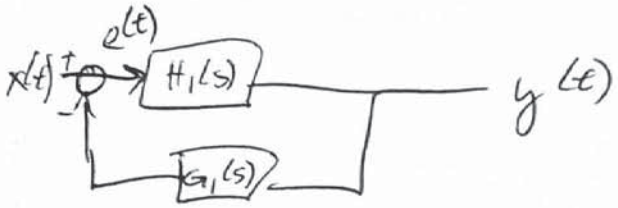
Dis 10 solutions

1a) $H(s) = \frac{Y(s)}{X(s)}$?



$$E(s) = X(s) - Y(s)G_1(s)$$

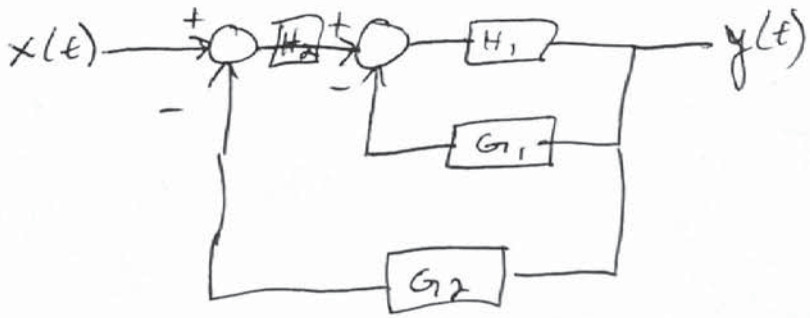
$$Y(s) = E(s)H_1(s) = H_1(s)\left(X(s) - Y(s)G_1(s)\right)$$

$$Y = H_1(s)X(s) - H_1G_1Y$$

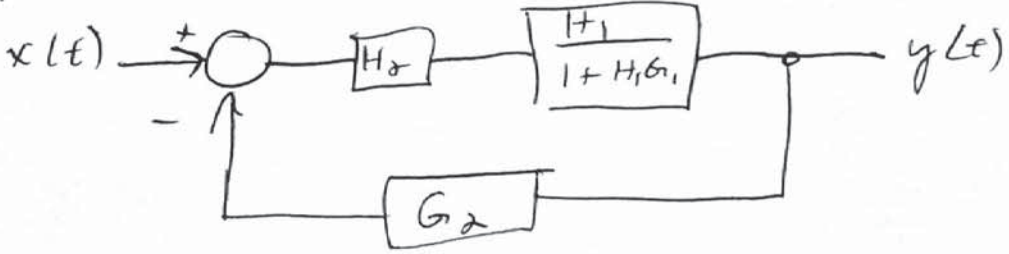
$$Y(1 + H_1G_1) = H_1X$$

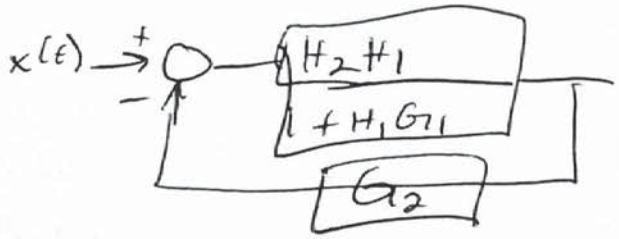
$$\boxed{\frac{Y}{X} = \frac{H_1}{1 + H_1G_1}}$$

1b)



Simplify:



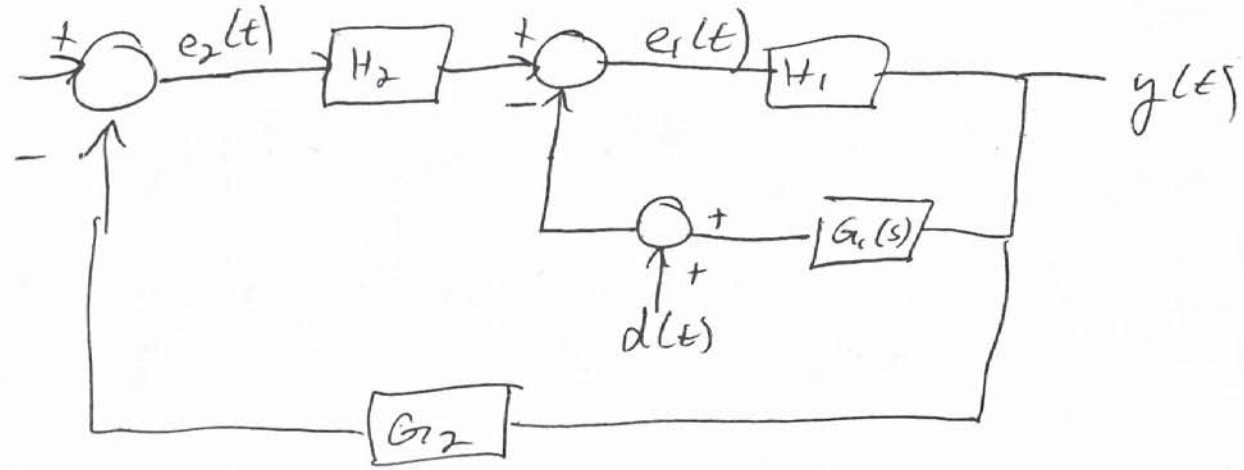


$$\frac{Y}{X} = \frac{\frac{H_2H_1}{1 + H_1G_1}}{1 + \frac{G_2H_2H_1}{1 + H_1G_1}}$$

Simplify:

$$\frac{H_2 H_1}{1 + H_1 G_1 + G_2 H_2 H_1} = \frac{Y}{X}$$

1c)



Let $x(t) = 0$

$$Y = E_1 H_1$$

$$E_1 = E_2 H_2 - (D + Y G_1)$$

$$E_2 = -G_2 Y$$

$$E_1 = -G_2 Y H_2 - D - Y G_1$$

$$Y = -H_1 H_2 G_2 Y - H_1 D - H_1 G_1 Y \rightarrow Y + H_1 H_2 G_2 Y + H_1 G_1 Y = -H_1 D$$

$$\frac{Y(s)}{D(s)} = \frac{-H_1}{1 + H_1 H_2 G_2 + H_1 G_1} = H_D(s)$$

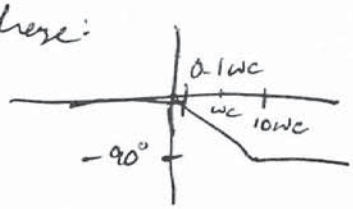
2a) $H(s) = \frac{200\pi}{s+20\pi} = \frac{10}{\frac{s}{20\pi}+1}$

$H(jw) = \frac{10}{1+\frac{jw}{20\pi}} = \frac{K}{1+\frac{jw}{w_c}}$ where $K=10$, $w_c = 20\pi$

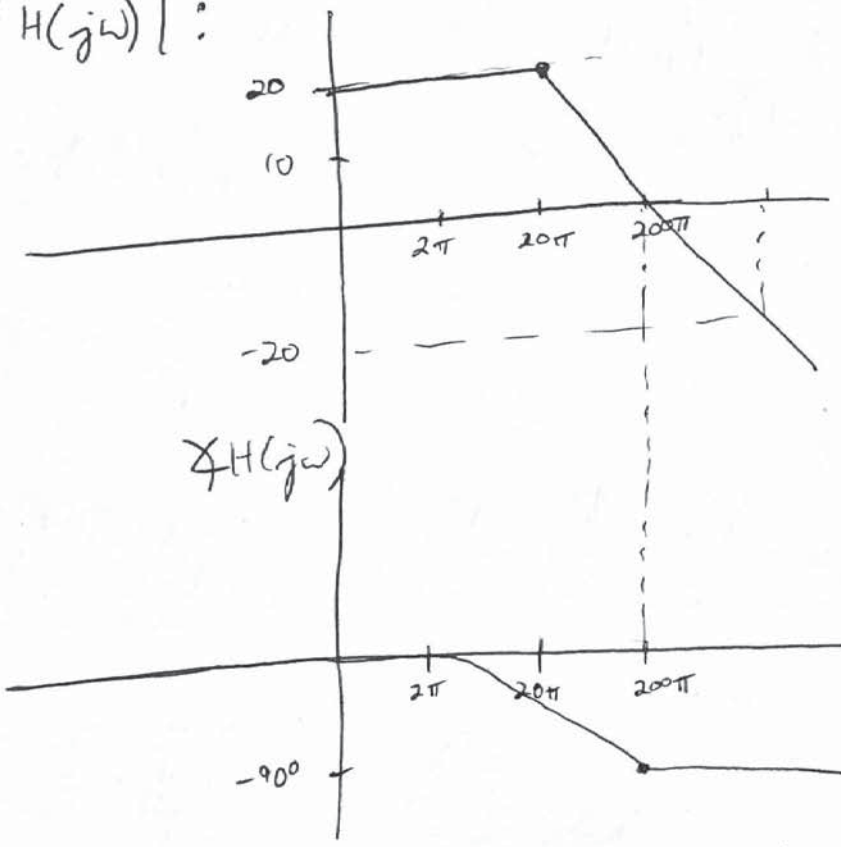
positive constant: $Mag = 20\log K$ [dB], $phase = 0°$

simple pole: Mag:

phase:



$|H(jw)|$:



$w_{unity} = 200\pi$

$\angle H(jw_{unity}) = -90°$

$w_{180°} = \infty$

$|H(jw_{180°})| \to 0$

$(-180) - \angle H(jw_{unity}) = -90° = PM$

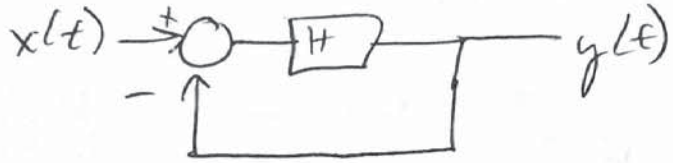
$GM = \infty$

GM → We can increase gain as much as we want, the phase will never hit -180°

PM → We can increase phase by 90° before we hit 180°.

2 b)

If:



$$H(s) = \frac{200\pi}{s+20\pi}$$

$$H_{CL} = \frac{H(s)}{1+H(s)} = \frac{200\pi}{s+20\pi+200\pi} = \boxed{\frac{200\pi}{s+220\pi}}$$