EE119 Introduction to Optical Engineering

Spring 2002

Final Exam

Name: Solutions

SID: ____________________

CLOSED BOOK. FOUR 8 1/2" X 11" SHEETS OF NOTES, AND SCIENTIFIC POCKET CALCULATOR PERMITTED.

TIME ALLOTTED: 180 MINUTES

Fundamental constants you might need:

Planck's constant, $h = 6.62 \times 10^{-34}$ J-s

Boltzmann's constant, $k = 1.38 \times 10^{-23}$ J/K

Permittivity of free space, $\varepsilon_o = 8.85 \times 10^{-12}$ F/m

Permeability of free space, $\mu_o = 1.26 \times 10^{-6}$ H/m

Speed of light in vacuum, $c = 2.998 \times 10^{8}$ m/s

Electron charge, $e = 1.6 \times 10^{-19}$ C

Free electron mass, $m_o = 9.1 \times 10^{-31}$ kg

Electron volt, 1 eV $= 1.6 \times 10^{-19}$ J

1 radian = 57.3 degrees

Total 200points

1. **True or False. (30points, 2 points for each question)**

T **A)** At Brewster's angle, the reflected light is fully polarized.

T **B)** A light beam that illuminates a perfectly reflecting surface exerts pressure upon that surface.

F **C)** A light beam inside a prism with n = 1.39, incident at an interface with air with an angle of incidence of 45 degrees will be totally reflected. $\theta_c = 46°$

T **D)** An object placed 10 cm in front of a +5 diopter lens will form a virtual image 20 cm in front of the lens.

F **E)** An achromatic doublet lens perfectly cancels chromatic aberrations over a specified wavelength range.

F **F)** The ratio of the exit numerical aperture to the entrance numerical aperture of an imaging system is equal to the transverse image magnification. $\frac{NA_{image}}{NA_{obj}} = \frac{1}{M}$

T **G)** The effective lens power for two thin lenses in contact is equal to the sum of the individual powers of the two lenses.

F **H)** The fovea is the region within your field of vision that is most sensitive to low light levels.

F **I)** Visual accommodation refers to the ability of the eye to adapt to differing light levels.

F **J)** The retina contains about six billion rod cells.

F **K)** Myopia is corrected using a positive lens. negative lens

T **L)** A telescope that has a combination of reflecting and refracting elements is called a **catadioptric** system.

T **M)** A singlet lens with perfect spherical surfaces made from glass with homogeneous index of refraction and having a circular aperture stop cannot have any coma aberration for an on-axis image point.

F **N)** In offset-reference holography, the reconstruction beam must illuminate the hologram plate at the same angle as the reference wave used to record the hologram.

F **O)** Stimulated emission only occurs when a population inversion exists between the two atomic levels participating in the transition.

2. **Short questions. (30points, 3 points for each question)**

   A) A Mach-Zender interferometer (λ=632nm) is set up with a sample of gas placed in one of the optical paths. The gas is in a thin-walled, glass cube with 2cm dimensions. As the gas is heated to a new temperature, a detector records the passing of 2000 fringes. How much has the index of refraction of the gas changed?
   B) Suppose that the objective lens of a telescope has a diameter of 35mm. How far apart (in light-years) must two stars be before they are theoretically resolvable by the lens? Assume monochromatic wavelength of 550nm. The stars are near the center of our galaxy, about 30,000 light-years away.
   C) A He-Ne laser with a flat output coupler generates an output beam with Gaussian beam radius w of 1 mm at the output. What is the Gaussian beam radius of the beam at a distance of 2 meters? (λ=633 nm)
   D) What causes dark current in a photomultiplier tube?
   E) Name four advantages of optical fibers compared to wires for transmitting information.
   F) Define the chief ray for a given object point in an imaging system.
   G) Define chromatic aberration.
   H) Define field curvature.
   I) Explain why people in their late 40s to early 50s who previously never required glasses start using "reading glasses".
   J) Describe the polarization of the following transverse, monochromatic waves:
      i) $\mathbf{E} = \mathbf{x}\sin(\omega t\text{-}kz) + \mathbf{y}\sin(\omega t\text{-}kz+180)$
      ii) $\mathbf{E} = \mathbf{x}\sin(\omega t\text{-}kz) + \mathbf{y}\sin(\omega t\text{-}kz+90)$

A) $m\lambda = \Delta n L \rightarrow \Delta n = m\lambda / L = \underline{0.0632}$

B) $\alpha_T = 5.5/CA(\text{inches}) = 3.99$ sec

$Y = 30{,}000 \cdot \sin(3.99\text{sec}) = 0.58$ light years ($\approx 5.6\times10^{15}$ m)

C) $W(z) = W_0\left[1+\left(\frac{z}{z_R}\right)^2\right]^{1/2} = W_0\left[1+\frac{z^2\lambda^2}{\pi^2 W_0^4}\right]^{1/2} = 1.73$ mm

D) Thermal emission of electrons from cathode + dynodes

E) No crosstalk, high bandwidth, low loss, no EM interference, small, light + flexible.

F) Ray that passes through center of stop + pupils

G) Variation of lens focus with changes in optical wavelength.

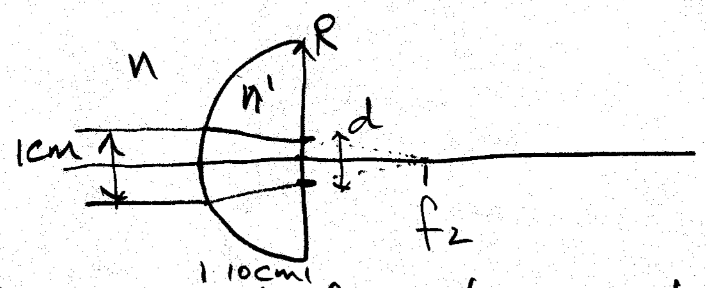
H) Imagepoints lie on a curved surface

I) Ability to accomodate is reduced, resulting in the near point moving too far away for comfortable reading

J) i) linear polarization, 45° to x, y axes

ii) right circular polarization

3. **Fun with lenses. (15 points)** A solid glass hemisphere is placed with its flat face down on a tabletop. The hemisphere has a radius of 10cm and a refractive index of 1.5. It is lit from above by a collimated beam 1cm in diameter centered on the hemisphere axis. What is the diameter of the circle of light on the table? [Hint: You can use the paraxial approximation.]



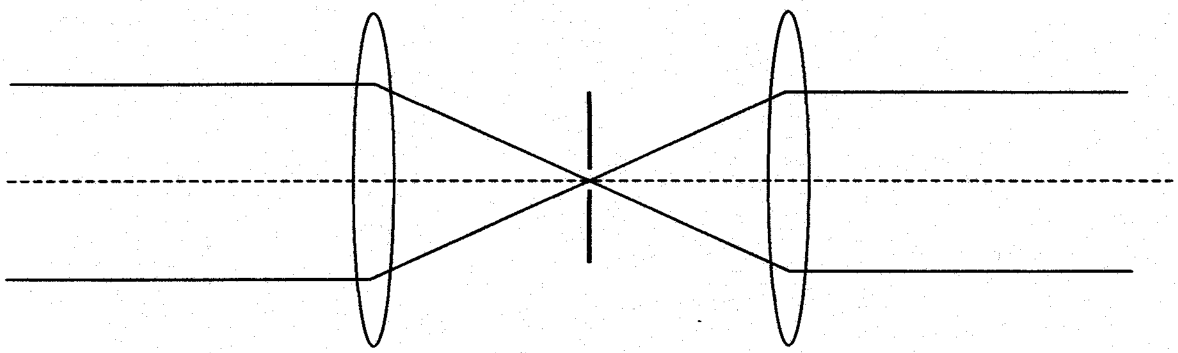
Back focal length for the single surface is:

$$f_2 = \frac{n'}{n'-n} R = \frac{1.5}{1.5-1.0} \cdot 10\,cm = 30\,cm$$

At a distance of 10 cm from the surface the beam diameter would be $\frac{2}{3}$ of its diameter at the surface

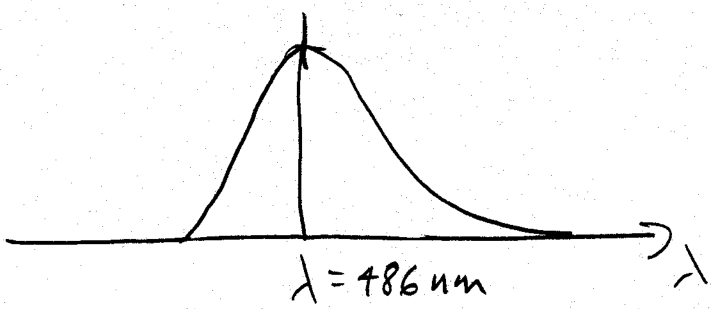
$$d = \frac{2}{3}\,cm$$

4. **Lens-based color filter. (20points)** Consider the arrangement of lenses and a small pinhole shown in the figure below. Lens 1 is made with Dense Flint glass, and Lens 2 is made with Spectacle Crown glass. The indices for these two glasses are indicated in the table. Both lenses are designed to have equal focal lengths at the 'F' wavelength (486 nm), they are separated by twice this focal length, and the pinhole is placed on the optical axis, halfway between the two lenses.



Glass type	n_C (656 nm)	n_D (589 nm)	n_F (486 nm)	n_G (434 nm)
Spectacle Crown	1.52042	1.52300	1.52930	1.53435
Dense Flint	1.64357	1.64900	1.66200	1.67456

For collimated white light, incident from the left on Lens 1, sketch the spectrum of the light that emerges from lens 2. Explain in words how the device works.



At 486 nm the light is focused on the pinhole and the maximum amount of light passes through and is collected by lens 2. As wavelength increases or decreases from 486 nm, chromatic aberration in lens 1 causes the focal spot size on the pinhole to increase, so less light passes through the pinhole. The spectrum is asymmetric because dispersion increases towards shorter λ.

5. **Illuminator. (15points)** List three functions of the illuminator optics in a projection system.

1. Uniformly illuminate the object.
2. Collect the maximum amount of light from the source
3. Redirect the light from the source into the entrance pupil of the projection optics.

6. **Camera lens (30 points).** A "telephoto" lens system for a camera consists of a convex lens of focal length12cm placed 8cm in front of a concave lens of focal length −6cm. The camera is focused on an object 6 meters in front of the convex lens.

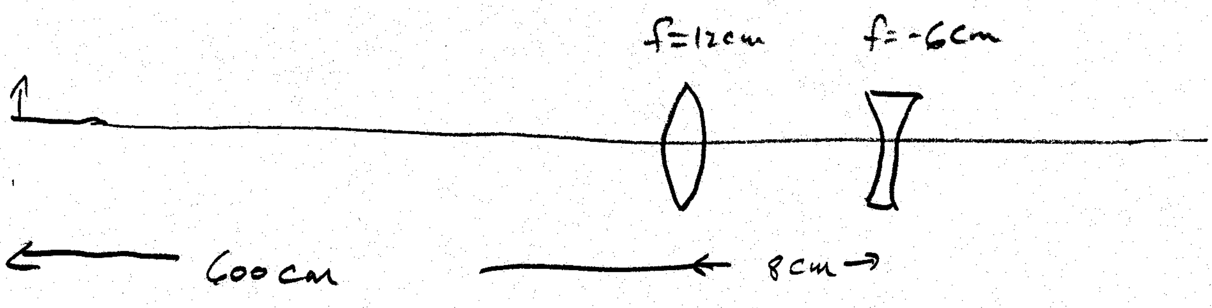
8 A) What will be the total length of the camera from convex lens to the photographic film when the image of the object is in focus.

8 B) What magnification is achieved?

Now, a single thin converging lens replaces this lens system. The focal length of this lens being so chosen that the same magnification results when the distance from the object to the lens remains at 6 meters.

7 C) What is the focal length required?

7 D) What will now be the total length of the camera when the image is in focus on the plate?



A) first image from convex lens $\frac{1}{d_2} = \frac{1}{12} - \frac{1}{600} \rightarrow d_2 = 12.245\text{cm}$

$M_1 = -0.0204$

image formed by concave lens: $d_1' = +4.245\text{ cm}$

$\frac{1}{d_2'} = -\frac{1}{6} + \frac{1}{4.245} \rightarrow d_2' = 14.513$

$M_2 = 3.419$

Camera length is $8 + 14.513 = 22.513\text{ cm}$

B) $M_{TOT} = M_1 \cdot M_2 = -0.0698$

c) $M = -0.0698 = \frac{d_2}{d_1} = \frac{d_2}{-600} \rightarrow d_2 = 41.85\text{ cm}$

$\frac{1}{f} = \frac{1}{d_2} - \frac{1}{d_1} = \frac{1}{41.85} - \frac{1}{-600} \rightarrow f = 39.12\text{ cm}$

D) Camera length is $d_2 = 41.85\text{ cm}$

7. **Lasers. (30 points)**

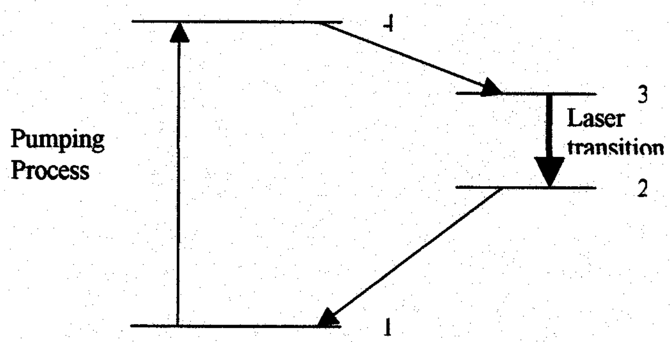
A) Calculate the longitudinal mode spacing for a He-Ne laser operating at 632 nm with 25 cm spacing between the mirrors. Assume n = 1 inside the He-Ne gas.

B) Suppose the laser is operating in a single longitudinal mode with a bandwidth of 50 MHz. What is the corresponding linewidth (in wavelength)? Give your answer in nm.

C) Consider the energy level diagram for a 4-level laser shown below. If N_1, N_2, N_3, N_4 represent the number densities of atoms in levels 1, 2, 3, and 4, respectively, state the condition for population inversion on the laser transition.

D) Write a differential equation for the population in level 3 that describes spontaneous emission from level 4 to level 3, and spontaneous and stimulated emission from level 3 to level 2.

E) Discuss gain saturation in the context of these energy levels.



A) $\Delta\nu = \frac{c}{2L} = \frac{3\times10^{10}\text{ cm/s}}{50\text{ cm}} = \underline{600\text{ MHz}}$

B) $\lambda = \frac{c}{\nu} \rightarrow \Delta\lambda = -\frac{c}{\nu^2}\Delta\nu = -\frac{\lambda^2}{c}\Delta\nu = 6.7\times10^{-14}\text{ m}$

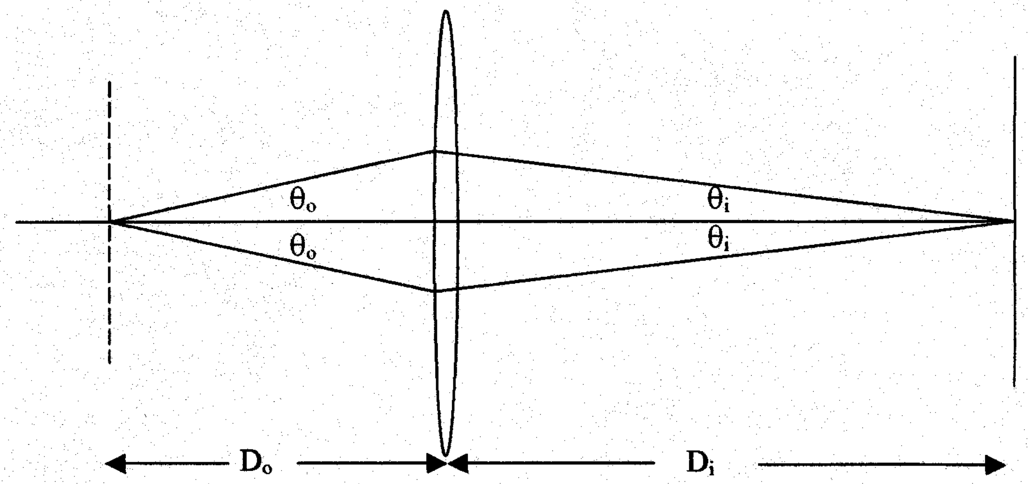
$= \underline{6.7\times10^{-5}\text{ nm}}$

c) $N_3 > N_2$

D) $$\frac{dN_3}{dt} = A_{43}N_4 - A_{32}N_3 - B_{32}(N_3 - N_2)I$$

E) Gain is proportional to $N_3 - N_2$. As laser intensity, I, increases, stimulated emission reduces N_3 and increases N_2. When the gain exactly equals the cavity losses, steady state is reached.

8. **Interference imaging. (30 points)** Consider a sinusoidal transmission grating with period d_g illuminated by a plane wave traveling parallel to the optic axis. The diagram below shows the ray directions for the ±1 plane wave diffraction orders at angles of $\pm\theta_o$. A positive lens with focal length f is placed a distance D_o behind the grating ($f < D_o$). The ray directions for the two diffracted orders on the image side are $\pm\theta_i$. A screen is placed at the image plane distance D_i after the lens, such that an image of the grating is formed.



A) Find θ_o in terms of d_g and λ.
B) Now find θ_i in terms of θ_o and M, the transverse image magnification, and then in terms of M, d_g and λ. ~~Assume the paraxial approximation.~~
C) Now calculate the period of the interference pattern formed on the screen due to the interference of the two plane waves traveling at $\pm\theta_i$.
D) Finally, if we simply consider this a standard imaging system, what would be the period of the image grating on the screen in terms of the object grating period and M?
E) Is the period of the interference pattern you calculated in C the same as the period of the image grating you calculated in D? F) Can you give an explanation for this result? (extra credit)

A) $\lambda = d_g \sin\theta_o \quad \therefore \quad \theta_o = \sin^{-1}\frac{\lambda}{d_g}$

B) $M = \frac{\sin\theta_o}{\sin\theta_i} \quad \theta_i = \sin^{-1}\left(\frac{\sin\theta_o}{M}\right) = \sin^{-1}\left(\frac{\lambda}{Md_g}\right)$

C) $d_i = \frac{\lambda}{2\sin\theta_i} = \frac{\lambda}{2(\lambda/Md_g)} = \frac{Md_g}{2}$

D) $d_i = Md_g$

E) No, they are different.

F) The image is actually due to interference of 0 and ±1 orders.