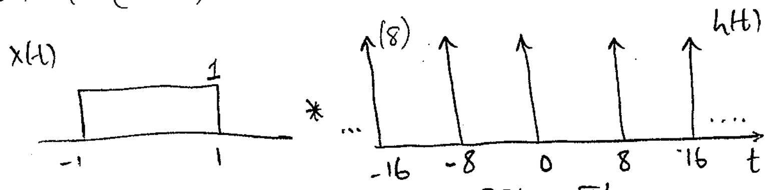
EE120 PS#4 Fall 2014

(1)

$\sum \delta(t/8-n) = 8\sum\delta(t-8n) \longleftrightarrow \frac{2\pi}{8}\cdot 8\cdot\sum\delta(\omega-\frac{2\pi k}{8})$

a) $\Pi(t/2) * \text{comb}(t/8) = y(t)$

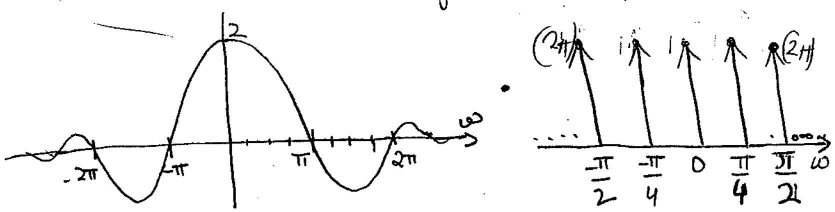


$\Downarrow$

$2\pi/8 = \pi/4$

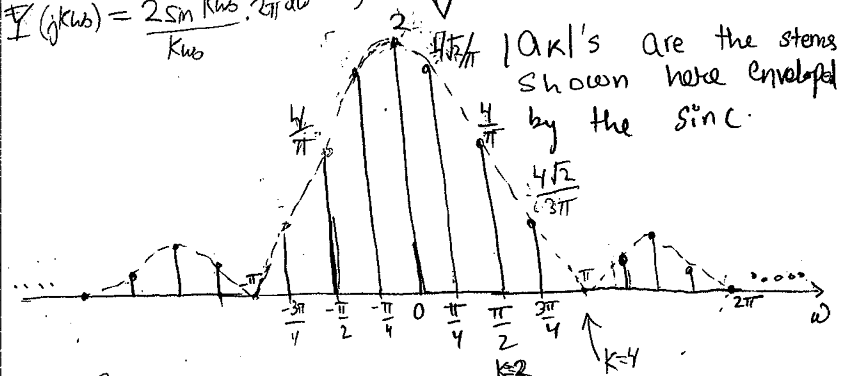
$X(j\omega) = \frac{2\sin\omega}{\omega}$

$X(j\omega) * 2\pi\sum\delta(\omega-2\pi k/8)$



$Y(jk\omega_0) = \frac{2\sin k\omega_0}{k\omega_0}\cdot 2\pi\,\delta(\omega-k\omega_0)$

$|a_k|$'s are the stems shown here enveloped by the sinc.



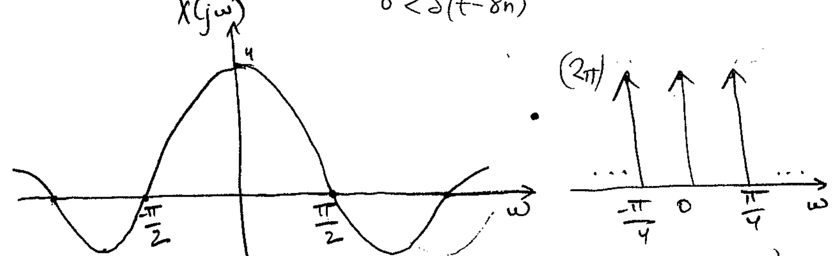
$\omega_0 = \frac{2\pi}{8} = \pi/4$

$a_k = \frac{2\sin k\pi/4}{k\pi/4} = \frac{1}{2\pi}\text{Area}\{Y(jk\omega_0)\} = \frac{8\sin k\pi/4}{k\pi}$

(1)

b) $\underset{x(t)}{\pi(t/4)} * \underset{h(t)}{\text{comb}(t/8)} = y(t)$

$8\sum\delta(t-8n)$

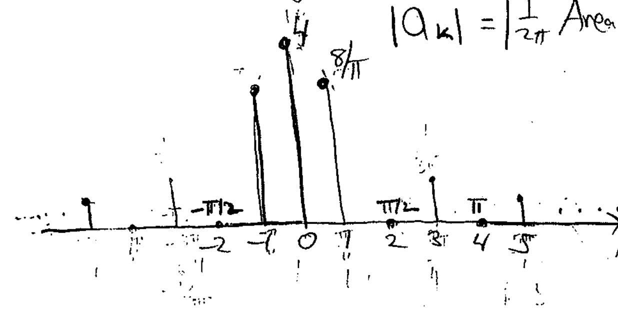


$X(j\omega) = \frac{2\sin 2\omega}{\omega}$

$\frac{2\pi}{8}\cdot 8\sum\delta(\omega-k\frac{2\pi}{8})$

$|a_k| = \left|\frac{1}{2\pi}\text{Area}\{Y(j\omega)\}\right|$

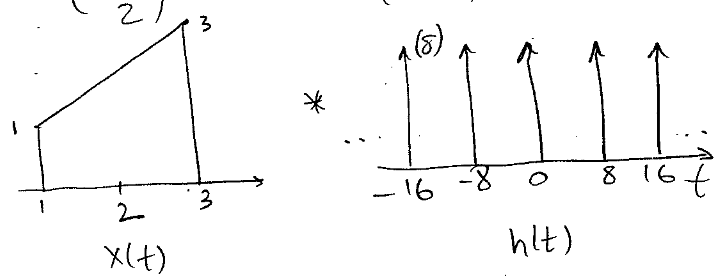
$\omega = k\frac{2\pi}{8}$



$Y(jk\omega_0) = \frac{2\sin 2\cdot k\pi/4}{k\pi/4}\cdot 2\pi\,\delta(\omega-k\pi/4)$

$a_k = \frac{8\sin k\pi/2}{k\pi}$

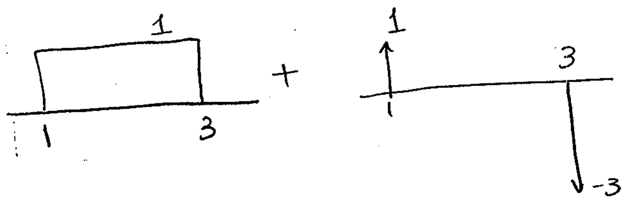
c) $t\,\Pi\left(\frac{t-2}{2}\right) * \text{comb}(t|8) = y(t)$

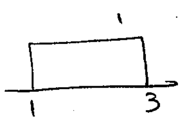


Evaluate $\mathcal{F}\left\{\frac{dx}{dt}\right\} = j\omega X(j\omega)$;

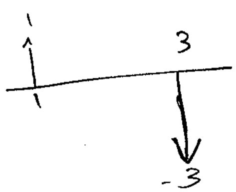
then, solve for $X(j\omega)$.

$\frac{dx}{dt} =$





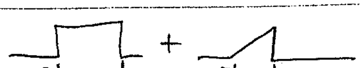
$\longleftrightarrow \; \frac{2\sin(\omega)}{\omega} e^{-j2\omega}$



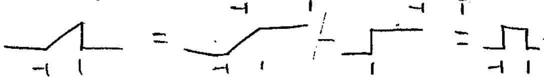
$\longleftrightarrow \left(e^{j\omega} - 3e^{-j\omega}\right) e^{-j2\omega}$

$= \left(-2\cos(\omega) + j4\sin(\omega)\right) e^{-j2\omega}$

x(t) not unique :



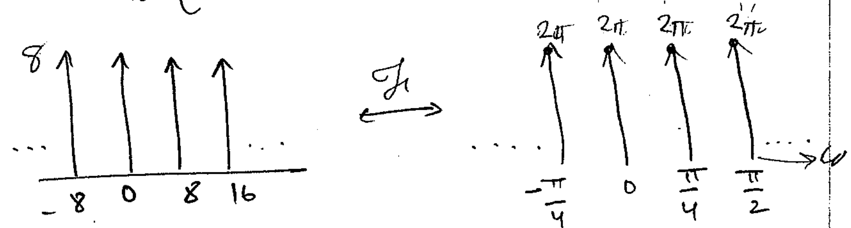
Where



$\therefore X(j\omega) = \frac{e^{-j2\omega}}{j\omega}\left[\frac{2\sin(\omega)}{\omega} - 2\cos(\omega) + j4\sin(\omega)\right]$

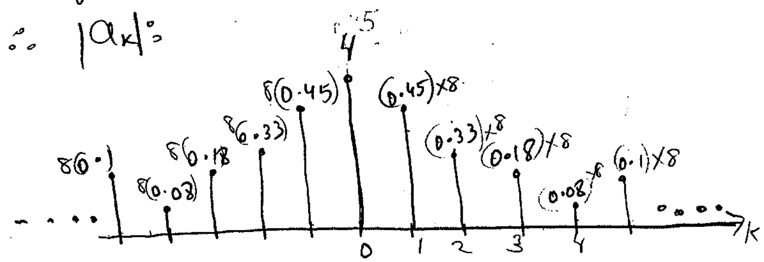
$|X(j\omega)|^2 = \frac{4}{\omega^2}\left[\left(\frac{\sin(\omega)}{\omega} - \cos(\omega)\right)^2 + 4\sin^2(\omega)\right]$

$= \frac{4}{\omega^2}\left[\frac{\sin^2(\omega)}{\omega^2} - \frac{2\sin(\omega)\cos(\omega)}{\omega} + \cos^2(\omega) + 4\sin^2(\omega)\right]$

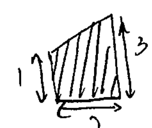


$|X(j\omega)| = \frac{2}{\omega}\sqrt{\frac{\sin^2(\omega)}{\omega^2} + \cos^2(\omega) + 4\sin^2(\omega) - \frac{2\sin(\omega)\cos(\omega)}{\omega}}$

$\therefore |a_k|$:

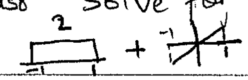


$|X(0)| =$



$= \frac{1}{2} \times 2 \times (1+3) = 4.$

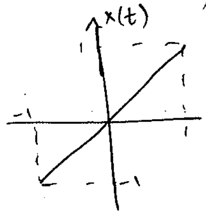
*. could also solve for $X(j\omega)$ assuming:
$x(t) =$

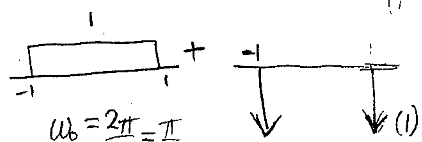


$\longrightarrow$ then applying shifting property.

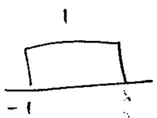
d) $\underbrace{t\,\Pi(t/2)}_{x(t)} * \underbrace{\text{comb}(t/8)}_{h(t)} = y(t)$

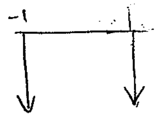
$8\sum\delta(t-8n)$



; $\dfrac{dx}{dt} =$ 

$\omega_0 = \frac{2\pi}{8} = \frac{\pi}{4}$

 $\longleftrightarrow$ $2\,\dfrac{\sin(\omega)}{\omega}$

 $\longleftrightarrow$ $-e^{-j\omega} + e^{+j\omega} = -2\cos(\omega)$

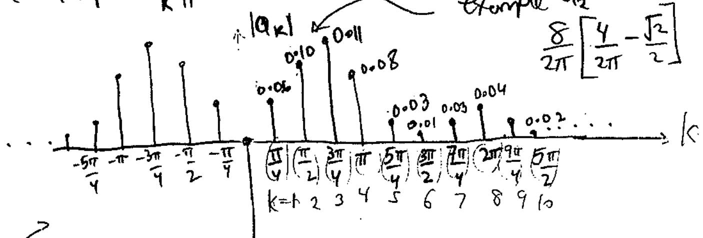
$Y(j\omega) = X(j\omega)\cdot 2\pi \sum\delta(k-k\omega_0)$.

$\therefore X(j\omega) = \dfrac{2}{j\omega}\left[\dfrac{\sin(\omega)}{\omega} - \cos(\omega)\right]$; $X(0) = 0$

$|X(j\omega)| = \dfrac{2}{\omega}\left(\dfrac{\sin(\omega)}{\omega} - \cos(\omega)\right)$

$a_k = \left|X\left(j\frac{k\pi}{4}\right)\right| = \dfrac{8}{k\pi}\left[\dfrac{4\sin(k\pi/4)}{k\pi} - \cos(k\pi/4)\right]$

example $a_2 = \dfrac{8}{2\pi}\left[\dfrac{4}{2\pi} - \dfrac{\sqrt{2}}{2}\right]$

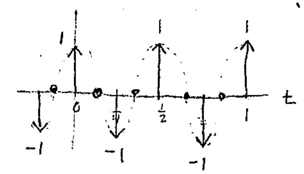


line spectrum for y(t).

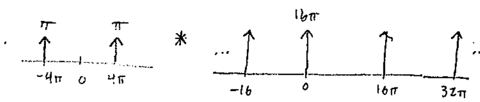
② a) $z(t) = x(t)\,w(t)$

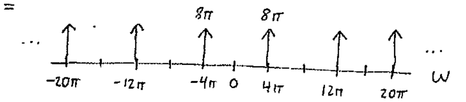
$= \cos(4\pi t)\cdot\sum_n \delta(t-n/8)$

$= \sum_{n=-\infty}^{\infty} \cos\left(\frac{\pi n}{2}\right)\cdot\delta(t-n/8)$



$Z(j\omega) = \frac{1}{2\pi} X(j\omega) * W(j\omega)$

$= \frac{1}{2\pi}\cdot$ 

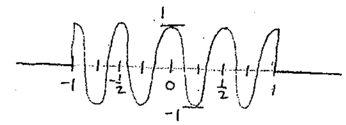
$=$ 

$y(t) = z(t) * \delta(t) = z(t)$ (same as above)

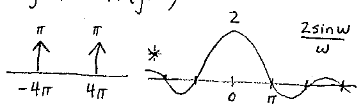
$Y(j\omega) = Z(j\omega)$ (same as above)

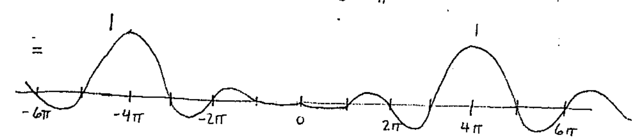
b) $z(t) = x(t)\,w(t)$

$= \cos(4\pi t)\cdot\Pi(t/2)$



$Z(j\omega) = \frac{1}{2\pi}\cdot X(j\omega) * W(j\omega)$

$= \frac{1}{2\pi}\cdot$ 

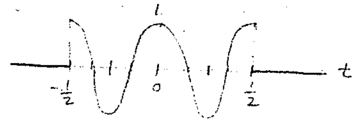
$=$ 

$y(t) = z(t)$

$Y(j\omega) = Z(j\omega)$ (same as above)

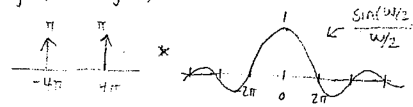
② c) $z(t) = x(t)\,w(t)$

$= \cos(4\pi t)\cdot \Pi(t)$



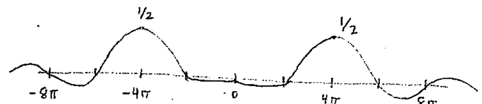
$Z(j\omega) = \frac{1}{2\pi} X(j\omega) * W(j\omega)$

$= \frac{1}{2\pi}\cdot$



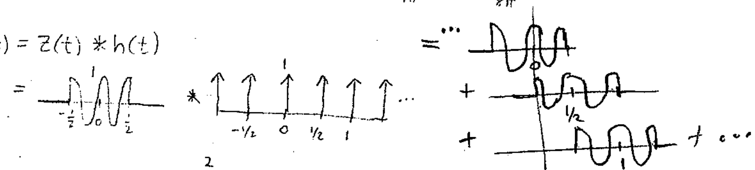
$\frac{\sin(\omega/2)}{\omega/2} = \frac{2\sin(\omega/2)}{\omega}$

$=$

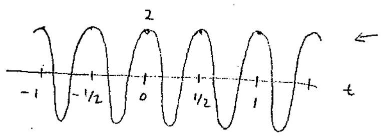


$y(t) = z(t) * h(t)$

$=$

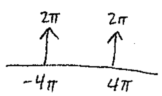


$=$



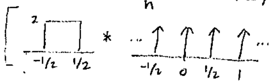
$\leftarrow 2\cos(4\pi t)$

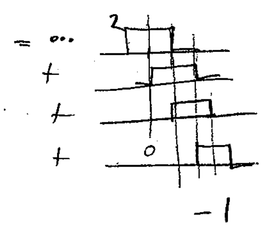
$Y(j\omega) =$



d) $w(t) = \left[2\,\Pi(t) * \sum_n \delta(t - n/2)\right] - 1$

$= \left[\ \ast\ \right] - 1$



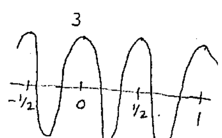


$= 4 - 1$

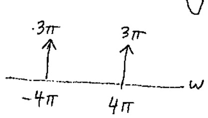
$= 3$

$z(t) = x(t)\,w(t)$

$= 3\cos(4\pi t)$



$Z(j\omega) =$

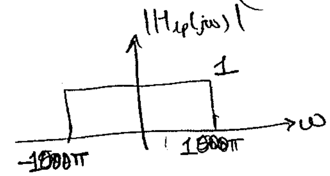


$y(t) = z(t)$

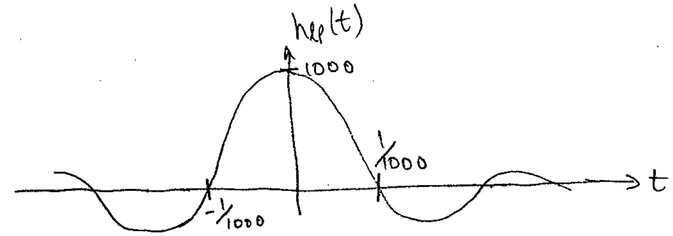
$Y(j\omega) = Z(j\omega)$ same as above

③

a) $H_{lp}(j\omega) = \Pi\left(\frac{\omega}{2\pi\cdot 1000}\right)$

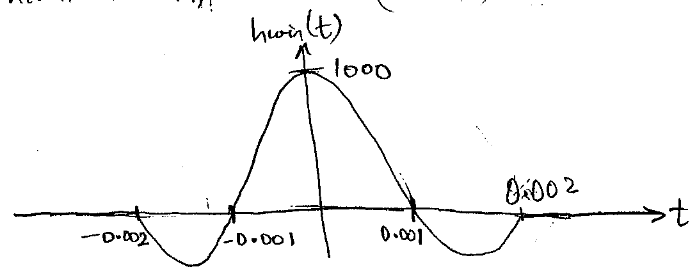


$h_{lp}(t) = \frac{1}{2\pi}\,\frac{2\sin(1000\pi t)}{t}$

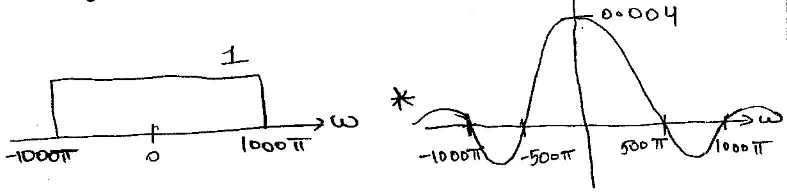


b) $h_{win}(t) = h_{lp}(t)\cdot \Pi\left(\frac{t}{0.004}\right)$

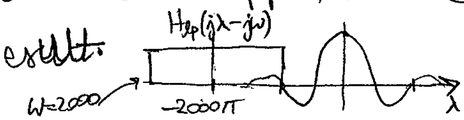
$=$



$$H_{win}(j\omega) = H_{lp}(j\omega) * \frac{2\sin(0.002\omega)}{\omega}$$



Convolving sinc with a rect results in ripples and over-shoot behavior. We will sketch an approximate convolution result.

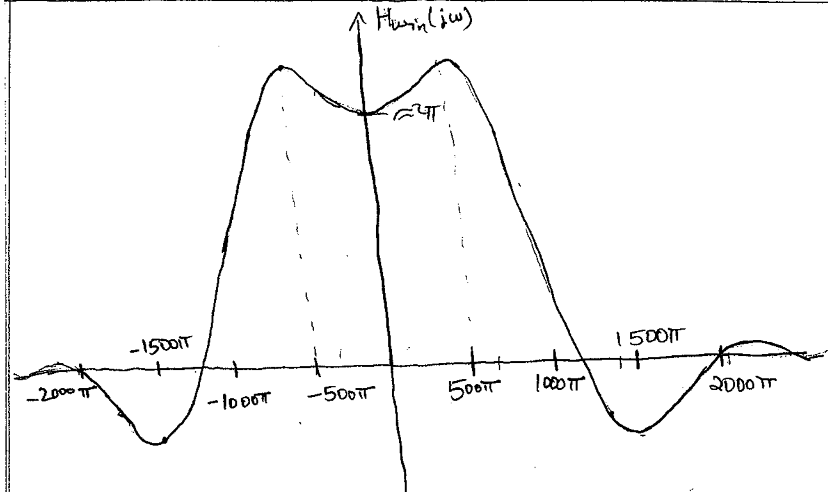


$|H_{win}(j\omega)|$ is the most negative at $\omega = -1500\pi$ because of the large negative area in the second lobe.

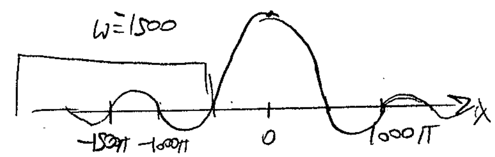
$-1500\pi \le \omega \le -500\pi$: integrated area increases and peaks @ $\omega = -500\pi$

$-500\pi \le \omega \le 0$: integrated area decreases a bit and drops a little when $\omega = 0$

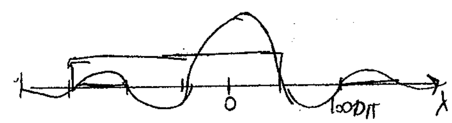
$0 \le \omega$: Result of the convolution is the same as that for $\omega < 0$.



$-1500\pi \le w \le -500\pi$

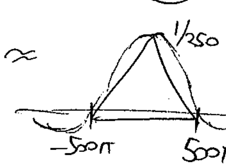


$w = -500\pi$



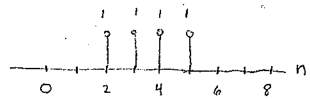
estimating $H_{win}(j0) \approx$

rough approximation



$$= \frac{1}{2}(1000\pi)\frac{1}{250}$$
$$= 2\pi.$$

④ a) $x[n] = u[n-2] - u[n-6]$



$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn}$$

$$= e^{-jw2} + e^{-jw3} + e^{-jw4} + e^{-jw5}$$

$$= e^{-jw\frac{7}{2}} \left[e^{jw\frac{3}{2}} + e^{jw\frac{1}{2}} + e^{-jw\frac{1}{2}} + e^{-jw\frac{3}{2}} \right]$$

$$= e^{-jw\frac{7}{2}} \left[2\cos\left(\frac{3w}{2}\right) + 2\cos\left(\frac{w}{2}\right) \right]$$

b) $x[n] = \sin\left(\frac{\pi}{2} n\right) + \cos(n)$

$$= \frac{1}{2j} e^{j\frac{\pi}{2}n} - \frac{1}{2j} e^{-j\frac{\pi}{2}n} + \frac{1}{2} e^{jn} + \frac{1}{2} e^{-jn}$$

Since $e^{jw_0 n} \xleftrightarrow{\text{DTFT}} \sum_{K=-\infty}^{\infty} 2\pi\,\delta(w - w_0 - 2\pi K)$

$$X(e^{jw}) = \sum_{K=-\infty}^{\infty} \left[\frac{1}{2j} \cdot 2\pi\,\delta\left(w - \frac{\pi}{2} - 2\pi K\right) - \frac{1}{2j} \cdot 2\pi\,\delta\left(w + \frac{\pi}{2} - 2\pi K\right) + \frac{1}{2} \cdot 2\pi\,\delta(w - 1 - 2\pi K) + \frac{1}{2} \cdot 2\pi\,\delta(w + 1 - 2\pi K) \right]$$

periodic $0 \le w < 2\pi$ $= X(e^{jw}) = \pi \left[-j\,\delta\left(w - \frac{\pi}{2}\right) + j\,\delta\left(w + \frac{\pi}{2}\right) + \delta(w-1) + \delta(w+1) \right]$

extra fun problem

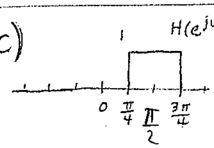
$$H(e^{jw}) = \cos^2 w + \sin^2 w$$

$$= 1$$

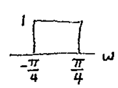
$$= \sum_{n=-\infty}^{\infty} h[n] e^{-jwn}$$

By inspection, $h[n] = \delta[n]$

c)



Since $\frac{\sin \frac{\pi n}{4}}{\pi n} \xleftrightarrow{\text{DTFT}}$



and $e^{jw_0 n} x[n] \longleftrightarrow X(e^{j(w - w_0)})$,

$$h[n] = e^{j\frac{\pi}{2}n} \cdot \frac{\sin\left(\frac{\pi n}{4}\right)}{\pi n}$$

e) $H(e^{jw}) = \delta\left(w + \frac{\pi}{3}\right) + \delta\left(w - \frac{\pi}{3}\right)$ for $-\pi \le w \le \pi$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[\delta\left(w + \frac{\pi}{3}\right) + \delta\left(w - \frac{\pi}{3}\right) \right] e^{jwn}\, dw$$

$$= \frac{1}{2\pi} \left[e^{-j\frac{\pi}{3}n} + e^{j\frac{\pi}{3}n} \right]$$

$$= \frac{1}{\pi} \cos\left(\frac{\pi}{3} n\right)$$

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⑤ - see web page for .ipynb file