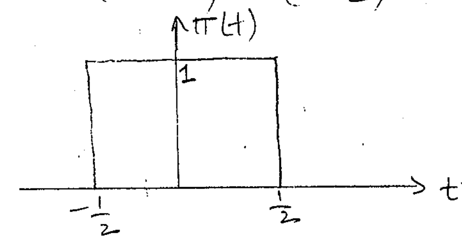
EE 120 HW 1, P. 1 — Fall 2014

? = to be calculated D = designed $\checkmark$ = known, measured

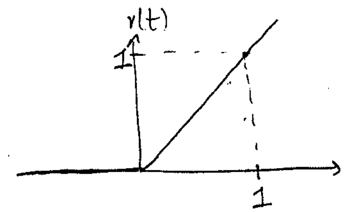
	$\underset{\sim}{x}$	$\underset{\sim}{H}$	$\underset{\sim}{y}$
a) radio surgery	D targeted radiation	$\checkmark$ brain tissue	D energy dumped in tissue
b) MRI	D magnetic field	? body organs	$\checkmark$ measurements molecule excitation
c) seismology	D explosion	? earth's subsurface	$\checkmark$ reflected energy waves
d) camera	? light waves	D/$\checkmark$ camera	$\checkmark$ pixels
e) Facebook	D new feature	? user population behavior	$\checkmark$ measurements user satisfaction

Different scenarios can be interpreted differently. Any answer that is justified will be accepted.

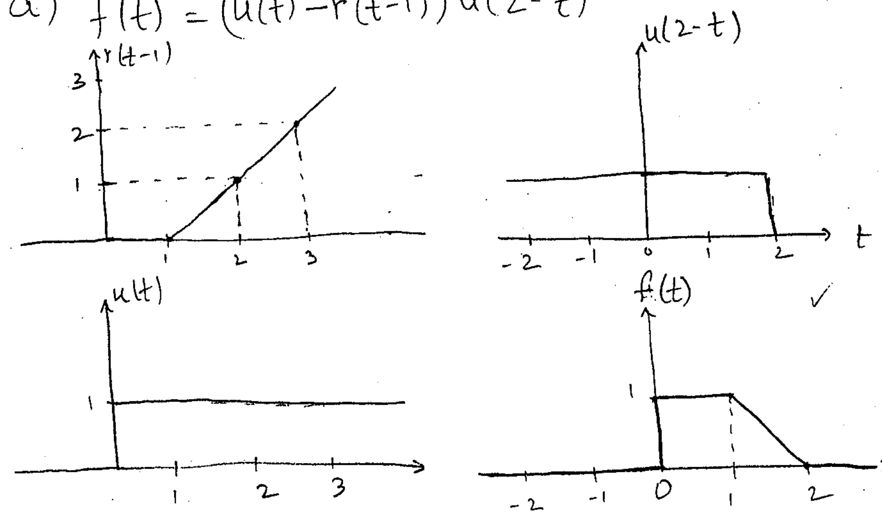
② $\Pi(t) = u\left(t+\frac{1}{2}\right) - u\left(t-\frac{1}{2}\right)$



$r(t) = t\,u(t)$

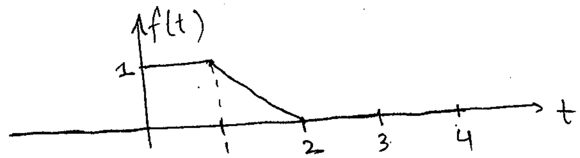


a) $f(t) = \left(u(t) - r(t-1)\right)u(2-t)$

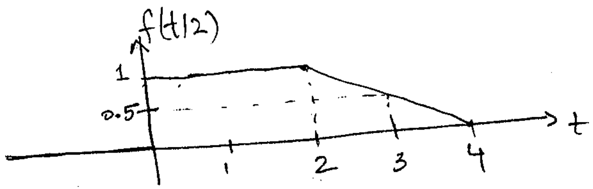


b) $f(t/2)$: $f(t)$ "slowed down"

$f(t)$:



$\therefore f(t/2)$:

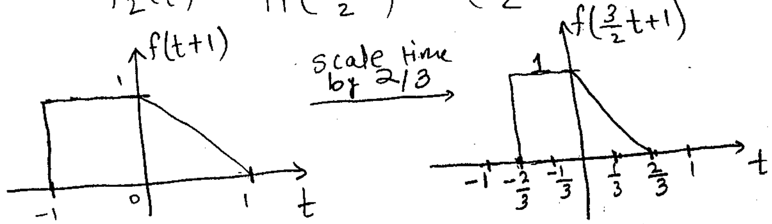


c)

Approach 1: $\boxed{\text{shift-then-scale}}$

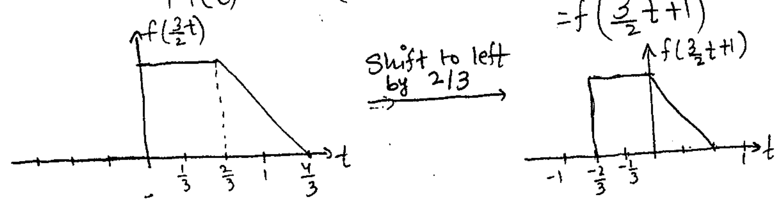
$f_1(t) = f(t+1)$

$f_2(t) = f_1\left(\frac{3}{2}t\right) = f\left(\frac{3}{2}t+1\right)$

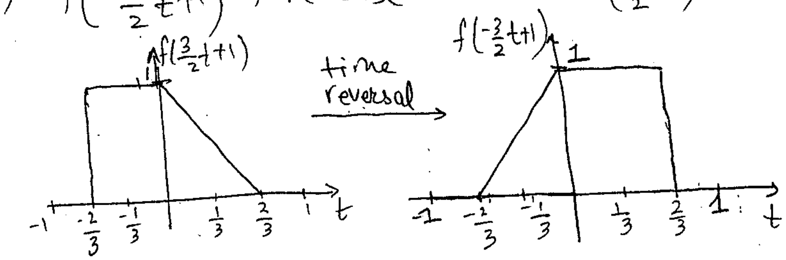


Approach 2: $\boxed{\text{Scale-then-shift}}$

$f_1(t) = f(3/2\, t)$; $f_2(t) = f_1\left(t+\frac{2}{3}\right)$
$= f\left(\frac{3}{2}t+1\right)$



d) $f\left(-\frac{3}{2}t+1\right) \Rightarrow$ reverse time in $f\left(\frac{3}{2}t+1\right)$

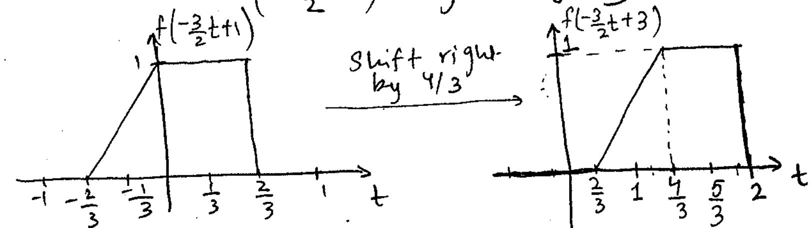


e) $f\left(3\left[-\frac{1}{2}t+1\right]\right) = f\left(-\frac{3}{2}t+3\right)$

$-\frac{1}{2}t+1 = \frac{2}{3}$
$-\frac{1}{2}t = -\frac{1}{3}$
$t = \frac{2}{3}$

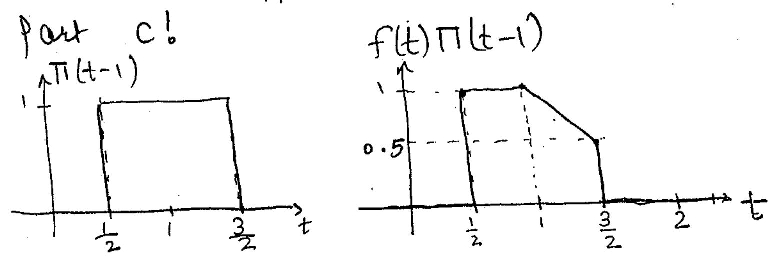
$= f\left(-\frac{3}{2}t+1+2\right) = f\left(-\frac{3}{2}\left(t-\frac{4}{3}\right)+1\right)$

$\hookrightarrow$ Shift $f\left(-\frac{3}{2}t+1\right)$ right by $\frac{4}{3}$



*. verify you get this same result with the approach shown in part c!

f)



③

a) $\delta_1(t) = \lim\limits_{a\to\infty} \frac{a}{2} e^{-a|t|}$

i) $\delta_1(t-t_0) = \lim\limits_{a\to\infty} \frac{a}{2} e^{-a|t-t_0|}$

if $t \neq t_0$, then $t - t_0 \neq 0$

$\therefore \lim\limits_{a\to\infty} \frac{a}{2} e^{-a|t-t_0|} = \infty e^{-\infty} = 0$

Additionally,

$\lim\limits_{t\to t_0} \delta_1(t-t_0) = \infty$

ii) $\int_{-\infty}^{\infty} \delta(t)\,dt = \int_{-\infty}^{0} \frac{a}{2} e^{at} dt + \int_{0}^{\infty} \frac{a}{2} e^{-at}\,dt$

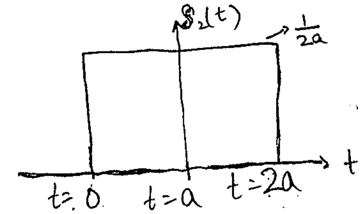
$= \frac{1}{2} e^{at}\Big|_{t=-\infty}^{t=0} - \frac{1}{2} e^{-at}\Big|_{t=0}^{t=\infty}$

$= \frac{1}{2} + \frac{1}{2} = 1$

$\therefore \boxed{\int_{-\infty}^{\infty} \delta(t)\,dt = 1}$

b) $\delta_2(t) = \lim\limits_{a\to 0} \frac{1}{2a} \Pi\left(\frac{t-a}{2a}\right)$

i)



$\frac{1}{2a} \pi\left(\frac{\overset{\neq 0}{t-t_0}}{2a} - \frac{1}{2}\right)$

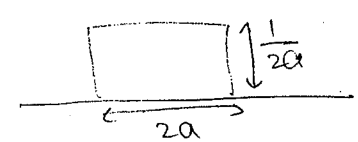
~~if $t \neq t_0$~~: $\delta_2(t-t_0) = \lim\limits_{a\to 0} \frac{1}{2a} \Pi\left(\frac{t-a-t_0}{2a}\right)$

$= (\infty)[\Pi(\infty)] = 0$

Also:

$\lim\limits_{t\to t_0} \delta_2(t-t_0) = \infty$.

ii) $\int_{-\infty}^{\infty} \delta_2(t)\,dt$ = Area of rectangle:



$\therefore \int_{-\infty}^{\infty} \delta_2(t)\,dt = (2a)\left(\frac{1}{2a}\right) = 1$

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6/.

EE120 HW 1, P.4

$$z \cdot z^* = r \cdot e^{j\theta} \cdot r \cdot e^{-j\theta} = r^2$$

$$\frac{z}{z^*} = \frac{re^{j\theta}}{re^{-j\theta}} = e^{j2\theta}$$

$$(z_1 z_2)^* = (r_1 e^{j\theta_1} r_2 e^{j\theta_2})^* = \left[r_1 r_2 e^{j(\theta_1+\theta_2)}\right]^* = r_1 r_2 e^{-j(\theta_1+\theta_2)}$$

$$= r_1 e^{-j\theta_1} \cdot r_2 e^{-j\theta_2} = z_1^* z_2^*$$

$$\left(\frac{z_1}{z_2}\right)^* = \left(\frac{r_1 e^{j\theta_1}}{r_2 e^{j\theta_2}}\right)^* = \left[\frac{r_1}{r_2} \cdot e^{j(\theta_1-\theta_2)}\right]^* = \frac{r_1}{r_2} e^{-j(\theta_1-\theta_2)} = \frac{r_1 e^{-j\theta_1}}{r_2 e^{-j\theta_2}} = \frac{z_1^*}{z_2^*}$$

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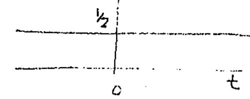
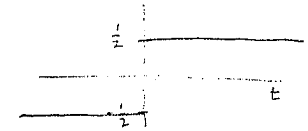
EE120 HW 1, P. 5

a) $$x_e(t) = \frac{x(t)+x(-t)}{2} = \frac{u(t)+u(-t)}{2} = \frac{1}{2}$$

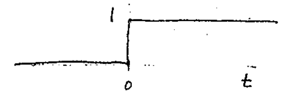


$$x_o(t) = \frac{x(t)-x(-t)}{2} = \frac{u(t)-u(-t)}{2} = \begin{cases} \frac{1}{2}, & t>0 \\ -\frac{1}{2}, & t<0 \end{cases}$$



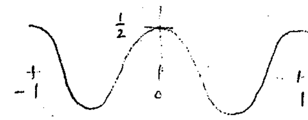
$$= \frac{\mathrm{sgn}(t)}{2}$$

$$u(t) = x_o(t) + x_e(t)$$

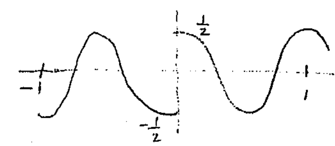


or $u(t) = \frac{1}{2}(1+\mathrm{sgn}(t))$. If $\mathrm{sgn}(0) \triangleq 0$, then $u(0) = \frac{1}{2}$. O/W does not define $u(0)$.

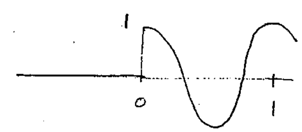
b) $$x_e(t) = \frac{x(t)+x(-t)}{2} = \frac{\cos(2\pi t)u(t) + \cos(-2\pi t)u(-t)}{2} = \frac{\cos(2\pi t)[u(t)+u(-t)]}{2} = \frac{1}{2}\cos(2\pi t)$$



$$x_o(t) = \frac{x(t)-x(-t)}{2} = \frac{\cos(2\pi t)u(t) - \cos(-2\pi t)u(-t)}{2} = \frac{\cos(2\pi t)[u(t)-u(-t)]}{2} = \begin{cases} \frac{1}{2}\cos(2\pi t), & t>0 \\ -\frac{1}{2}\cos(2\pi t), & t<0 \end{cases}$$



$$x(t) = x_o(t) + x_e(t)$$



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EE120 HW 1, P. 6

a) not BIBO stable $\int_0^{\infty} |e^{t}| dt \to \infty$

Let $x(t) = 1$.

then $y(t) = \int_{-\infty}^{\infty} h(l)\,x(t-l)\,dl$

$= \int_0^{\infty} e^{l}\,dl \to \infty$

b) BIBO stable $e \int_0^{t} |(t-1)^2 e^{-t}| dt < \infty$

$y(t) = \int_{-\infty}^{\infty} h(l)\,x(t-l)\,dl$

$|y(t)| \leq \int_{-\infty}^{\infty} |h(l)|\,|x(t-l)|\,dl$

$\leq B \cdot \int_{-\infty}^{\infty} |h(l)|\,dl$

$\leq B \int_0^{\infty} (l-1)^2 e^{(1-l)}\,dl$

$\leq B \int_{-1}^{\infty} s^2 e^{-s}\,ds$

which is finite

c) not BIBO stable since $\sum_{n=4}^{\infty} |u[n-4]| \to \infty$

Let $x[n] = 1$

then $y[n] = \sum_{K=-\infty}^{\infty} h[K]\,x[n-K]$

$= \sum_{K=-\infty}^{\infty} u[K-4]$

$= \sum_{K=4}^{\infty} 1 \to \infty$

d) not BIBO stable since $\cos 2\pi n\, u[n] = u[n]$, $\sum_{n=0}^{\infty} u[n] \to \infty$

Let $x[n] = 1$

then $y[n] = \sum_{K=-\infty}^{\infty} h[K]\,x[n-K]$

$= \sum_{K=-\infty}^{\infty} \cos[2\pi K]\,u[K]$

$= \sum_{K=-\infty}^{\infty} u[K]$

$= \sum_{K=0}^{\infty} 1 \to \infty$

e) not BIBO stable

Let $x(t) = 1$

then $y(t) = \int_{-\infty}^{\infty} h(l)\,x(t-l)\,dl$

$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(l-2n)\,dl$

$= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(l-2n)\,dl$

$= \sum_{n=-\infty}^{\infty} 1 \to \infty$

$\int |h(t)| dt = \int_{-\infty}^{\infty} \sum_n \delta(t-2n)\,dt$

$= \sum_{n=-\infty}^{\infty} \int_{2n-\epsilon}^{2n+\epsilon} \delta(t-2n)\,dt$

where $0 < \epsilon < 1$

$= \sum_{n=-\infty}^{\infty} 1 \to \infty$

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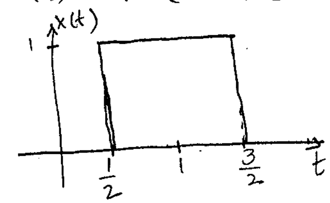
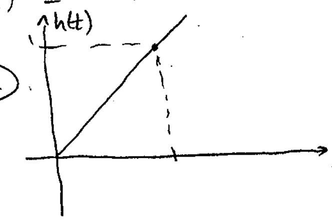
(7)

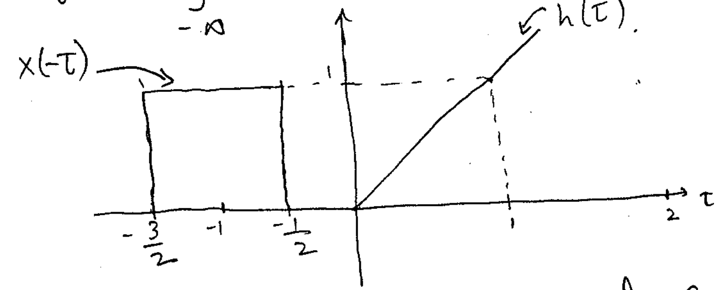
a) $x(t) = \Pi(t-1)$; $h(t) = r(t)$.



$*$

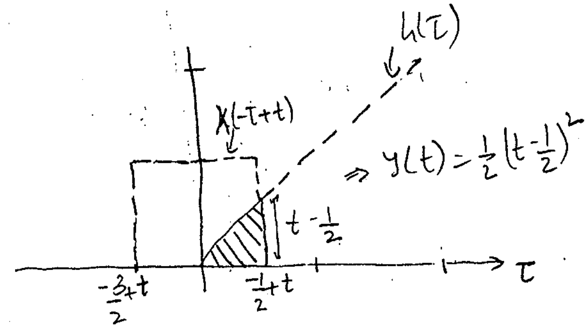


$y(t) = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau$



Shift $x(-\tau)$ by time t and calculate integral as $x(-\tau+t)$ slides through $h(\tau)$.

$\frac{1}{2} < t < \frac{3}{2}$:



$\Rightarrow y(t) = \frac{1}{2}\left(t-\frac{1}{2}\right)^2$

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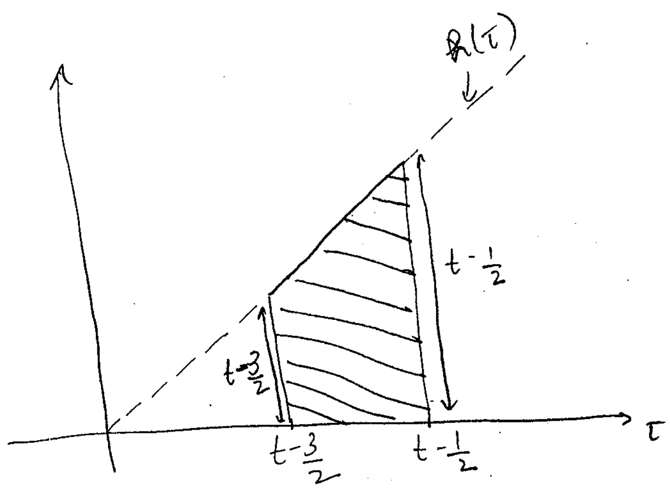
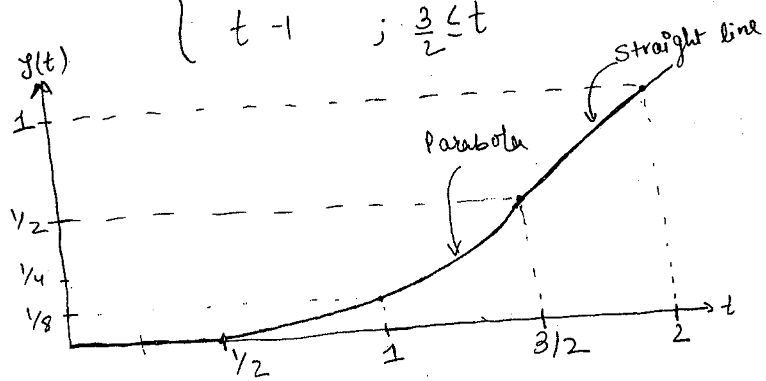
$\frac{3}{2} \le t$:

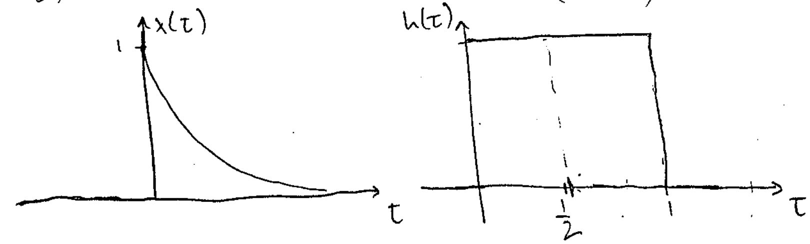


$$y(t) = \text{Area of shaded region}$$
$$= \frac{1}{2}\left[t-\frac{3}{2}+t-\frac{1}{2}\right] = t-1$$

$$\therefore\ y(t) = \begin{cases} 0 & ;\ t < 1/2 \\ \frac{1}{2}\left(t-\frac{1}{2}\right)^2 & ;\ \frac{1}{2} \le t < 3/2 \\ t-1 & ;\ \frac{3}{2} \le t \end{cases}$$



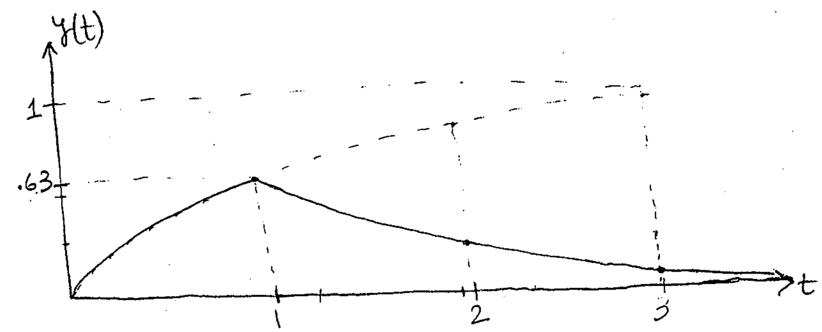
b) $x(t) = e^{-t}u(t)$; $h(t) = \Pi\left(t-\frac{1}{2}\right)$



$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(-\tau+t)\,d\tau$$

$0 \le t < 1$: $y(t) = \int_0^t e^{-\tau}\,d\tau = 1-e^{-t}$

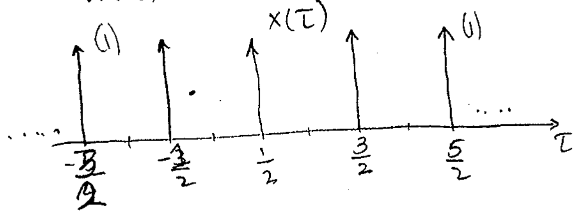
$1 \le t$: $y(t) = \int_{t-1}^{t} e^{-\tau}\,d\tau = -(e^{-t}-e\cdot e^{-t})$
$= (e-1)e^{-t}$

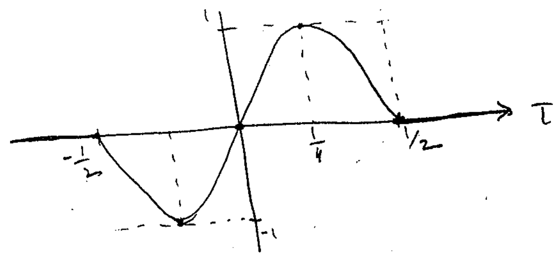


(24)

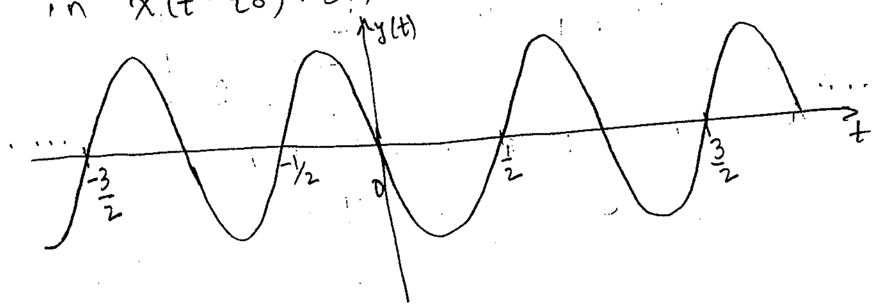
c) $x(t) = \sum_{n=-\infty}^{\infty} \delta(t - \frac{1}{2} - n)$

$h(t) = \Pi(t)\sin(2\pi t)$





First find $y(t)$ graphically. Convolving any signal $x(t)$ with $\delta(t-t_0)$ results in $x(t-t_0)$. So, we have $y(t)$:



$\therefore\ y(t) = -\sin(2\pi t)$

Now, solving algebraically:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(-\tau + t)\, d\tau$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta\left(\tau - \left(n + \tfrac{1}{2}\right)\right) \Pi(t-\tau) \sin\left(2\pi(t-\tau)\right) d\tau$$

$$= \sum_{n=-\infty}^{\infty} \Pi\left(t - \left(n+\tfrac{1}{2}\right)\right) \sin\left(2\pi\left(t - \left(n + \tfrac{1}{2}\right)\right)\right)$$

$$= -\sin(2\pi t)$$