Bipolar Junction Transistor (BJT)

\[ I_c = -\alpha_F I_E = -\alpha_F (-I_c - I_B) \]
\[ I_c = \frac{\alpha_F}{1-\alpha_F} I_B = \beta I_B \]
\[ \alpha_F \approx 0.99 \]
\[ \beta \approx 100 \]

**Ebers-Moll Model**

emitter saturation current
\[ I_f = I_{Es} \left( e^{\frac{V_{BE}}{V_{th}} - 1} \right) \]
\[ I_r = I_{Cs} \left( e^{\frac{V_{BC}}{V_{th}} - 1} \right) \]
collector saturation current
\[
\begin{cases}
  I_E = -I_F + \alpha_R I_R \\
  I_C = -I_R + \alpha_F I_F
\end{cases}
\]
I-V characteristics

I_c
0
1 μA
2
3
4
5 μA

V_C E

Cut-off
(no current)

Forward active region
→ ~ constant current region

Saturation region

In "Forward Active" region

\{ BE junction is forward-biased

BC junction is reverse-biased

Ebers-Moll model reduces to

\[ I_B \rightarrow \begin{array}{c}
\beta_F I_B \\
\end{array} \rightarrow I_c \]

\[ B \rightarrow I_B \]

\[ B \rightarrow 0,7V \]

\[ E \rightarrow 0,7V \]

\[ E \rightarrow \beta_F I_B \]
In "Saturation" region.

Both BE and BC junctions are forward biased.

\[ V_{CE} > 0 \quad V_{CE} = V_{BE} - V_{BC} > 0 \]

\[ \Rightarrow \text{BE junction is slightly more forward biased} \]

E-M model

\[ B \quad C \]
\[ 0.7V \quad V_{CE} \approx 0.1V \]
\[ E \]

Small-Signal Model (for Forward Active region)

\[ B \]
\[ V_{be} \]
\[ V_{th} \]
\[ V_{ce} \]

\[ g_m = \left. \frac{\partial I_C}{\partial V_{BE}} \right|_{Q} = \beta_0 \left. \frac{\partial I_B}{\partial V_{BE}} \right|_{Q} = \beta_0 \frac{I_B}{V_{th}} = \frac{I_C}{V_{th}} = \frac{g_1 I_C}{kT} \]

\[ \text{dynamic resistance of pn junction} \]

\[ r_{th} = \left( \frac{\partial I_B}{\partial V_{BE}} \right)^{-1} = \left( \frac{g_m}{\beta_0} \right)^{-1} = \frac{\beta_0}{g_m} \]
\[ I_c = I_s e^{\frac{V_{BE}}{V_{th}} \left( 1 + \frac{V_{CE}}{V_{An}} \right)} \]

\( V_{An} \): Early voltage

(due to base width modulation; equivalent to channel length modulation in MOSFET, though the physical mechanism is different)

\[ R_0 = \frac{V_{An}}{I_c} \]

**Frequency Response**

**Capacitances:**

\( C_b = \) Base charging capacitance

\[ C_b = T_F \cdot g_m \]

\[ T_F = \frac{W_b^2}{2D_{nb}} \]

\( W_b \): base width

\( D_{nb} \): Diffusion Coef.

\[ (\frac{I_c}{T_F} = \frac{C_b \cdot V_{be}}{T_F} \Rightarrow C_b = \frac{I_c}{V_{be}} \cdot T_F = \frac{\frac{I_c}{V_{be}}}{\frac{\partial I_c}{\partial V_{BE}}} \Rightarrow T_F = g_m(T_F) ) \]
Depletion Capacitance

Base-Emitter junction.

\[ V_{BE} = 0.7 \, V \]

\[ C_{JE} \approx \sqrt{2} \, C_{JE0} \]

Junction capacitance at \( V_{BE} = 0 \)

\[ C_{II} = C_b + C_{JE} \]

BC junction: reverse-biased.

\[ C_u = \frac{C_{uo}}{\sqrt{1 + \frac{V_{CB}}{\Phi_{BC}}}} \]

\( \Phi_{BC} \): Built-in potential of BC junction

Collector-Substrate: also a reverse-biased junction

\[ C_{cs} = \frac{C_{cs0}}{\sqrt{1 + \frac{V_{CS}}{\Phi_{BS}}}} \]

\( \Phi_{BS} \): Built-in potential of collector-substrate junction

\[ \begin{array}{c}
\text{Collector} \\
\text{Emitter} \\
\text{Base} \\
\end{array} \]

\[ R_b, C_u, C_{II}, V_{II}, r_i, r_o, C_{cs}, Y_{ex} \]
BJT Cross-Section (Simplified)

B

E

C

P+ \text{n+} \text{n} \text{CJE} \text{C_E}

\text{n+} \text{n} \text{Cn}

Ccs \text{Cs}

P-substrate