

# EE119 Homework 9: LCDs and Lasers

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Due Friday, April 10, 2009

- The electro-optic characteristics for a TN-LCD and STN-LCD are given in the figure 1. Two vertical lines in each graph indicate the OFF- ON rms-operating voltages for 240:1 multiplexing. (This is a dual scan VGA display, so  $M = 240$  instead of 480.)

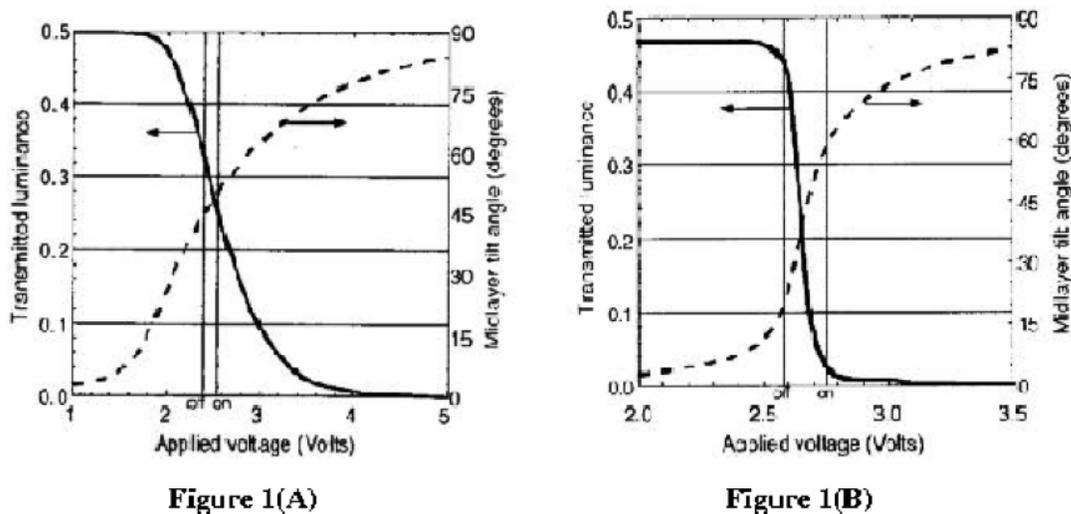


Figure 1: electro-optic characteristics for TN-LCD and STN-LCD

- Which graph corresponds to the characteristics of a TN-LCD? Why?

**Solution:**

Figure 1A corresponds to a TN-LCD because the luminance-voltage curve is less steep, reflecting the lesser sensitivity of the twisted nematic liquid crystal compared to the super-twisted nematic LCD.

- The Pixel Contrast Ratio is defined to be

$$PCR = \frac{L_{on} + (M - 1)L_{off}}{ML_{off}}$$

Where  $M$  is the number of display rows and  $L$  is the transmitted luminance. Calculate the PCR for the TN-LCD and STN-LCD displays, using the values shown on the graph in the figure. Explain any difference between the two values.

**Solution:**

For the TN-LCD, the PCR is

$$\frac{0.33 + 239 \times 0.25}{0.33} = 1.0013$$

$$\frac{0.43 + 239 \times 0.02}{0.02} = 1.085$$

The PCR for the STN-LCD is higher because the super-twisted liquid crystal is more sensitive to voltage changes.

2. A nematic liquid crystal has  $n_0=1.52$  and  $n_e=1.75$  at  $\lambda=577$  nm. Find the half-wave-plate thickness at this wavelength.

**Solution:**

The difference in optical path length between the two polarizations is  $\Delta nd$  where  $d$  is the thickness of the plate. We want this distance to equal half a wavelength, which is equivalent to requiring that

$$d = \frac{\lambda}{2\Delta n} = \frac{1}{2 * 0.23} = 1.25 \times 10^{-6} m$$

So the half-wave thickness is 1.25 microns.

3. Spontaneous and Stimulated Emission

- (a) [Hecht 13.16] Show that for a system of atoms and photons in equilibrium at a temperature  $T$  the ratio of the transition rates of stimulated to spontaneous emission is given by

$$\frac{1}{e^{h\nu/k_B T} - 1}$$

**Solution:**

When a system of atoms and photons is in equilibrium, the number of atoms in the ground state or any excited state is not changing. Let's consider a 2-level system, where the atom only has 1 accessible excited state, and where the only way in which an atom can go from the ground state to the excited state is through emission or absorption of radiation. We'll denote the number of atoms in the ground state as  $N_g$  and the number of atoms in the excited state as  $N_e$ . The energy difference between the excited and ground states is  $h\nu$ . The Maxwell-Boltzmann distribution tells us that, at thermal equilibrium, the ratio between the number of atoms in the ground and excited states is

$$\frac{N_e}{N_g} = e^{-h\nu/k_B T}$$

The only processes that change  $N_e$  and  $N_g$  are stimulated emission, spontaneous emission, and absorption. Let's call  $A$  the probability of spontaneous emission from the excited state to the ground state and let's let  $B_{12}$  be the probability of stimulated emission. From quantum mechanics (the argument involves time-reversal symmetry) we know that the probability of (stimulated) absorption is the same as the probability of stimulated emission, so  $B_{21} = B_{12}$ . Since absorption and stimulated emission require light to be present, we'll call  $\rho(\nu)$  the number of photons of frequency  $\nu$  that are interacting with the atoms. We can write the rate of change of the excited state population as

$$\frac{dN_e}{dt} = N_g B \rho(\nu) - N_e A - N_e B \rho(\nu) = 0$$

We can set this rate of change to equal zero because we are at thermal equilibrium. Dividing by  $N_g$  gives

$$B\rho(\nu) = \frac{N_e}{N_g}A - \frac{N_e}{N_g}B\rho(\nu) = e^{-h\nu/k_B T} [A - B\rho(\nu)]$$

Collecting terms,

$$B\rho(\nu)(1 + e^{-h\nu/k_B T}) = e^{-h\nu/k_B T} A$$

$$B\rho(\nu) = \frac{e^{-h\nu/k_B T} A}{1 + e^{-h\nu/k_B T}} = \frac{A}{1 + e^{h\nu/k_B T}}$$

The ratio of the rates of stimulated emission to spontaneous emission is

$$\frac{B\rho(\nu)}{A} = \frac{1}{1 + e^{h\nu/k_B T}}$$

- (b) [Hecht 13.17] A system of atoms in thermal equilibrium is emitting and absorbing photons with energy of 2 eV. Determine the ratio of the transition rates of stimulated emission to spontaneous emission at a temperature of 300 K. Discuss the implications of your answer. [Hint: See the previous problem.]

**Solution:**

Plugging these numbers into the expression of part (a), and using  $k_B T = 1/40$  at 300 K, gives

$$\frac{1}{1 + e^{2\text{eV}/1/40}} = \frac{1}{1 + e^{80}} = 1.8 \times 10^{-35}$$

This means that the overwhelming majority of photons that are emitted will be emitted by spontaneous emission. This is why you don't normally see stimulated emission, and why lasers are not at thermal equilibrium.

#### 4. Resonator cavities

A cavity, such as the space between two end mirrors on a laser, will allow light to resonate in it if an integer number of wavelengths of light fit into the cavity. The end mirrors of a laser are separated by 25 cm.

- (a) What is the spacing between longitudinal modes in the laser cavity?

**Solution:**

Light of wavelength  $\lambda$  will resonate in a cavity if an integer number of wavelengths will fit into a length of twice cavity; this sets up a resonant wave. This requirement is equal to

$$m\lambda = 2L = 2 \times 25 = 50\text{cm}$$

$$\lambda = \frac{c}{\nu} = \frac{50\text{cm}}{m}$$

This means that the resonant frequencies are given by

$$\nu = \frac{3 \times 10^8}{5 \times 10^{-1}} = 0.6 \times 10^9 = 6 \times 10^8$$

So the spacing between adjacent modes is 600 MHz.

- (b) The bandwidth of the gain curve is approximately 1500 Mhz. What is the maximum number of possible frequencies that can lase?

**Solution:**

1500/600=2.5, so you can fit 3 frequencies into the cavity.

- (c) If the length of the cavity were to decrease, how would your answers to (a) and (b) change?

**Solution:**

If the length of the cavity decreased, the spacing between modes would decrease, and then fewer modes would be allowable; It would not change the gain curve.

5. To make short pulses of laser light, the different modes in a cavity are fixed relative to one another so that the electric fields add constructively in one place and destructively everywhere else, giving a pulse of light that is as close to a delta function (in time) as possible. The lower limit on the duration of the pulse is the energy-time uncertainty principle, which states that  $\Delta E \Delta t \geq \hbar$  or, equivalently  $\Delta \nu \Delta t \geq 1/2\pi$ . If you want to make a pulse centered at 600 nm that is 10 femtoseconds long, what is the range of wavelengths at which your lasing medium must have gain?

**Solution:**

Since  $\Delta t$  is  $10 \times 10^{-15}$  seconds, we need a  $\Delta \nu$  of

$$\Delta \nu = \frac{1}{2 \times 3.14 \times 10^{-14}} = 1.6 \times 10^{13} \text{ Hz}$$

Our center frequency is  $\nu_0 = c/(6 \times 10^{-7}) = 5 \times 10^{14}$ , so we need lasing between  $4.92 \times 10^{14}$  Hz and  $5.08 \times 10^{14}$  Hz. These frequencies correspond to wavelengths of 620 nm and 581.4 nm. So we need lasing between 590 nm and 610 nm, or a bandwidth of about 20 nm.

6. [Hecht 13.22] Make a rough estimate of the amount of energy that can be delivered by a ruby laser whose crystal is 5.0mm in diameter and 0.050 m long. Assume the pulse of light lasts  $5.0 \times 10^{-6}$  s. The density of aluminum oxide ( $\text{Al}_2\text{O}_3$ ) is  $3.7 \times 10^3$  kg/m<sup>3</sup>. Use the data in the discussion of Fig. 13.6 and the fact that the chromium ions make a 1.79eV lasing transition. How much power is available per pulse?

You can assume that there is one  $\text{Cr}_2\text{O}_3$  unit for  $2 \times 10^4$  units of  $\text{Al}_2\text{O}_3$ .

**Solution:**

The crystal has volume of  $\pi \times 6.25 \times 10^{-6} \times 5 \times 10^{-2} = 98 \times 10^{-8} = 9.8 \times 10^{-7} \text{ m}^3$ . There are  $9.8 \times 10^{-7} \text{ m}^3 \times 3.7 \times 10^3 \text{ kg/m}^3 = 3.6$  grams of Aluminum Oxide in this crystal.

The concentration of Chromium Oxide is 0.05 % by mass, so there are  $1.8 \times 10^{-3}$  grams of  $\text{Cr}_2\text{O}_3$ . The atomic mass of  $\text{Cr}_2\text{O}_3$  is 152 amu, so there are  $1.8 \times 10^{-3} / 152 = 1.19 \times 10^{-5}$  moles of  $\text{Cr}_2\text{O}_3$  in the crystal. Multiplying by Avogadro's number gives  $1.19 \times 10^{-5} \times 6.022 \times 10^{23} = 7.17 \times 10^{18}$  molecules of  $\text{Cr}_2\text{O}_3$ .

Each molecule has 2 Chromium atoms, so there are  $1.4 \times 10^{19}$  Chromium atoms that could possibly contribute to a lasing transition at 1.79 eV. This gives a total maximum output energy of  $2.56 \times 10^{19} \text{ eV} = 4.1$  Joules. Since this energy would be emitted in 5 microseconds, the total power is  $4/5 \times 10^{-6} = 8.2 \times 10^5$  Watts = 0.8 MWatts.

This is the absolute maximum power you could get out of the crystal. In reality, the power will be a lot lower because we've made lots of unrealistic assumptions.