EE119 Introduction to Optical Engineering
Fall 2009
Final Exam

Name: Solutions

SID: _____

CLOSED BOOK. THREE 8 1/2" X 11" SHEETS OF NOTES, AND SCIENTIFIC POCKET CALCULATOR PERMITTED.

TIME ALLOTTED: 180 MINUTES

Fundamental constants you might need:

Planck's constant, $h = 6.6261 \times 10^{-34}$ J-s

Boltzmann's constant, $k = 1.3807 \times 10^{-23}$ J/K

Permittivity of free space, $\varepsilon_o = 8.8542 \times 10^{-12}$ F/m

Permeability of free space, $\mu_o = 1.2566 \times 10^{-6}$ H/m

Speed of light in vacuum, $c = 2.9979 \times 10^{8}$ m/s

Electron charge, $e = 1.6022 \times 10^{-19}$ C

Free electron mass, $m_o = 9.1094 \times 10^{-31}$ kg

Electron volt, 1 eV = 1.6022×10^{-19} J

1 radian = 57.3 degrees

Total 145 points

16 students

1. **True or False. (30points, 2 points for each question)**

F A) A photograph taken with a lens aperture setting of f/4 and exposure time of 1/500 sec will have a larger depth of focus (DOF) than one taken with a lens aperture setting of f/8 and exposure time of 1/250 sec.

F B) When a beam of light passes from air into a material with index n=1.5, the wavelength becomes longer and the photon energy doesn't change.

F C) The exit pupil of an imaging system coincides with the second principle plane

T D) The number of longitudinal modes that will oscillate in a laser depends on whether the gain transition is homogeneously or inhomogeneously broadened.

F E) Light incident on a glass plate at Brewster's angle will have 100% transmission for s-polarization

F F) The retina is the region within your field of vision that is the center of sharp vision.

F G) Visual accommodation refers to the ability of the eye to adapt to differing light levels.

T H) In most people, the near point moves farther away as they age.

T I) In diode lasers, laser action takes place under forward bias.

T J) Cosmic rays and ambient radioactivity can generate dark current in a photomultiplier tube.

F K) Stimulated emission only occurs when a population inversion exists between the two atomic levels participating in the transition.

T L) The wavefront at the minimum beam waist for a Gaussian laser beam is always flat.

F M) Brewster's angle (measured from the surface normal) is always less than 57 degrees.

F N) If the work function of a metal is only 1 eV, then each 3 eV photon could eject up to 2 electrons from the surface.

T O) Gaussian optics applies to spherical surfaces only in the paraxial approximation.

2. **Short questions. (~~40~~ 20 points, 4 points for each question)**

A) Suppose that the objective lens of a standard visible light microscope has magnification of 15X and NA of 0.85. The eyepiece has magnification 5X. If a person with normal vision uses this microscope what will be the minimum feature size they can resolve?

B) A 10 cm long chamber with flat, parallel windows is placed in one arm of a Michelson interferometer illuminated with 633 nm laser light. The chamber is initially filled with air at standard pressure and temperature, with a refractive index of 1.00029. As all of the air is pumped out of the chamber, the fringes are recorded by a detector. How many light/dark fringe pairs will be counted?

C) Draw a simple diagram of a Galilean Telescope. Show lenses and the locations of their focal lengths.

D) You are detecting a light beam at 580 nm wavelength with an average power of 1 nanoWatts with 60% quantum efficiency. What is the shot-noise limited signal/noise ratio of the detector?

E) Explain in words how to find the entrance aperture in an optical imaging system.

A) Diffraction limit is $0.61\lambda/NA$

for $\lambda = 500$ nm (green light), $NA = 0.85$ limit is ~360 nm

But the eye can only resolve ~75 μm unaided.
The total magnification is $(15)\cdot(5) = 75$ so the
minimum resolvable by eye with magnification is ~1 μm

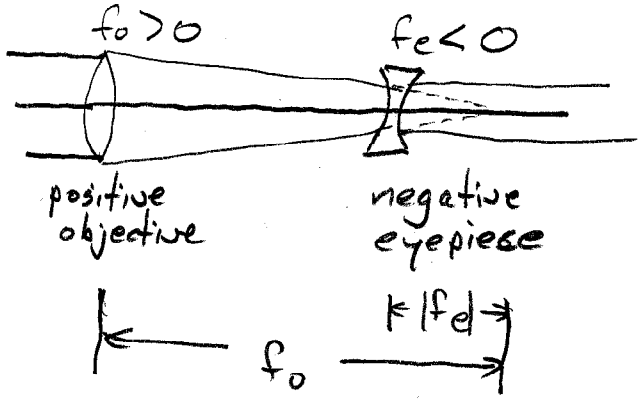
B) $\Delta = \Delta d = N(\lambda_0/2)$

$\Delta = n_{air}X - n_{vac}X$

$N = 2\Delta/\lambda_0 = [2(0.00029)\cdot(0.1\text{m})]/633\text{ nm}$

$= 91.6$

c)



$M = -\frac{f_o}{f_e}$

D) 580 nm → 2.14 eV

$1nW = 10^{-9} W = 10^{-9} J/s = 6.24 \times 10^{9}$ eV/sec

$= 2.92 \times 10^{9}$ ph/sec

QE is 60% so detected count rate is 1.75×10^{9} counts/sec

In 1 sec, shot noise is $\sqrt{1.75 \times 10^{9}} = 4.18 \times 10^{4}$

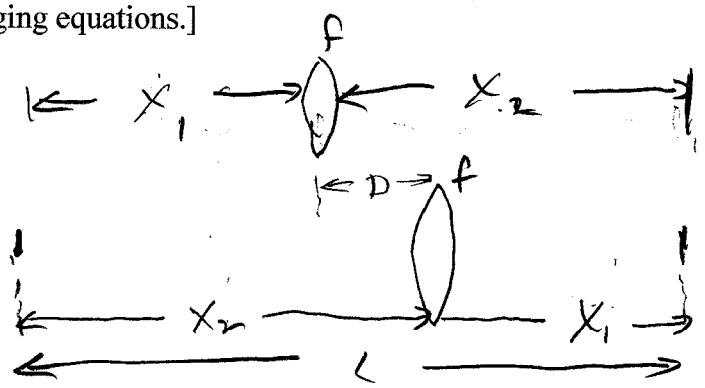
$S/N = \frac{1.75 \times 10^{9}}{4.18 \times 10^{4}} = \underline{4.18 \times 10^{4}}$

E) The entrance pupil is the image of the aperture as seen from the object side

3. **Bessel's method (20points).** As you studied in lab, Bessel's method is a convenient way to measure the focal length of a positive lens. For a pair of conjugate object and (real) image points, separated by a distance L > 4f, there will be two locations of the lens, a distance D apart, which image the same conjugate points. It is fairly easy to measure this distance D. Then the focal length f can be found by:

$$f = (L^2 - D^2)/4L$$

Prove this. [Hint: instead of deriving this expression starting from what you know about object and image distances, it might be easier to simply prove it is correct by substituting into the expression relations you can obtain about L and D from the imaging equations.]



With object distance defined as positive, the lens law is $\frac{1}{X_1} + \frac{1}{X_2} = \frac{1}{f}$

for conjugates X_1, X_2 as shown above

now, $L = X_1 + X_2$, $D = X_2 - X_1$

$$f = (L^2 - D^2)/4L$$

$$= \frac{(X_1+X_2)^2 - (X_2-X_1)^2}{4L} = \frac{4X_1X_2}{4(X_1+X_2)}$$

$$f = \frac{X_1X_2}{X_1+X_2}$$

then $\frac{1}{f} = \frac{X_1+X_2}{X_1X_2} = \frac{1}{X_1} + \frac{1}{X_2}$ lens law!

5 10

4. **Aberration. (30points)** The interferometer is widely used to test aberrations of an optical system or a component. Answer following questions.
   A) What kind of interferometer is in Figure 1?
   B) If you want to test a glass plate, where should you place it? Place the test plate in Fig1.
   C) If the glass plate is not in place, what should the interferogram look like? Choose from the bottom pictures. Assume all the components, except the glass plate, are free from aberrations, and they are in perfect alignment.
   D) If the glass plate is not in place and the M1 is slightly tilted in x-direction, what should the interferogram look like? Choose from the bottom pictures.
   E) Now, the glass is in place and the M1 is in its original position. If one of the surfaces of the glass plate has some small amount of cylindrical curvature, what should the interferogram look like? Choose from the bottom pictures.
   F) The glass plate is in place and the M1 is in its original position. If the glass plate has some astigmatism, what should the interferogram look like? Choose from the bottom pictures.

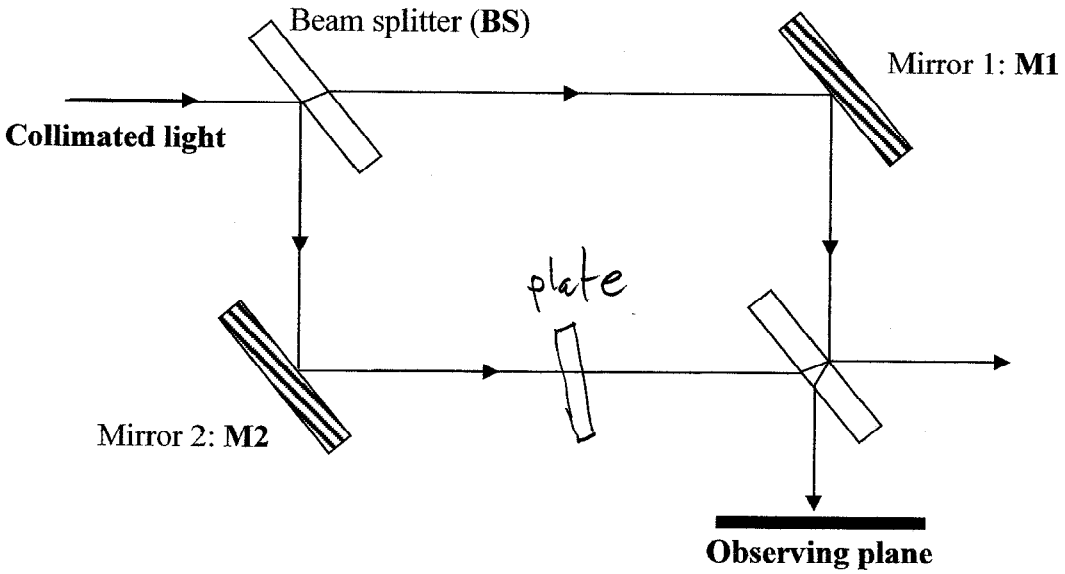
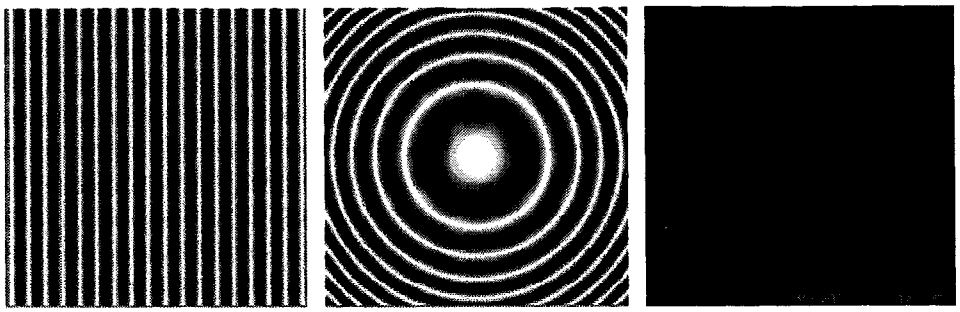
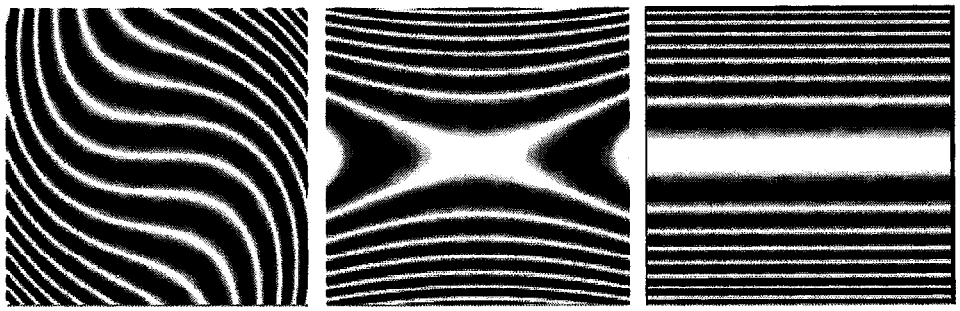


Fig. 1



(1) (2) (3)



(4) (5) (6)

A) Mach Zender

B) See diagram

C) 3

D) 1

E) 6

F) 5

6. **Fiber modes. (10 points)** An optical fiber has n1 = 1.42, n2 = 1.45. It is used with a laser at wavelength 1.35 microns. The core d = 8.5 microns.

A) Is this fiber single-mode for the given wavelength? If not, how many modes does it support?
B) What is the NA of the fiber?

$$\cos\theta_c = \frac{n_1}{n_2} \rightarrow \theta_c = 11.68^\circ$$

$$\sin\theta_m = (m+1)\frac{\lambda}{2n_2 d} \cong 0.2 \quad \text{for } \theta_c$$

$$\text{so} \quad m \leq (0.2)\frac{2n_2 d}{\lambda} - 1$$

$$m \leq 2.7$$

So $m = 0, 1, 2$ are guided → 3 modes

B) $$NA = (n_2^2 - n_1^2)^{1/2} = 0.29$$

7. **Laser cavity (20 points).** A TEM00 (Transverse Electric Mode) He-Ne laser (λ=632.8nm) has a cavity that is 0.45 m long, a fully reflecting mirror of radius R=5m (concave), and a flat output mirror.
   A) Determine the location of the beam waist in the cavity. Set z=0 at this location to be the reference plane.
   B) Determine the beam waist radius (w_0).
   C) Determine the beam spot size w(z) at the left and right cavity mirrors.
   D) Determine the half-angle beam divergence (θ) for this laser.

$R = 5m$ ($0.45m$ |

A) Waist is at the output mirror

B) $R(z) = z + z_R^2/z$ $\qquad z_R = \left[(R(z) - z)\, z\right]^{1/2}$

we have $R = 5m$ at $z = 0.45m$

$z_R = 1.43\ m = \dfrac{\pi w_0^2}{\lambda}$

$w_0 = \left(\dfrac{\lambda z_R}{\pi}\right)^{1/2} = 5.37 \times 10^{-4}\ m$

$= 0.537\ mm$

C) $w(z=0) = w_0 = 0.537\ mm$

$w(z=5m) = w_0\left[1 + \left(z/z_R\right)^2\right]^{1/2}$

$= 0.563\ mm$

D) $\theta_d = \dfrac{\lambda}{\pi w_0} = 3.75 \times 10^{-4}\ rad$

8. **Laser design. (20 points)** You are trying to build a new type of vertical cavity surface emitting laser operating at a beautiful violet color (λ_{vac}=444.4nm). It operates using a stack of thin semiconductor films to make the mirrors and the active medium, and emits light out of the top surface of the stack. Assume the material is a 4 level system with suitable energy levels and decay rates to allow population inversion to be accomplished. Given the following specs, why does the laser not work and how would you fix it? You must explain your answer fully.

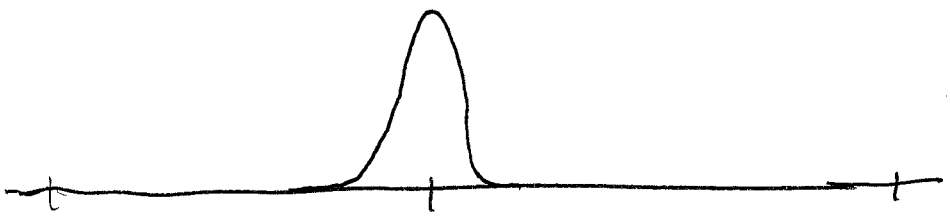
- Energy of laser transition: 2.7899 eV
- The gain medium has index n = 3.50
- The gain curve is homogeneously broadened with a Gaussian shape, with $g(\lambda) = g_0 * \exp[-(\lambda-\lambda_0)/\Delta\lambda]^2$
- The peak gain $g_0 = 0.1\ \mu m^{-1}$, centered at λ_0=444.40 nm with a bandwidth $\Delta\lambda$ of 0.6 nm.
- The physical cavity length is also equal to the gain length of 10μm
- Cavity loss (From output mirror): 10%

Mode spacing is $c/2nL = 4.2827$ THz

laser gain peaks at $\nu_0 = c/\lambda_0 = 674.595$ THz

gain bandwidth is

$$\Delta\nu_0 = \nu_0 \frac{\Delta\lambda}{\lambda} = 0.911 \text{ THz}$$



modes occur at $\nu_j = j\frac{c}{2nL}$

The modes do not overlap the gain curve!

Gain at closest mode (j=158) ~ 2.1 THz away is

$g = g_0 e^{-(2.1/0.9)^2} \approx 0.0004$ way too low

To fix, adjust the cavity length