EE119 Introduction to Optical Engineering

Spring 2003

Final Exam

Name: Solutions

SID: ______________________

CLOSED BOOK. THREE 8 1/2" X 11" SHEETS OF NOTES, AND SCIENTIFIC POCKET CALCULATOR PERMITTED.

TIME ALLOTTED: 180 MINUTES

Fundamental constants you might need:

Planck's constant, $h = 6.62 \times 10^{-34}$ J-s

Boltzmann's constant, $k = 1.38 \times 10^{-23}$ J/K

Permittivity of free space, $\varepsilon_o = 8.85 \times 10^{-12}$ F/m

Permeability of free space, $\mu_o = 1.26 \times 10^{-6}$ H/m

Speed of light in vacuum, $c = 2.998 \times 10^{8}$ m/s

Electron charge, $e = 1.6 \times 10^{-19}$ C

Free electron mass, $m_o = 9.1 \times 10^{-31}$ kg

Electron volt, 1 eV $= 1.6 \times 10^{-19}$ J

1 radian = 57.3 degrees

Total 200points

1. **True or False. (30points, 2 points for each question)**

F A) For normal dispersion, the index of refraction is smaller for blue light than for red light.

T B) 12,000Å = 1.20 μm

T C) For a polarizing prism, such as the Nicol or Glan Air Prism, the transmitted component is p-polarized, and the reflected component is s-polarized.

F D) The chief ray always passes through the exact center of the exit window of an imaging system.

T E) For a light beam of given wavelength and average power, the signal/noise ratio due to shot noise from a PIN photodiode will increase as the quantum efficiency increases.

F F) In contrast to a photomultiplier tube, a channel electron multiplier can be operated in air.

F G) A camera exposure setting with shutter speed of 1/256 sec and aperture setting f/4 gives an equivalent film exposure to 1/64 sec shutter speed and f/2 aperture setting.

F H) The minimum gain required for a laser to reach threshold and begin to oscillate depends on whether the laser transition is homogeneously or inhomogeneously broadened.

T I) In myopia, the power of the eye is too large, and the sufferer cannot focus at infinity.

T J) Right-hand circularly polarized light is incident on a linear polarizer. 50% of the incident power is transmitted.

F K) Light incident on a glass plate at Brewster's angle will have 100% transmission for s-polarization.

T L) Hyperopia is corrected using a positive lens.

F M) The main function of the illuminator in a projection system is to make sure that the image is sharp.

F N) A CCD chip consists of an array of p-i-n diodes.

T O) In a photodiode, only photons that are absorbed within the diode depletion region contribute to the photocurrent. (Assume a depletion approximation model.)

2. **Short questions. (30points, 3 points for each question)**
   A) Write down mathematical expressions for a plane wave and a spherical wave.
   B) Write down a mathematical expression for the transverse intensity variation of a Gaussian laser beam. Give a definition in words for each of the parameters that you use.
   C) When light passes through two polarizers whose axes of transmission are parallel, a photodetector reads 30units. If the second polarizer is then turned through 30degrees, what will the detector read?
   D) A HeNe laser, operating at 633 nm wavelength, has a cavity length of 20 cm. The laser is operating in a single longitudinal mode. If the cavity length is increased by 2%, what will be the fractional change in the output wavelength and in which direction (shorter or longer wavelength)?
   E) Give a brief description of laser "Q-switching".
   F) How many wavelengths wide must a single slit be if the first Fraunhofer diffraction minimum occurs at an angular distance of 30 degrees from the optic axis?
   G) Write down one advantage of PIN photodetector over a simple pn junction detector.
   H) Eddy's father has worn glasses for years. As he is getting older he has trouble reading, and he has a habit of peeking under his glasses to read better. What was his original problem (near-sighted or far-sighted), and are his old glasses made with positive or negative lenses?
   I) State the gain threshold condition for steady-state laser oscillation in words.
   J) Give the definition for a "marginal ray" in an optical imaging system.

A) plane wave: $\vec{E}(\vec{r}) = \vec{E}_0 \exp[-i(\vec{k}\cdot\vec{r} - \omega t)]$

spherical wave: $\vec{E}(\vec{r}) = \vec{E}_0 \frac{1}{r} \exp[-i(\vec{k}\cdot\vec{r} - \omega t)]$

B) $I(r) = I_0 \exp[-2r^2/\omega^2]$

r: radial distance ; I_0: peak intensity on axis;
ω: beam radius

c) $30 \cdot \cos^2(30°) = 22.5$ units

D) 2% longer wave length

E) A light modulator is inserted in the laser cavity. With the modulator set for low transmission the laser cannot lase, so population builds up to a very high level in the upper laser state. Then the modulator switches to high transmission. Now the laser is way above threshold. Laser action builds up rapidly releasing the stored energy in a short, intense pulse.

F) $\sin\Theta = \frac{\lambda}{d}$. $d = \frac{\lambda}{\sin 30^\circ} = \frac{\lambda}{1/2} = 2\lambda$

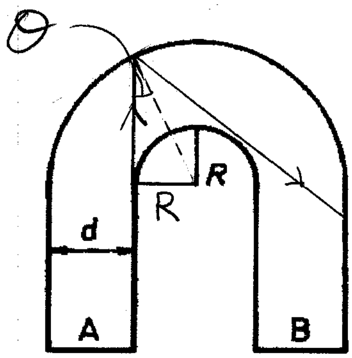
G) i-layer can be made thick compared to depletion layer. Increased thickness causes more light to be absorbed in the active region, increasing detection efficiency

H) Near-sighted. Negative lens

I) round trip gain is equal to round trip losses

J) Rays that pass right at the edges of the pupils and aperture stop

3. **Light guide. (15 points)** A glass rod of rectangular cross-section is bent into the shape shown as the figure below. A parallel beam of light falls perpendicularly on the flat surface A. Determine the minimum values of the ratio R/d for which all light entering the glass through surface A will emerge from the glass through surface B. The index of refraction of the glass is 1.5.



We need to consider the limiting ray that passes along the inside edge as shown.

From the geometry, $\sin\theta = \frac{R}{d+R}$

The critical angle is $\sin\theta_c = \frac{1}{1.5} = 2/3$

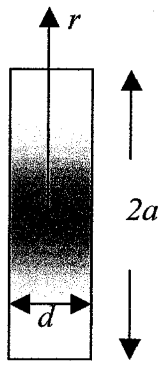
So we require

$$\frac{R}{d+R} \geq \frac{2}{3}$$

$$2(d+R) < 3R$$

$$\frac{R}{d} \geq 2$$

4. **GRIN lens. (20 points)** A gradient index lens, shown below, is made from a glass disk of constant thickness in which there is an index of refraction variation in the radial direction, *n(r)*. Given a disk of radius *a*, and thickness *d*, find the radial variation of the index that will produce the equivalent of a conventional lens of focal length, *f*. [Hints: Assume a suitable thin-lens approximation. Try to duplicate the thin-lens phase shift derived in class for a conventional lens in the paraxial approximation.]



We want to mimic the thin-lens phase shift, which is:

$$\phi(r) = \phi_0 - \frac{2\pi}{\lambda}\frac{r^2}{2f}$$

The phase shift passing through the GRIN lens is:

$$\phi(r) = \frac{2\pi}{\lambda} n(r) d$$

So we want $n(r) = n_0 - \frac{r^2}{2fd}$

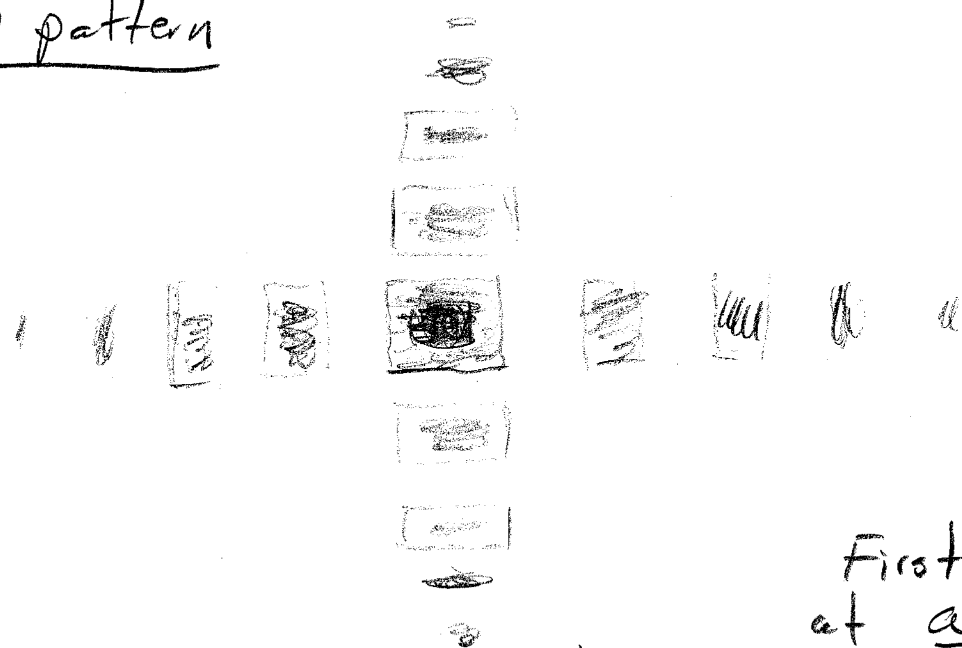
The value of n_0 just affects the overall constant phase shift and can be chosen for convenience.

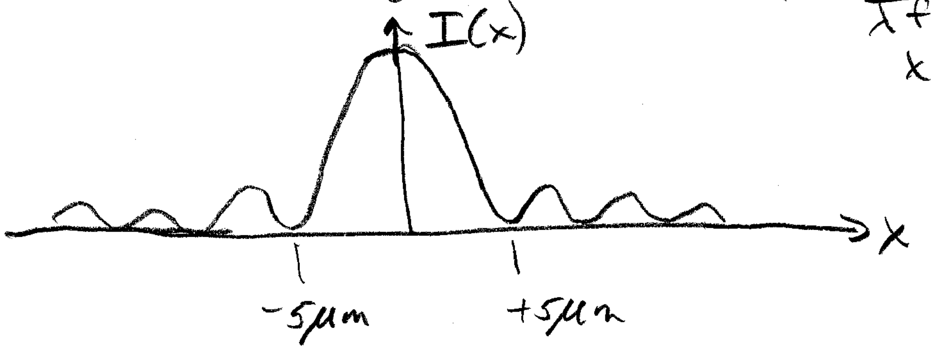
5. **Diffraction theory of a lens (15 points)** Consider a 100 mm focal length lens with a 1 cm square aperture (instead of a circular one). For a plane wave incident on this lens with 500 nm wavelength, sketch the intensity pattern at the focus. Show two sketches, one a 2D sketch of the pattern, and the second being a plot of intensity vs. position along a horizontal line through the center of the pattern. Give the dimensions of the central lobe of the pattern.

We get the Fraunhofer diffraction pattern from a square aperture

$$I(z=f) = I_0 \, \mathrm{sinc}^2\left(\frac{ax}{\lambda f}\right) \mathrm{sinc}^2\left(\frac{ay}{\lambda f}\right) \qquad a = 1\,\mathrm{cm}$$

<u>2D pattern</u>





First null occurs at $\frac{ax}{\lambda f} = 1$

$$x = \frac{\lambda f}{a} = 5\,\mu m$$

Full width of the central lobe is

10μm x 10μm

6. **Optical fibers. (15points)** An optical fiber has cladding index n1 = 1.42, and core index n2 = 1.45. It is used with a laser at wavelength 1.3 μm. The core d = 8.5 μm.

A) Is this fiber single-mode for the given wavelength? If not, how many modes does it support? [5 points]
B) What is the maximum d for single mode operation at this wavelength? [5 points]
C) For the given d = 8.5 μm, what is the minimum wavelength for single mode operation? [5 points]

A) $\cos\theta_c = \frac{n_1}{n_2} \rightarrow \theta_c = 11.68°$

$\sin\theta_m = (m+1)\frac{\lambda}{2n_2 d} \cong 0.2$

$m = \frac{(0.2)\cdot 2n_2 d}{\lambda} - 1$

$= 2.84$

So $m = 0, 1, 2$ are guided → 3 modes

B) we need $m < 1$

$\frac{(0.2)2n_2 d}{\lambda} - 1 < 1$

$d < \frac{\lambda}{0.2 n_2} = 4.48 \mu m$

c) $\lambda > 0.2 n_2 d$

$\lambda > 2.465 \mu m$

7. **Microscope (15 points).** A compound microscope has magnification of 600 (viewing with a fully relaxed eye). The microscope has standardized tube-length, and a 100X objective lens with NA=0.75.
   A) What is the focal length of the objective lens? [5 points]
   B) What is the focal length of the eyepiece lens? [5 points]
   C) Would you say that the magnification is excessive? Justify your answer. [5 points]

A) $f_o = \frac{16}{M_o} = \frac{16}{100} = 0.16 \text{ cm} = \boxed{1.6 \text{ mm}}$

B) $f_e = \frac{25}{M_e}$ $\quad M_e = 6$

$f_e = \boxed{4.17 \text{ cm}}$

c) $M_{max} = 240 \, NA = 180$

So the magnification of 600 is excessive.

8. **Thin film interference. (10points)**
   A) Design an antireflection coating for a normal incident light of 633nm wavelength on a glass substrate with the refractive index of 1.5. Specify the thickness and refractive index of the coating material. [5 points]
   B) A thin film of ethanol (n = 1.36) that is spread on a flat glass plate and illuminated with white light shows a color pattern in reflection. One region in the film strongly reflects green light (500 nm). What is the thickness of the film at that point? [5 points]

A) $nd = \lambda_0/4$

$n = \sqrt{n_g} = \boxed{1.225}$ $d = \frac{\lambda_0}{4n} = \boxed{129.2\ nm}$

B) In this case $\delta = \frac{4\pi n_f d}{\lambda_0}$

assuming normal incidence.

There is no "extra" relative π phase shift arising from the refections since at both the air-ethanol interface and the ethanol-glass interface, the light goes from lower index to higher index. So the condition for constructive interference is $\delta = 2m\pi$; $m = 1, 2, \ldots$

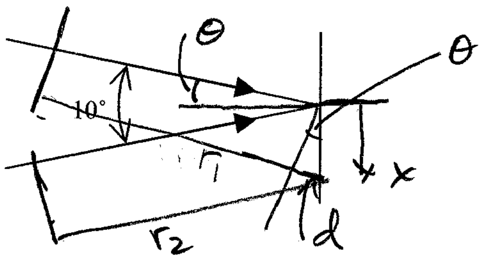
The minimum thickness film is then

$$\frac{4\pi n_f d}{\lambda_0} = 2\pi$$

$$d = \frac{\lambda_0}{2n_f} = \boxed{183.8\ nm}$$

9. **Interference and interferometry. (20 points)**

A) The beam from a ruby laser emitting red light of wavelength 694.3 nm is used with a beamsplitter to produce two coherent beams. Both are reflected from plane mirrors and brought together on the same photographic plate. If the angle between the two beams is 10° and the plate normal bisects this angle, find the fringe separation of the interference fringes on the plate. [10 points]



$\sin\Theta = \frac{x}{d}$ $\Theta = 5°$

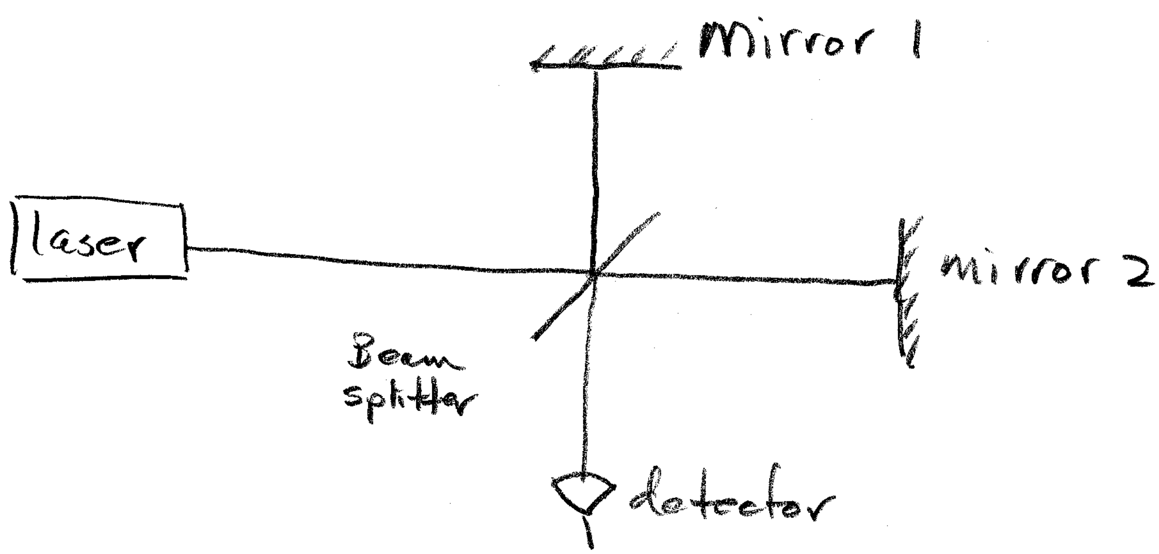
The net path difference $(r_1 - r_2)$ at point x is $2d = \frac{2x}{\sin\Theta}$

The fringe pattern has the form:

$I(x) = 2I_0 + 2I_0 \cos\frac{2\pi}{\lambda}(r_2 - r_1) = 2I_0 + 2I_0\cos\left(\frac{4\pi x}{\lambda \sin\Theta}\right)$

Fringe spacing is $x = \frac{\lambda}{2\sin\Theta} = 3983\,nm = 3.983\,\mu m$

B) Draw a diagram showing a Michelson interferometer. Be sure to clearly label all of the critical components. [10 points]



10. **Laser beams (30 points)** A laser cavity is 125 cm long. The beam waist occurs at the laser output with radius w_o=1 mm. It is a lowest order Gaussian beam. The reflectivity of the back mirror is R_1=100% and the reflectivity of the output mirror R_2 = 95%. The laser wavelength is λ= 532 nm.
   A) What is the radius of curvature of the output mirror? [5 points]
   B) What is the Rayleigh range, z_R of the beam? [5 points]
   C) At what distance from the laser output mirror is the Gaussian spot radius equal to 1 cm? [5 points]
   D) What is the radius of curvature of the wavefront of the beam at a distance of 1, mm, 1 m and 100 m from the output mirror? [5 points]
   E) A 100 mm focal length lens is placed at a distance of 10m from the output of the laser. Assume the lens diameter is large enough to capture the entire beam. At what distance from the lens will the focused beam waist occur, and what will be the value of the Gaussian beam radius at the focus? [10 points]

A) $R = \infty$

B) $z_R = \frac{\pi \omega_o^2}{\lambda} = \boxed{5.905 \text{ m}}$

C) $\omega(z) = \omega_o \left[1 + \left(z/z_R\right)^2\right]^{1/2}$

$\frac{\omega(z)}{\omega_o} = 10 = \left[1 + \left(z/z_R\right)^2\right]^{1/2}$

$\left(\frac{z}{z_R}\right)^2 = 99$

$z = z_R\sqrt{99} = \boxed{58.75 \text{ m}}$

D) $R(z) = z + z_R^2/z$

$z = 1\text{mm}$ $R = \boxed{34.87 \text{ Km!}}$

$z = 1\text{m}$ $R = \boxed{35.87 \text{ m}}$

$z = 100\text{ m}$ $R = \boxed{100.35 \text{ m}}$

E) At $z = 10$ m, $R_i = 13.487$ m; $\omega_i = 1.967$ mm

by lens law $\frac{1}{R_o} = \frac{1}{f} - \frac{1}{R_i} \Rightarrow R_o = 10.075$ cm

→ con't on next page

Approximate distance to the waist as equal to $R_0 = \boxed{10.075\,cm}$

Approximate new waist as

$$\omega(z) = \omega_0\left[1+\left(\frac{z\lambda}{\pi\omega_0^2}\right)^2\right]^{1/2}$$

$$\cong \omega_0 \frac{z\lambda}{\pi\omega_0^2}$$

$$\omega(z) \cong \frac{z\lambda}{\pi\omega_0}$$

$$\omega_0 \cong \frac{z\lambda}{\pi\omega_i} \cong \frac{R_0\lambda}{\pi\omega_i} = \boxed{8.67\,\mu m}$$

Now check $z_R = \frac{\pi\omega_0^2}{\lambda} = 0.44\,mm$

Since $z \cong 10.075\,cm$, the approximations are quite accurate.