac{1}{N} \sum_{n=0}^{N-1} X[k] Y[k] \]

\[ = \sum_{k=0}^{N-1} X[k] Y[k] \delta_{k,0} + \sum_{k=0}^{N-1} X[k] Y[k] \delta_{k,k_2} \]

\[ \delta_{k,0} \text{ if } k \neq k_2 \]

\[ \alpha X[k] Y[k_2] \text{ if } k = k_2 \]
2. Phasor analysis

Time domain

\[ V(t) = i(t)R \]
\[ i(t) = C \frac{dV_c(t)}{dt} \]
\[ V_c(t) = L \frac{di(t)}{dt} \]

Derivation of phasor:

\[ V_c(t) = \text{Re} \{ V_c e^{j\omega t} \} \]
\[ = V_c \cos \omega t \]
\[ i_c(t) = -C V_c \omega \sin \omega t \]
\[ = \text{Re} \{ jC V_c e^{j\omega t} \} \]
\[ Z_c = \frac{V_c}{I_c} = \frac{V_c}{jC V_c \omega} = \frac{1}{jC \omega} \]

Impedance (phasor domain)

\[ V_c = I_c = \frac{1}{jC \omega} \]

\[ V_+ = V_- = 0 \]

\[ V_{out+} = -\frac{V_{in}}{R} \]

\[ H(w) = \frac{V_{out}}{V_{in}} = -\frac{1}{jwCR} \]

\[ \log |H(w)| = \log \left( \frac{10^4}{\omega} \right) = 4 - \log \omega \]

\[ 20\log |H(w)| \approx -20 \text{dB/dec} \]