EE 120. GSI: Ming

OH: Tu 2-3pm, Cory 406

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HW due Friday (9/2)

- Ice-breaking
- DFT basics
- Phasor analysis

time domain $\qquad$ freq. domain

$$x[n] \rightleftarrows X[k]$$

$$X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi nk}{N}}$$

$$x[n] = \sum_{n=0}^{N-1} X[k]\, e^{j\frac{2\pi nk}{N}}$$

Why DFT?

- Signal processing: low-pass filter
- communication: AM
- image compression

(a). $x[n] = \cos\left(\frac{2\pi K}{N} n\right)$

$$= \frac{1}{2} e^{j\frac{2\pi K}{N}n} + \frac{1}{2} e^{-j\frac{2\pi K}{N}n}$$

$$X[k] = \frac{1}{N}\sum_{n=0}^{N-1}\left(\frac{1}{2}e^{j\frac{2\pi K n}{N}} + \frac{1}{2}e^{-j\frac{2\pi K n}{N}}\right)e^{-j\frac{2\pi kn}{N}}$$

$$= \frac{1}{2N}\sum_{n=0}^{N-1} e^{j\frac{2\pi(K-k)n}{N}} + \frac{1}{2N}\sum_{n=0}^{N-1} e^{-j\frac{2\pi(k+K)n}{N}}$$

$$= \begin{cases} \frac{1}{2} & k = K \text{ or } N-K \\ 0 & \text{o.w.} \end{cases}$$

$$\sum_{n=0}^{N-1} e^{j\frac{2\pi kn}{N}} = \frac{1-e^{j\frac{2\pi k N}{N}}}{1-e^{j\frac{2\pi k}{N}}} = 0 \text{ if } k \neq 0$$

(b) Signal superposition:

$$\frac{1}{N}\sum_{n=0}^{N-1}(\alpha x[n] + \beta y[n])\, e^{-j\frac{2\pi nk}{N}}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1}\alpha x[n]\, e^{-j\frac{2\pi nk}{N}} + \frac{1}{N}\sum_{n=0}^{N-1}\beta y[n]\, e^{-j\frac{2\pi nk}{N}}$$

$$= \alpha X[k] + \beta Y[k]$$

(c). $\frac{1}{N}\sum_{n=0}^{N-1} x'[n]\, e^{-j\frac{2\pi nk}{N}}$ $\qquad$ Note: $x[n]$ periodic

$$= \frac{1}{N}\sum_{n=0}^{N-1} x[n-M]\, e^{-j\frac{2\pi(n-M)k}{N}}\, e^{-j\frac{2\pi Mk}{N}}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi nk}{N}}\, e^{-j\frac{2\pi Mk}{N}} = X[k]\, e^{-j\frac{2\pi Mk}{N}}$$

(d) $\frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{j\frac{2\pi nm}{N}}\, e^{-j\frac{2\pi nk}{N}}$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi n(k-m)}{N}} = X[k-m]$$

(e). $$\frac{1}{N}\sum_{n=0}^{N-1} x[n]\, y^*[n] = \frac{1}{N}\sum_{n=0}^{N-1}\underbrace{\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1} X[k_1]\, e^{j\frac{2\pi nk_1}{N}}\, Y^*[k_2]\, e^{-j\frac{2\pi nk_2}{N}}}_{\begin{cases} 0 & \text{if } k_1 \neq k_2 \\ N X[k_1] Y^*[k_1] & \text{if } k_1 = k_2 \end{cases}}$$

$$= \sum_{k=0}^{N} X[k]\, Y^*[k]$$

2. phasor analysis

	time domain	phasor domain	impedance ($\frac{V(w)}{I(w)}$)
R	$v(t) = i(t) \cdot R$	$V(w) = I(w) \cdot R$	R
C	$i(t) = C \frac{dv(t)}{dt}$	$V(w) = I(w) \frac{1}{jwC}$	$\frac{1}{jwC}$
L	$v(t) = L \cdot \frac{di(t)}{dt}$	$V(w) = I(w) \cdot jwL$	jwL

Derivation of phasor:

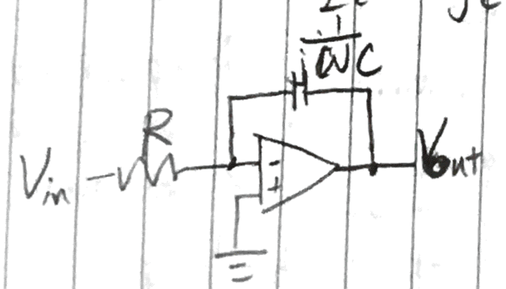
$V_c(t) = Re\{V_c e^{jwt}\}$

$= V_c \cos wt$

$i_c(t) = -C V_c w \sin wt$

$= Re\{jCV_c w e^{jwt}\} \Rightarrow I_c = jCV_c w$

$\Rightarrow Z_c = \frac{V_c}{I_c} = \frac{V_c}{jCV_c w} = \frac{1}{jwC}$



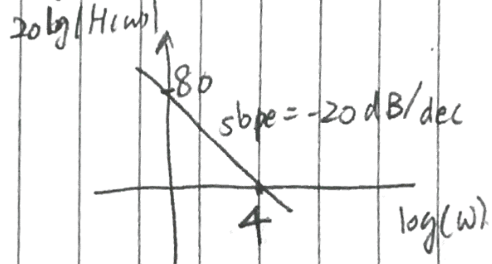
$V_- = V_+ = 0$

$\frac{V_{out}}{\frac{1}{jwC}} = -\frac{V_{in}}{R}$

$H(w) = \frac{V_{out}}{V_{in}} = -\frac{1}{jwCR}$

$H(w) = -\frac{10^4}{jw}$

$\log |H(w)| = \log \frac{10^4}{w} = 4 - \log w$



(c).

KCL:

$-\frac{V_{in}}{R} = C \cdot \frac{dV_{out}}{dt}$

$V_{out} = -\frac{1}{RC}\int V_{in} dt$

integrates V_{in}