EE105
Microelectronic Devices and Circuits

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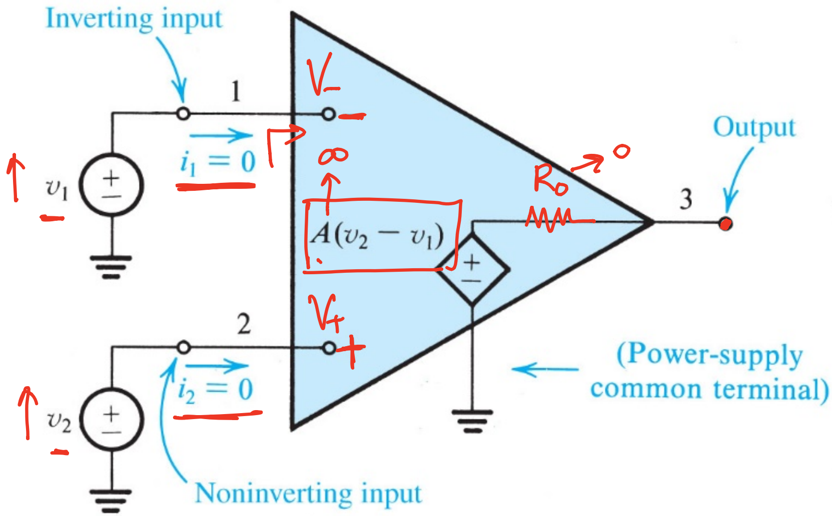
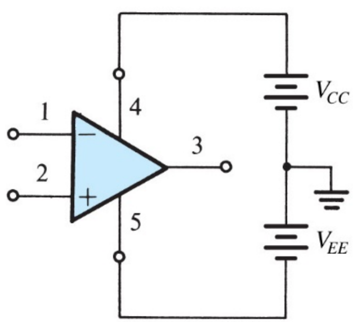
Ideal Op Amp



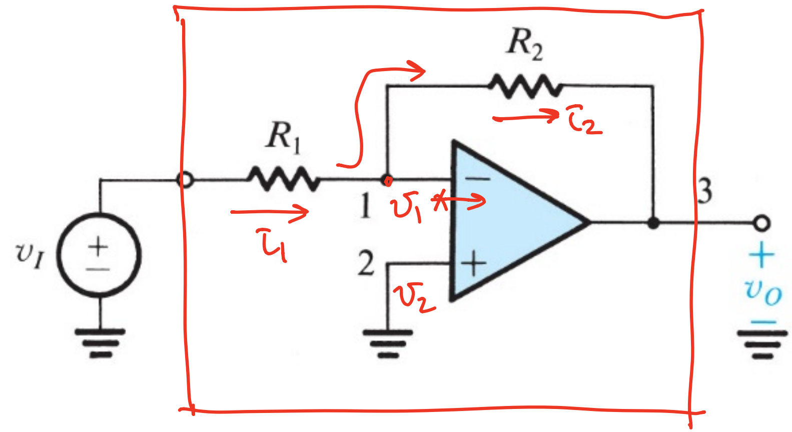
Golden Rules:

- Infinite open-loop gain, $A = \infty$
- Infinite input impedance
  - No current goes in
- Zero output impedance
- $V_- = V_+$ with feedback circuit
- Infinite bandwidth
- Infinite common-mode rejection

Op-Amp with dc bias



Inverting Amplifier



Ideal Op Amp

$v_1 = v_2 = 0$

$$v_o = v_1 - i_2 R_2$$
$$= 0 - \frac{v_I - 0}{R_1} \cdot R_2$$
$$= v_I \cdot \left(-\frac{R_2}{R_1}\right)$$

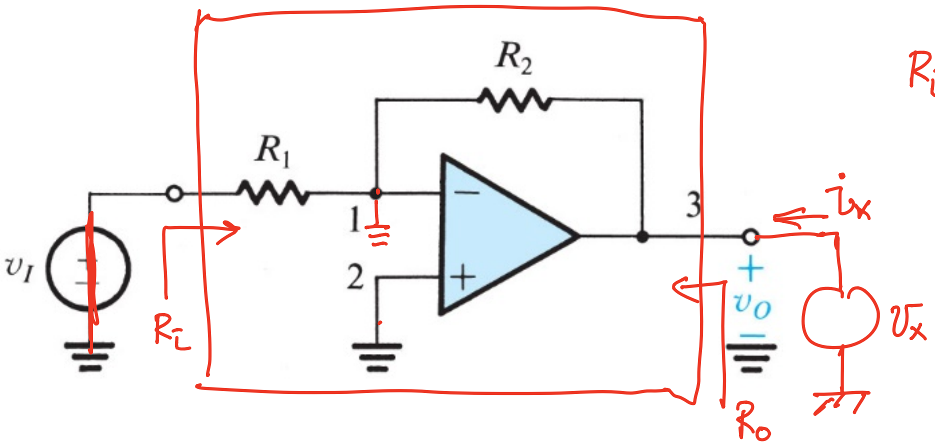
$$G = \frac{v_o}{v_I} = -\frac{R_2}{R_1}$$

Gain is determined by external circuit elements

Inverting Amplifier: Input and Output Resistances

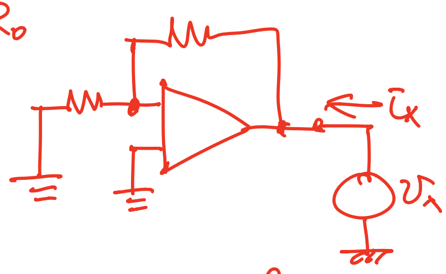
R_i

R_o



$R_i = R_1$

R_o



$v_1 = v_2$ ideal Op Amp

$$v_o = v_x = A\,(v_1 - v_2) = 0$$

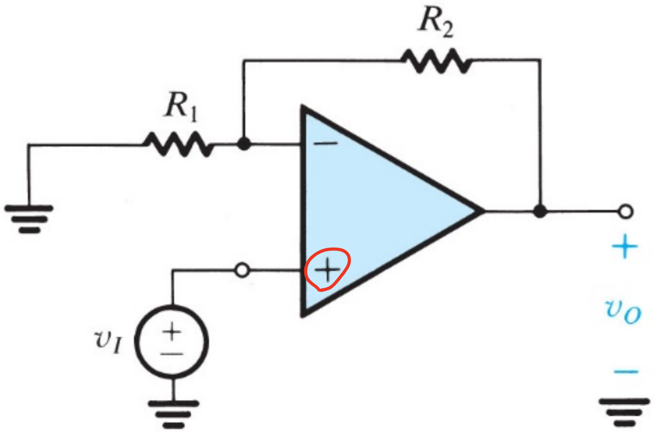
$A \to \infty \to \sim 10^6$, $\quad (v_1 - v_2) \to 0$

$$R_o = \frac{v_x}{i_x} = 0$$

To solve R_o:

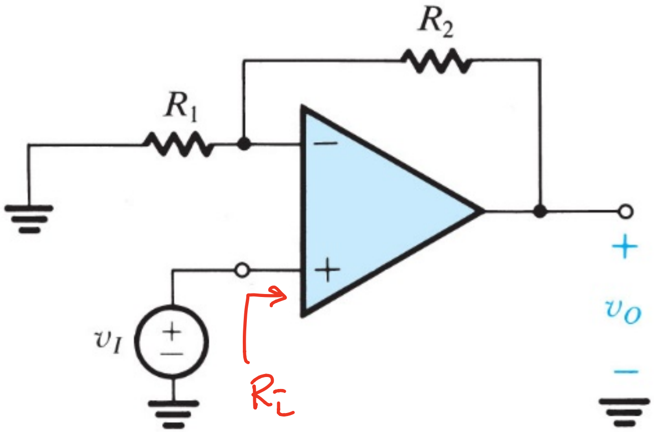
1. Replace v_I by short ckt
2. Apply test voltage source v_x
3. Solve i_x using KCL, KVL
4. $R_o = \dfrac{v_x}{i_x}$

Non-Inverting Amplifier



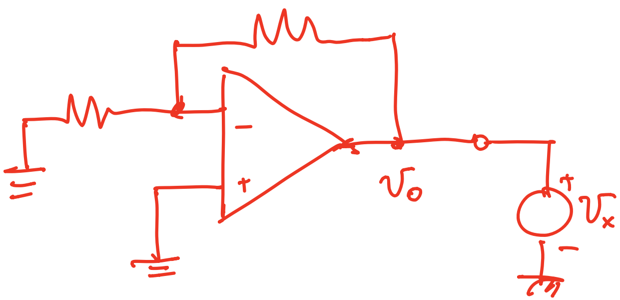
$$G = \frac{v_o}{v_i} = \left(1 + \frac{R_2}{R_1}\right)$$

Non-Inverting Amplifier: Input and Output Resistances



$R_i = \infty$

R_o



Identical to Inverting Amp

$R_o = 0$

$$v_x = v_o = \underbrace{A}_{10^6}\,(\underbrace{v_+ - v_-}_{=0}) = 0$$

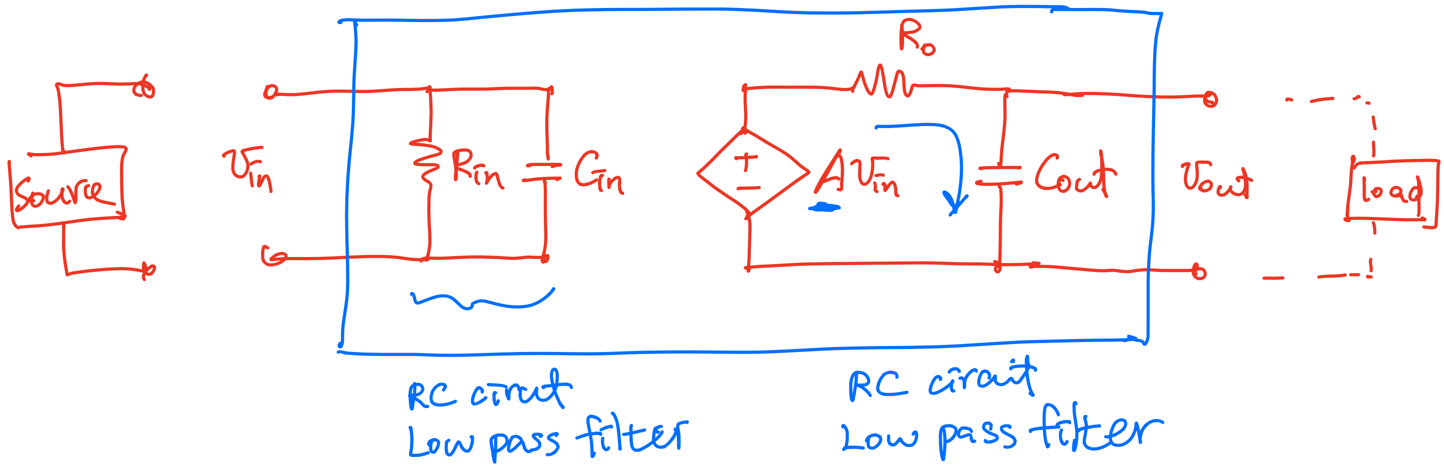
$$R_o = \frac{v_x}{i_x} = 0$$

Practical Op-Amps

- **Linear Imperfections:**
  - Finite open-loop gain ($A_0 < \infty$)
  - Finite input resistance ($R_i < \infty$)
  - Non-zero output resistance ($R_o > 0$)
  - Finite bandwidth / Gain-BW Trade-off
- **Other (non-linear) imperfections:**
  - Slew rate limitations
  - Finite swing
  - Offset voltage
  - Input bias and offset currents
  - Noise and distortion

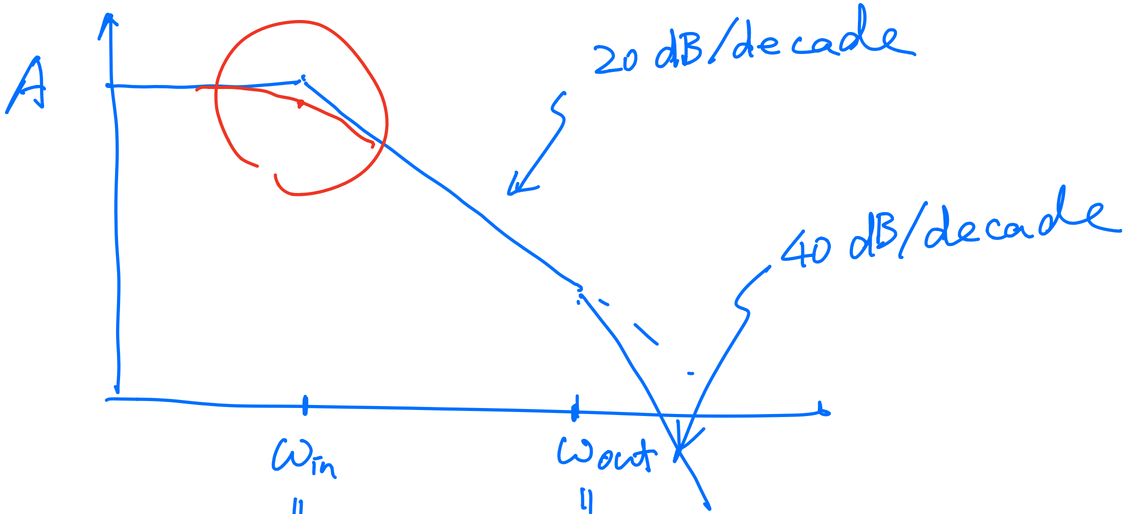
Two-Port

Simple Model of Amplifier



- **Input and output capacitances are added**
- **Any amplifier has input capacitance due to transistors and packaging / board parasitics**
- **Output capacitance is usually dominated by load**
  - **Driving cables or a board trace**

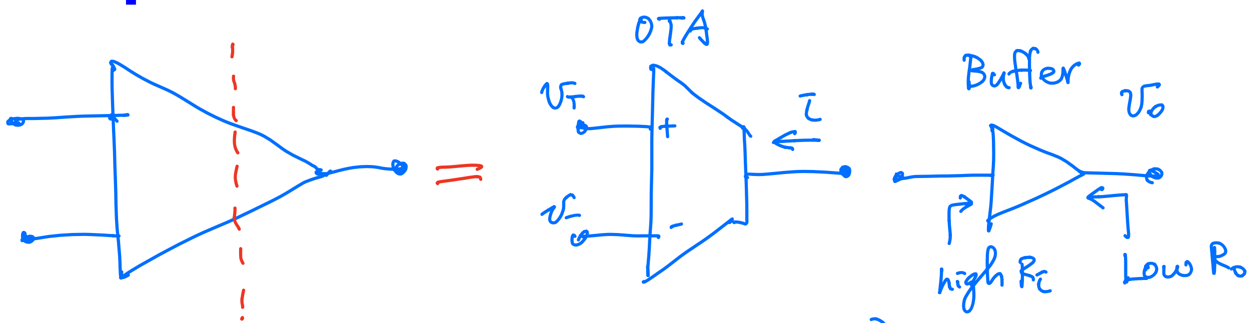
Transfer Function



$$\omega_{in} = \frac{1}{\tau_{in}} = \frac{1}{R_{in} C_{in}} \qquad \omega_{out} = \frac{1}{R_o \cdot C_{out}} = \frac{1}{\tau_{out}}$$

- **Using the concept of impedance, it's easy to derive the transfer function**

Operational Transconductance Amp



$$i = G_m (v_+ - v_-)$$

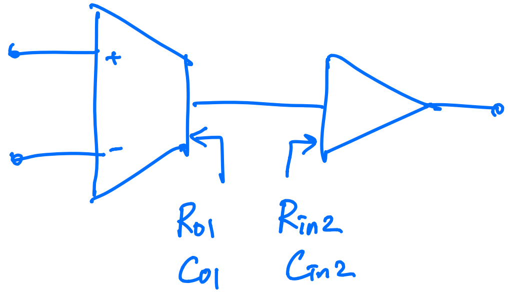
↑

Transconductance

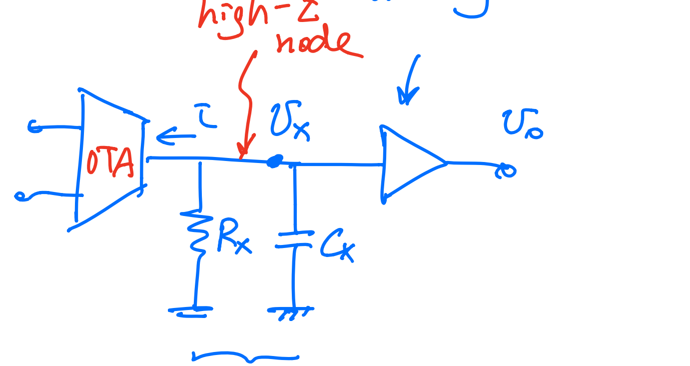
$[℧] = [S]$

- **Also known as an "OTA"**
  - **If we "chop off" the output stage of an op-amp, we get an OTA**
- **An OTA is essentially a *G*$_m$ amplifier. It has a current output, so if we want to drive a load resistor, we need an output stage (buffer)**
- **Many op-amps are internally constructed from an OTA + buffer**

Op-Amp Model



$\Rightarrow$



$i = G_m(v_+ - v_-)$

$v_x = i R_x = G_m R_x (v_+ - v_-)$

$v_o = v_x$

RC time $= R_x C_x$

$\omega_{3dB} = \dfrac{1}{R_x C_x}$

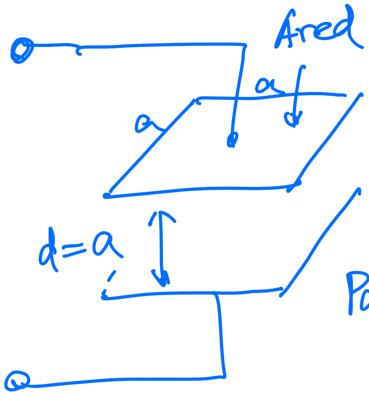
Gain $= G_m \cdot R_x$ $\quad$ R_x should be large

- **The model closely resembles the insides of an op-amp**
- **The input OTA stage drives a high Z node to generate a very large voltage gain**
- **The output buffer then can drive a low impedance load and preserve the high voltage gain**

Op-Amp Gain / Bandwidth

- **The dominant frequency response of the op-amp is due to the time constant formed at the high-Z node**

Transcondance of OTA



voltage gain $G = G_m R_x$

$$\omega_{3dB} = \omega_b = \frac{1}{R_x C_x}$$

Parallel Plate Cap $C = \frac{\epsilon A}{d} = \frac{\epsilon \cdot a^2}{a} = \epsilon \cdot a$

↳ Cube $A = a \times a$, $d = a$

- **An interesting observation is that the gain-bandwidth product depends on G_m and C_x only**

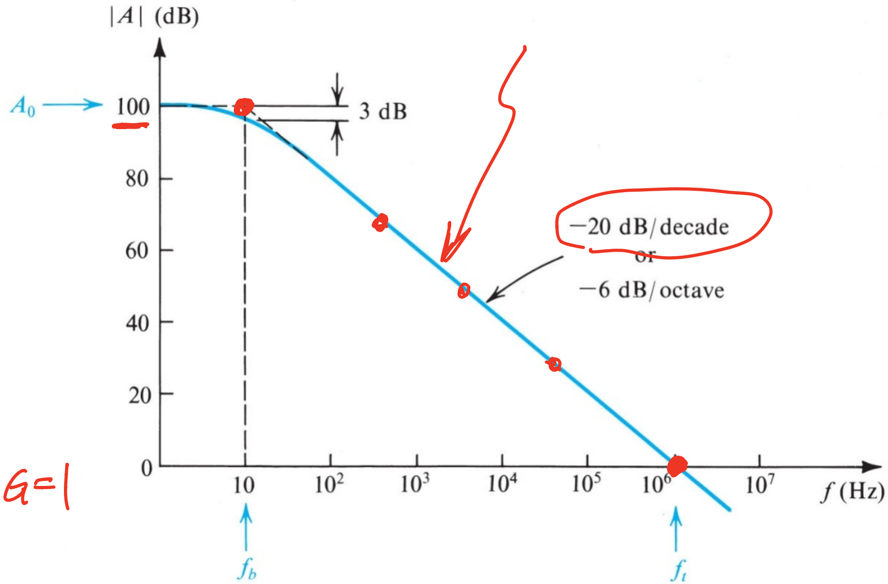
$$G \times \omega_{3dB} = (G_m \cancel{R_x})\left(\frac{1}{\cancel{R_x} C_x}\right) = \frac{G_m}{C_x}$$

Figure of Merit

Gain-Bandwidth Trade-off

Frequency Response of Open-Loop Op Amp

Same Gain × BW



$f_b = \frac{1}{2\pi}\omega_b = \frac{1}{2\pi}\omega_{3dB} = \frac{1}{2\pi}\frac{1}{R_x C_x}$ $\quad f_t = \frac{1}{2\pi}\omega_t$

Single pole response with a dominant pole at ω_b

$$\boxed{A(j\omega) = \frac{A_0}{1 + j\omega/\omega_b}}$$

A_0 : dc gain $= G_m R_x$ large $< \infty$

ω_b : 3dB frequency

$\omega_t = A_0\omega_b$: unity-gain bandwidth

(or "gain-bandwidth product")

For high frequency, $\omega >> \omega_b$

$$A(j\omega) = \frac{\omega_t}{j\omega}$$

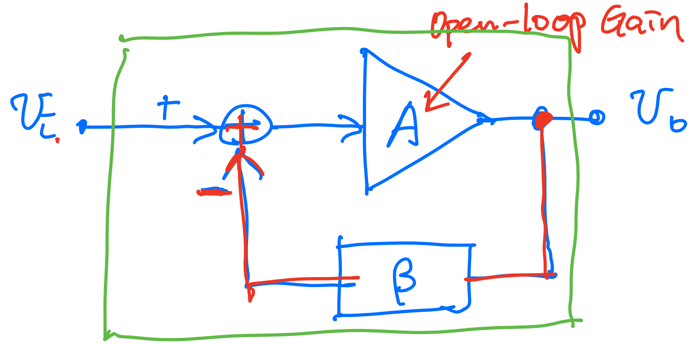
$\omega \gg \omega_b = \omega_{3dB}$ $\quad A(j\omega) \approx \frac{A_0}{j\left(\frac{\omega}{\omega_b}\right)}$

$|A(j\omega)| \to \left|\frac{A_0\omega_b}{j\omega}\right| \equiv 1$

$\omega = \omega_t = A_0 \cdot \omega_b = \text{Gain} \times BW$

Bandwidth Extension with Feedback

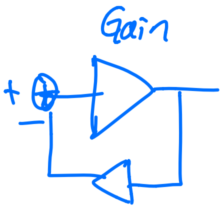
- **Overall transfer function with feedback:**



$$v_o = A(v_i - \beta v_o)$$

$$G = \frac{v_o}{v_i} = \left(\frac{A}{1+A\beta}\right)$$

close-loop Gain



$$v_o = A(j\omega)(v_i - \beta v_o);$$

$$\boxed{A(j\omega) = \frac{A_o}{1 + j\dfrac{\omega}{\omega_b}}}$$

$$G(j\omega) = \frac{\dfrac{A_o}{1+j\frac{\omega}{\omega_b}}}{1+\dfrac{A_o\beta}{1+j\frac{\omega}{\omega_b}}} = \frac{A_o}{1+j\frac{\omega}{\omega_b}+A_o\beta}$$

$$= \left(\frac{A_o}{1+A_o\beta}\right)\cdot\frac{1}{1+j\dfrac{\omega}{(1+A_o\beta)\omega_b}}$$

$$\omega_{3dB} = (1+A_o\beta)\cdot\omega_b$$

Bandwidth Extension and Gain Reduction

extension

- **Bandwidth increase:**

$$BW = (1 + A_o\beta)\omega_b$$

- **Gain reduces:**

$$G = \frac{A_o}{1 + A_o\beta}$$

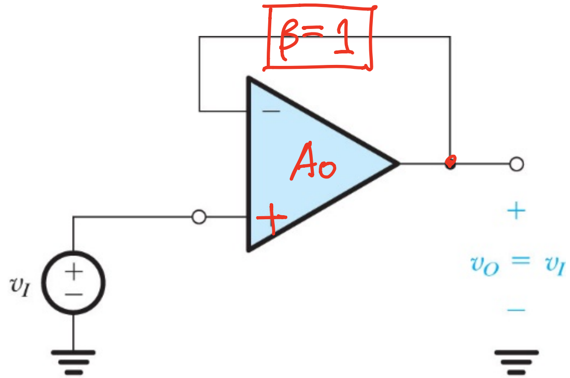
- **Gain-Bandwidth Product remains constant:**

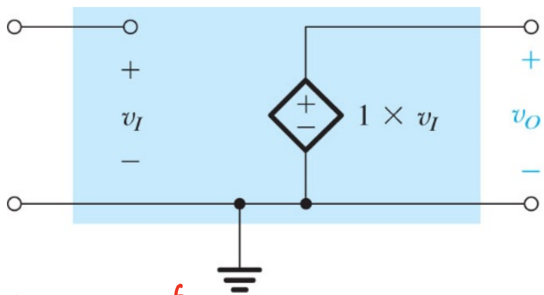
$$G \times BW = A_o\omega_b$$

Gain – Bandwidth Trade-off

Unity Gain Feedback Amplifier

- **An amplifier that has a feedback factor β = 1, such as a unity gain buffer, has the full GBW product frequency range**



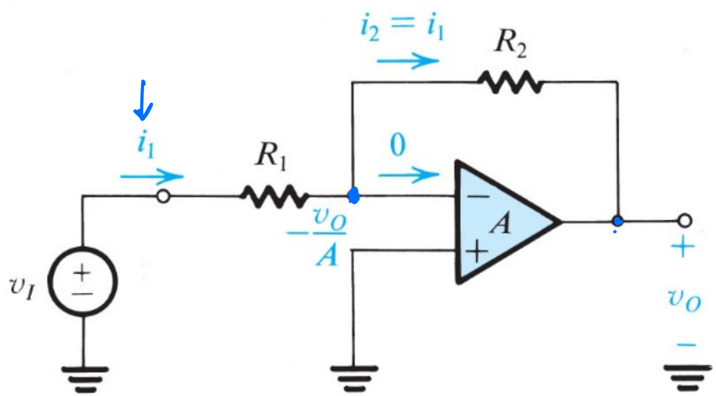


think $A_0 \sim 10^6$

$$G = \frac{A_o}{1 + A_o \underset{\approx 1}{\beta}} = \frac{A_o}{\cancel{1} + A_o} \approx 1$$

$$BW = (1 + A_o\beta)\omega_b = (1 + A_o)\omega_b \approx \underline{A_o\omega_b}$$

Voltage Gain of Inverting Amplifier with Finite Open-Loop Gain



$v_+ \neq v_-$, $v_+ = 0$

$\rightarrow v_o = A(v_+ - v_-) = -A(v_-) = -Av_-$

$i_1 = [v_i - (-\frac{v_o}{A})]/R_1$

$i_2 = i_1 = \frac{v^- - v_o}{R_2}$

$$\frac{v_i + \frac{v_o}{A}}{R_1} = \frac{-\frac{v_o}{A} - v_o}{R_2}$$

$$\left(v_i + \frac{v_o}{A}\right)\frac{1}{R_1} = \left(-\frac{v_o}{A} - v_o\right)\frac{1}{R_2}$$

$$v_o + \frac{v_o}{A} = \left(-\frac{R_2}{R_1}\right)\left(\frac{v_o}{A} + v_i\right)$$

$$v_o + \frac{1}{A} + \left(\frac{R_2}{R_1}\right)\frac{1}{A} = \left(-\frac{R_2}{R_1}\right)v_i$$

$$\frac{v_o}{v_i} = \frac{1}{1 + \frac{1}{A}\left(1 + \frac{R_2}{R_1}\right)}\left(-\frac{R_2}{R_1}\right)$$

$$\rightarrow G = \frac{v_o}{v_i} = \left(-\frac{R_2}{R_1}\right)\frac{1}{1 + \frac{1}{A}\left(1 + \frac{R_2}{R_1}\right)}$$

$A \rightarrow \infty$

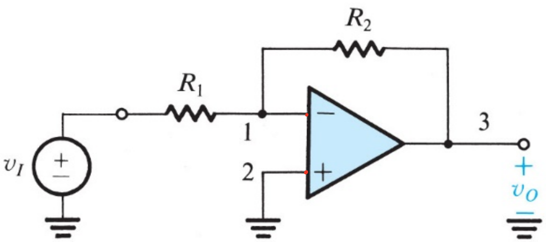
$G \rightarrow -\frac{R_2}{R_1}$ same Ideal Op Amp

BSAC

Frequency Response of Closed-Loop Op Amp

$A < \infty$

- $v_0 = A(v_+ - v_-)$



Steps to find frequency response of closed-loop amplifiers:

1. Find the transfer function with finite open-loop gain. For example, for inverting amplifier:

$$G = \frac{v_o}{v_I} = \left(-\frac{R_2}{R_1}\right)\frac{1}{1+\dfrac{(1+R_2/R_1)}{A}}$$

$A_0 \sim 10^6$

2. Substitute A with $A(j\omega) = \dfrac{A_0}{1+j\omega/\omega_b}$

3. Simplify the expression

$$G(\omega) = \left(-\frac{R_2}{R_1}\right)\frac{1}{1+(1+R_2/R_1)\dfrac{1+j\omega/\omega_b}{A_0}}$$

$$= \left(-\frac{R_2}{R_1}\right)\frac{1}{1+\cancelto{0}{\dfrac{(1+R_2/R_1)}{A_0}}+\dfrac{j\omega}{\left(\dfrac{A_0\omega_b}{1+R_2/R_1}\right)}}$$

Frequency Response of Closed-Loop Inverting Amplifier Example

Example
$A_0 = 100\,dB$
$= 10^5$

$\left|\frac{R_2}{R_1}\right| \sim 30\,dB \rightarrow 10^{1.5} \sim 30$

$\frac{f_{3dB}}{f_b} = \frac{A_0}{\left(\frac{R_2}{R_1}\right)} = 70\,dB \sim 10^{3.5}$
$\sim 3{,}000\times$

$$G(\omega) \approx \left(-\frac{R_2}{R_1}\right)\frac{1}{1+\dfrac{j\omega}{\omega_{3dB}}} \qquad \text{where } \omega_{3dB} = \frac{A_0\omega_b}{1+R_2/R_1}$$

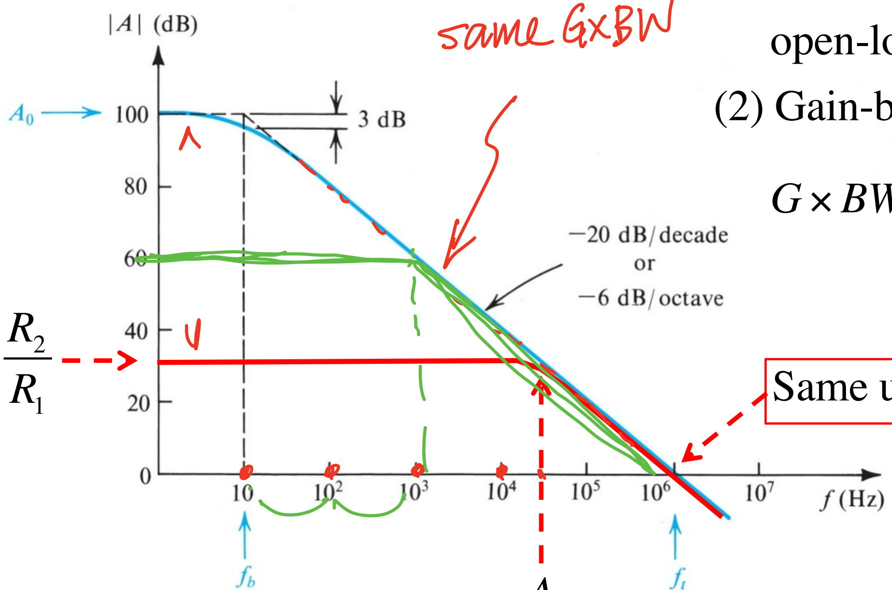
Note:

(1) 3-dB frequency is higher than open-loop bandwidth, ω_b

(2) Gain-bandwidth product remains unchanged:

$$G\times BW = \frac{R_2}{R_1}\,\frac{A_0\omega_b}{1+R_2/R_1} \approx \frac{R_2}{R_1}\,\frac{A_0\omega_b}{R_2/R_1} = A_0\omega_b = \omega_t$$

same GxBW



$$f_{3dB} \approx \frac{A_0}{R_2/R_1} f_b$$

$\boxed{\text{Same unity-gain frequency: } f_t}$

Example 2: $\left(\frac{R_2}{R_1}\right) = 1000 \rightarrow 60\,dB$

$\frac{A_0}{\left(\frac{R_2}{R_1}\right)} = 100\,dB - 60\,dB = 40\,dB$
$\rightarrow 100\times$