Discussion 12: Minimum-Phase/All-Pass Decomposition and Steady-State Error

1. Minimum-Phase/All-Pass Decomposition

In many applications, we want to cancel the effect of a system on the system input, so we would like to apply a filter that is the exact inverse of the system in order to “undo” the distortion. However, systems with zeros outside the unit circle (in Z-transform) will result in inverted transfer functions that are not causal. By using the Min-phase/all-pass decomposition, we can find a filter that at least “undoes” the magnitude of the distortion, but leaves some phase distortion.

Exercise:

a) Consider the transfer function, \( H_1(z) = \frac{1+3z^{-1}}{1+2z^{-1}} \), which might represent the distortion on a transmission line. Draw the pole-zero diagram for \( H_1(z) \).

b) What is the “inverse” filter, or, in other words, what function \( H_{inv}(z) \) exactly gives \( H_{inv}(z)H_1(z) = 1 \)? Why can’t we use this function, \( H_{inv}(z) \), in a practical application?

c) Find some \( H_{AP}(z) \) (all-pass filter) and \( H_{MP}(z) \) (“min-phase” filter) such that \( H_1(z) = H_{AP}(z)H_{MP}(z) \).

d) Using this “all-pass” decomposition, construct a new inverse filter such that \( |H_{inv}(z)H_1(z)| = 1 \).

Extra Practice: consider the transfer function, \( H_2(z) = \frac{(1+3z^{-1})(1+3e^{-j\pi/4}z^{-1})}{(1-\frac{1}{2}z^{-1})} \). Find the all-pass/min-phase decomposition for \( H_2(z) \).
2. Steady-State Error

Please refer to the “Steady State Error” Handout for some notes.

The “Type” of loop refers to the number of poles at $s = 0$ for the open loop gain, $DG$. Type 0 loops have constant steady state error for step response. Type 1 loops have zero steady state error for step response.

Exercise:

Consider the system below, where the input voltage $x(t)$ controls the angular velocity, $\omega(t)$, of the motor with moment of inertia $J$. The angular velocity is related to current by the diff. eq.: $J \frac{d\omega}{dt} = ki(t)$

a) Find $H_P(s)$, the transfer function from input voltage to output angular velocity.

b) Find the steady state error of the system under proportional control with gain $K$, and approximately sketch the steady state error vs. $K$.

c) What is the steady state error if we use integral control ($\frac{K}{s}$)?