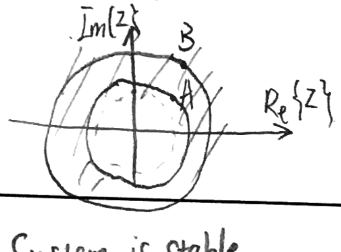
EE120 GSI: Ming

- ROC and system properties
- z-transform
- Filter

Prob 1.

1) causal system ⇒ $h[n]=0$ for $n<0$ ⇒ $\underline{C}$

$H(z)=\sum_{n=0}^{\infty} h[n]\, z^{-n}$



Circle. if $Z_A=r_A e^{j\omega}$ for $\forall\omega$ is included

$\left|\sum_{n=0}^{\infty} h[n] r_A^{-n} e^{-j\omega n}\right| < \infty$

$r_A<r_B$, then $Z_B=r_B e^{j\omega}$ for $\forall\omega$ is included

$\left|\sum_{n=0}^{\infty} h[n] r_B^{-n} e^{-j\omega n}\right| \le \left|\sum_{n=0}^{\infty} h[n] r_A^{-n} e^{-j\omega n}\right| \left|\sum_{n=0}^{\infty}\left(\frac{r_B}{r_A}\right)^{-n}\right| < \infty$ (Holder's inequality * assume $h[n]\geq 0$)

A C

2). $H(z)$ rational ⇒ $H(z)=\frac{B(z)}{A(z)}=\frac{b_0+b_1z^{-1}+\cdots+b_Mz^{-M}}{a_0+a_1z^{-1}+\cdots+a_Nz^{-N}}$

$=\frac{b_0}{a_0}z^{-M+N}\frac{z^M+\frac{b_1}{b_0}z^{M-1}+\cdots+b_M/b_0}{z^N+\left(\frac{a_1}{a_0}\right)z^{N-1}+\cdots+a_N/a_0}$

$=\frac{b_0}{a_0}z^{-M+N}\frac{\prod(z-z_k)}{\prod(z-p_k)}$

- M finite zeros, N finite poles

if $N-M\neq 0$ if $N-M>0$ ⇒ $|N-M|$ zeos at origin, poles at infy

$N-M\le 0$ ⇒ $|N-M|$ poles at origin

ROC cannot include poles! therefore $\underline{N-M\le 0}$ ($\underline{B}$)

B D

3). System is stable

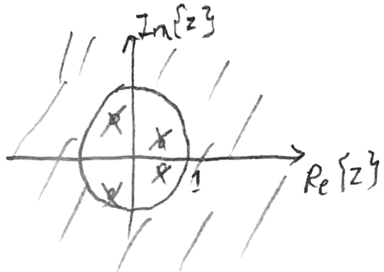
⇒ $\sum_{n=-\infty}^{\infty}|h(n)|<\infty$ absolutely summable

$|H(z)|=\left|\sum_{n=-\infty}^{\infty} h(n)z^{-n}\right| \le \sum_{n=-\infty}^{\infty}|h[n]z^{-n}|$

$=\sum_{n=-\infty}^{\infty}|h(n)|<\infty$ for $|z|=1$

⇒ ROC contains unit circle $\underline{D)}$

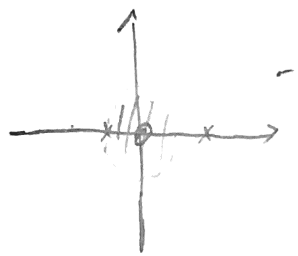
4)



$\underline{A}$

Prob 2

$X(z)=\frac{z^2}{(z-\frac{1}{2})(z-1)}$



left-sided: $x[n]=0$ for $n\geq 0$

⇒ ROC inside a circle.

rational ⇒ $|z|<\frac{1}{2}$ ROC

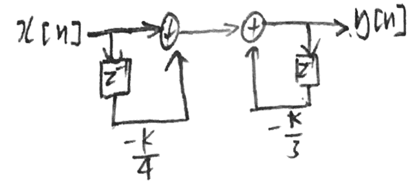
$X(z)=\frac{a}{1-\frac{1}{2}z^{-1}}+\frac{b}{1-z^{-1}}=\frac{a+b-(a+\frac{1}{2}b)z^{-1}}{(1-\frac{1}{2}z^{-1})(1-z^{-1})}$

⇒ $a=-1$, $b=2$

$x[n]=\left(\frac{1}{2}\right)^n u[-n-1]-2u[-n-1]$

Prob 3.

Method 1



$x[n] - \frac{k}{4}x[n-1] - \frac{k}{3}y[n-1] = y[n]$

$X(z) - \frac{k}{4}z^{-1}X(z) - \frac{k}{3}z^{-1}Y(z) = Y(z)$

$H(z) = \frac{Y(z)}{X(z)} = \frac{1-\frac{k}{4}z^{-1}}{1+\frac{k}{3}z^{-1}}$

zeros: $\frac{k}{4}$, poles: $-\frac{k}{3}$

rational, causal ⇒ ROC

$|z| > |\frac{k}{3}|$ to be stable.

we should include unit circle ⇒ $|\frac{k}{3}| < 1 \Rightarrow |k| < 3$

(c). $X(z) = \frac{z}{z-\frac{2}{3}}$, $Y(z) = H(z)X(z) = \frac{1-\frac{1}{4}z^{-1}}{(1+\frac{1}{3}z^{-1})(1-\frac{2}{3}z^{-1})}$

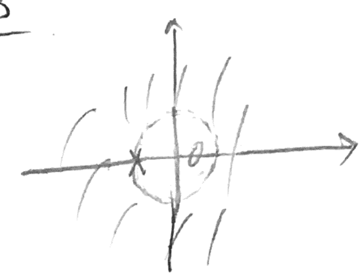
$= \frac{\frac{7}{12}}{1+\frac{1}{3}z^{-1}} + \frac{\frac{5}{12}}{1-\frac{2}{3}z^{-1}}$

$y[n] = \frac{7}{12}(-\frac{1}{3})^n u[n] + \frac{5}{12}(\frac{2}{3})^n u[n]$

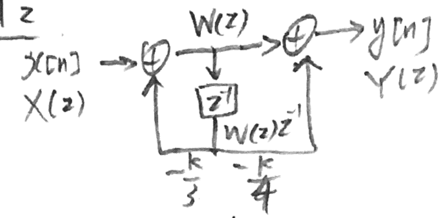
A

C

Prob3



Method 2



$Y(z) = W(z) - \frac{k}{4}z^{-1}W(z)$,

$X(z) - \frac{k}{3}z^{-1}W(z) = W(z)$ $\Rightarrow H(z) = \frac{1-\frac{k}{4}z^{-1}}{1+\frac{k}{3}z^{-1}}$

B

D