Discussion 2

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1 PN Junctions

1.1 Forward Bias

By forward biasing the junction (making the p-side more positive than the n-side), we reduce the built-in potential and decrease the depletion width. If we reduce the barrier sufficiently, electrons and holes (majority carriers) can be injected across the junction by diffusion. These excess carriers will recombine as they enter the opposite side of the junction, reducing exponentially from the edge of the depletion region.

Solving for the total current (which must be constant at each point across the junction) gives:

\[ I_D = I_S \left( e^{\frac{V_D}{V_T}} - 1 \right) \]

\[ I_S = Aq n_0^2 \left( \frac{D_n}{N_A L_n} + \frac{D_p}{N_D L_p} \right) \]

Note that although the total current is constant, the components due to electron/hole drift/diffusion vary throughout the junction. We often ignore the \(-1\) in the equation for \(I_D\), since it becomes negligible under typical forward bias conditions.

1.2 Reverse Bias

Recall that in reverse bias, ionized donors and acceptors create an electric field across the depletion pointing from the n-type side to the p-type side. In a typical PN junction, we can get a small amount of drift current across the junction due to thermally generated carriers in junction and a small number of minority carriers diffusing into the depletion region.

1.3 Small-Signal Model

Very often, we only care about small signals in our circuits (What counts as small? Well, it depends on the device. For a diode, we require \(V_D << V_T\). Why?) Although a diode is generally a non-linear device, we can model it as linear when operating with small signals around a certain bias point. In this case, we can define an equivalent resistance given by \(r_d = \frac{\partial V_D}{\partial I_D} = \left( \frac{\partial I_D}{\partial V_D} \right)^{-1} = \frac{V_T}{I_D}\).

1.4 Sample Problem

1. Construct the small-signal model for the following circuit:
Let $I_S = 1 \text{ fA}$ and assume $T = 300 \text{ K}$. (Solution: $I_D = 857 \mu \text{A}$, $r_d = 30.3 \Omega$.)

## 2 Bipolar Junction Transistor (BJT)

### 2.1 Structure

A BJT can be constructed in two ways: NPN or PNP. In an NPN device, we have sandwiched layers of n-type silicon, p-type silicon, and n-type silicon (and likewise for the PNP structure). Figure 1 shows a typical BJT structure (not to scale) with the terminals labeled. Figure 2 shows the associated circuit symbol for an NPN BJT. Note the convention that for any two voltages $V_X$ and $V_Y$, $V_{XY} = V_X - V_Y$.

### 2.2 Cutoff

When both the base-collector and base-emitter junctions are reverse biased, the device is said to be in cutoff. In this mode, we get (effectively) no current flowing through the device. Recall that a reverse-biased PN
junction flows a current of about $I_S$, which is on the order of $10^{-15}$ A.

2.3 Forward Active

In forward active, the base-emitter junction is forward biased and the base-collector junction is reverse biased (i.e., $V_{CE} > V_{BE}$). Recall that in a forward biased junction, we get minority carrier injection. In this case, the emitter injects electrons into the base and the base injects holes into the emitter. The electrons that are injected into the base diffuse across the base toward the collector. Since the base-collector junction is reverse biased, there is an electric field pointing from the collector to the base, causing the electrons diffusing through the base to be swept into the collector (assuming the base width is small relative to the diffusion length of the electrons in the base).

We’d like injection in the base to be much less than injection into the emitter (i.e., we want the base current to be much less than the collector current). This allows us to achieve a high current gain $\beta$, which is one of the essential parameters determining BJT performance. Here is a summary of equations describing the large-signal behavior of BJTs (we’ll talk more about large vs. small signal in a moment):

\[
I_C = I_S \left( e^{V_{BE}/V_T} - 1 \right)
\]

\[
I_C = \beta I_B
\]

\[
I_E = I_C + I_B = (1 + \beta) I_B
\]

We also like to define a parameter $\alpha = I_C/I_E = \beta/(1 + \beta)$.

2.3.1 Sample Problems

1. (Razavi 4.17) Compute $I_X$ and $I_Y$ in the following circuit. Use $I_{S1} = 3 \times 10^{-16}$ A, $I_{S2} = 5 \times 10^{-16}$ A, $\beta_1 = \beta_2 = 100$, $R_1 = 5 \text{ k}\Omega$, and $V_B = 800$ mV. (Solution: $I_X = 509 \text{ \mu A}$, $I_Y = 848 \text{ \mu A}$.)

![Figure 4: Schematic for Problem 1.](image)

2. (Razavi 4.11) Compute the voltage at node $X$ in the following circuit. Use $I_S = 6 \times 10^{-16}$ A and $\beta = 100$. (Solution: $V_X = 763$ mV.)

![Figure 5: Schematic for Problem 2.](image)

3. Compute the current gain $I_C/I_B$ of this “compound” transistor, known as a Sziklai pair. (Solution: $I_C/I_B = \beta_1(1 + \beta_2)$.)
2.4 Saturation

In saturation, both the base-emitter and base-collector junctions are forward biased. We typically divide this mode of operation into finer degrees: "soft" saturation, "heavy" saturation, “deep” saturation (these terms are used in Razavi). In soft saturation, the base-collector junction has a bias of less than 400 mV (approximately). Recall that a diode doesn’t really turn on hard until around 600 mV, meaning that the base-collector junction still acts reasonably reverse biased in this region. In this class, we will treat devices in soft saturation as having the same behavior as those in forward active.

In heavy saturation, the base-collector junction is more heavily forward biased. In this case, we not only get carrier injection from the base-emitter junction, but also from the base-collector junction. Since the base is more heavily doped than the collector, we will get significant injection of holes from the base into the collector, causing a rapid increase in base current and decrease in collector current (since the holes injected into the collector generate current in the opposite direction of the electrons normally collected by the collector). This drops the transistor’s $\beta$ and heavily degrades performance, as shown in Figure 7.

![Saturation Diagram](image)

Figure 7: BJT I-V characteristic showing degraded collector current in saturation.

In deep saturation, both junctions are very heavily forward biased. We won’t use the device in this mode, but as you can imagine, it begins to act like two forward-biased diodes, with the base-emitter one clamped at 800 mV and the base-collector one clamped at 200 mV.

2.5 Small-Signal Model

Just like for a diode, we can define a linear model for a BJT (which is inherently an exponential device) that represents its behavior when operating with small signals only. We have a base current that depends on the base-emitter voltage, allowing us to define a small-signal base-emitter resistance $r_e$. We have a collector current dependent on the base-emitter voltage, allowing us to define a small-signal transconductance $g_m$ (this
is a measure of how much the base-emitter voltage controls the collector current. This week, you'll also learn about the Early effect, which gives us a relationship between the collector current and the collector-emitter voltage, allowing us to define a small-signal collector-emitter resistance $r_o$. These parameters are derived as follows:

$$
g_m = \frac{\partial I_C}{\partial V_{BE}} = \frac{1}{V_T} I_S e^{V_{BE}/V_T} = \frac{I_C}{V_T}$$

$$
r_\pi = \frac{\partial V_{BE}}{\partial I_B} = \left( \frac{\partial I_B}{\partial V_{BE}} \right)^{-1} = \beta \frac{V_T}{I_C} = \beta g_m$$

$$
r_o = \frac{\partial V_{CE}}{\partial I_C} = \left( \frac{\partial I_C}{\partial V_{CE}} \right)^{-1} = \left[ \frac{1}{V_A} I_S e^{V_{BE}/V_T} \right]^{-1} = V_A \frac{I_C}{I_S}$$

We can then draw the small-signal model as follows:

![Small-signal model of a BJT](image)

**Figure 8: Small-signal model of a BJT.**

### 2.5.1 Sample Problem

1. For each of the circuits in Figure 9, determine the operating point and small-signal model. Assume $I_S = 8 \times 10^{-16}$ A, $\beta = 100$, and $V_A = \infty$. 
2.6 PNP BJT

We can also consider a PNP BJT, which operates much in the same manner as an NPN BJT. The important points to note about the PNP BJT are that some signs are flipped: for example, we typically define $I_C$ to be flowing into the collector in an NPN transistor, but for a PNP it flows out of the collector. The emitter current sees a similar reversal. The other important thing to remember about a PNP is that its small-signal model is the same as that of an NPN transistor (Why?). Figure 10 shows a PNP transistor schematic.

![PNP transistor schematic](image)

Figure 10: Schematic of a PNP transistor with the terminals, voltages, and currents labeled.

2.6.1 Sample Problem

1. For each of the circuits in Figure 11, determine the operating point and small-signal model. Assume $I_S = 3 \times 10^{-16}$ A, $\beta = 100$, and $V_A = \infty$.

![Circuits for Problem 1](image)

Figure 11: Circuits for Problem 1.

3 Biasing

In order for our transistors to operate properly, we need them biased at the appropriate current (recall, this sets our small-signal parameters, which, as we will see, affect gain, impedance matching, frequency response, etc.). Ideally, we’d like our biasing to be consistent over changes in supply voltage, temperature, and even device parameters if possible.

In Chapter 5, Razavi discusses various methods of biasing and solves a number of examples with various combinations of resistors. We can consider the most general case, with a resistive divider at the base, a resistor at the collector, and a resistor at the emitter, then generalize it to the special cases that Razavi considers. Figure 12 shows such a general biasing condition.
Figure 12: A biased NPN transistor.

Assume we are given the resistor values, the supply voltage, and the transistor parameters and we want to find the transistor currents and voltages. We can write the following:

\[ I_B = \frac{V_{CC} - V_X}{R_1} - \frac{V_X}{R_2} \]
\[ I_C = \beta I_B \]
\[ V_{BE} = V_X - I_E R_E \]
\[ = V_X - \frac{1+\beta}{\beta} I_C R_E \]
\[ = V_T \ln \frac{I_C}{I_S} \]

If we replace \( I_C \) with the expression involving \( V_X \), we have a solvable equation for \( V_X \) (though it requires iteration), from which we can obtain all other relevant currents and voltages. As an example of generalizing the result, if we want to consider what Razavi calls ‘simple’ biasing, we can just set \( R_2 = \infty \) and \( R_E = 0 \). Why would we use this more complicated biasing network over simple biasing? (Answer: It is less sensitive to variations in \( \beta \) and \( V_{CC} \).) Note that Razavi often solves biasing problems with Thévenin equivalent circuits, which you may find easier.