Lecture 17

Bipolar Junction Transistors (BJT)
Reading: 7.1 – 7.5

- NPN or PNP sandwich (Two back-to-back diodes)
- How does current flow? Base is very thin.
- A good BJT satisfies the following
  \[
  I_C \approx -I_E \\
  I_C \gg I_B \\
  I_C \approx I_e e^{\frac{qV_{BE}}{kT}}
  \]
Actual BJT Cross Section

- Vertical npn sandwich (pnp is usually a lateral structure)
- n+ buried layout is a low resistance contact to collector
- Base width determined by vertical distance between emitter diffusion and base diffusion

BJT Layout

- Emitter area most important layout parameter
BJT Schematic Symbol

- Collector current is controlled by base current linearly.
- Collector current is an exponential function of the base-emitter voltage.

BJT Collector Characteristic

- Ground emitter
- Fix $V_{CE}$
- Drive base with fixed current $I_B$
- Measure the collector current
Collector Characteristics ($I_B$)

Base-Emitter Voltage Control
Transistor Action

- Base-emitter junction is forward biased and collector-base junction is reverse biased
- Electrons “emitted” into base much more than holes since the doping of emitter is much higher
- Magic: Most electrons cross the base junction and are swept into collector
- Why? Base width much smaller than diffusion length. Base-collector junction pulls electrons into collector
**Diffusion Currents**

- Minority carriers in base form a uniform diffusion current. Since emitter doping is higher, this current swamps out the current portion due to the minority carriers injected from base.

**BJT Currents**

Collector current is nearly identical to the (magnitude) of the emitter current ... define

\[ I_C = -\alpha_F I_E \quad \alpha_F = .999 \]

KCL:

\[ -I_E = I_C + I_B \]

DC Current Gain:

\[ I_C = -\alpha_F I_E = \alpha_F (I_B + I_C) \]

\[ I_C = \frac{\alpha_F}{1-\alpha_F} I_B = \beta_F I_B \]

\[ \beta_F = \frac{\alpha_F}{1-\alpha_F} = \frac{.999}{.001} = 999 \]
Origin of $\alpha_F$

Base-emitter junction: some reverse injection of holes into the emitter $\rightarrow$ base current isn’t zero

![Diagram of transistor showing base-emitter junction and base current]

Some electrons lost due to recombination

Typical: $\alpha_F \approx 0.99$  $\beta_F \approx 100$

Collector Current

Diffusion of electrons across base results in

$$J_n^{\text{diff}} = qD_n \frac{dn_p}{dx} = \left( \frac{qD_n n_{pB0}}{W_B} \right) \frac{qV_{BE}}{kT} e^{\frac{qV_{BE}}{kT}}$$

$$I_S = \left( \frac{qD_n n_{pB0} A_E}{W_B} \right)$$

$$I_C = I_S e^{\frac{qV_{BE}}{kT}}$$
**Base Current**

Diffusion of holes across emitter results in

\[ J_{p\text{ diff}} = -qD_p \frac{dp_{nE}}{dx} = \left( \frac{qD_p p_{nE0}}{W_E} \right) \left( \frac{qV_{BE}}{e^{kT}} - 1 \right) = -2D_p \frac{p_{nE}}{W_E} \]

\[ I_B = \left( \frac{qD_p p_{nE0} A_E}{W_E} \right) \left( \frac{qV_{BE}}{e^{kT}} - 1 \right) \]

---

**Current Gain**

\[ \beta_p = \frac{I_C}{I_B} = \frac{\left( \frac{qD_n n_{pB0} A_E}{W_B} \right)}{\left( \frac{qD_p p_{nE0} A_E}{W_E} \right)} = \left( \frac{D_n}{D_p} \right) \left( \frac{n_{pB0}}{p_{nE0}} \right) \left( \frac{W_E}{W_B} \right) \]

Minimize base width

\[ \left( \frac{n_{pB0}}{p_{nE0}} \right) = \frac{n_{pB0}}{n_{nE0}} = \frac{N_{N_E}}{N_{A,B}} \]

Maximize doping in emitter
Ebers-Moll Equations

Derivation: Write emitter and collector currents in terms of internal currents at two junctions

\[
I_E = -I_{ES} \left(e^{\frac{V_{BE}}{V_{th}}} - 1\right) + \alpha_R I_{CS} \left(e^{\frac{V_{BC}}{V_{th}}} - 1\right)
\]

\[
I_C = \alpha_F I_{ES} \left(e^{\frac{V_{BE}}{V_{th}}} - 1\right) - I_{CS} \left(e^{\frac{V_{BC}}{V_{th}}} - 1\right)
\]

\[
\alpha_F I_{ES} = \alpha_R I_{CS}
\]

Ebers-Moll Equivalent Circuit

Building blocks: diodes and \(I\)-controlled \(I\) sources

![Ebers-Moll Equivalent Circuit Diagram]
**Forward Active Region**

B-C junction is not forward-biased → $I_R$ is very small

![Diagram showing forward active region]

Typical Values:

$V_{BE} = 0.7$

$V_{CE} > 0.2$

**Simplified Ebers-Moll**

**Forward-Active Case:**

$V_{BE} = 0.7$

$I_C = \beta F I_B$

Saturation: both diodes are forward-biases → batteries

$I_C = V_{FE} - V_{BE}$
Base-Emitter Voltage Control

- Saturation Region (Low Output Resistance)
- Reverse Active (Bad Transistor)
- Forward Active Region (High Output Resistance)

Actual BJT Cross Section

- Vertical npn sandwich (pnp is usually a lateral structure)
- n+ buried layout is a low resistance contact to collector
- Base width determined by vertical distance between emitter diffusion and base diffusion
Small-Signal Models

Analogy from MOSFET s.s. model:

\[ i_D = f(v_{GS}, v_{DS}, v_{BS}) \quad i_C = f(v_{BE}, v_{CE}) \]

Transconductance \( g_m \)

- The transconductance is analogous to diode conductance
Transconductance (cont)

- **Forward-active large-signal current:**
  \[ i_C = I_s e^{\frac{v_{BE}}{V_{th}}} (1 + v_{CE}/V_A) \]

- Differentiating and evaluating at \( Q = (V_{BE}, V_{CE}) \)
  \[ \frac{di_C}{dv_{BE}} \bigg|_Q = \frac{q}{kT} I_s e^{\frac{qV_{BE}}{kT}} (1 + V_{CE}/V_A) \]
  \[ g_m = \frac{di_C}{dv_{BE}} \bigg|_Q = \frac{qI_C}{kT} = \frac{I_C}{v_{th}} \]

Comparison with MOSFET

- **Typical bias point:** drain/coll. current = 100 \( \mu \text{A} \);
  Select \((W/L) = 8/1, \mu_n C_{ox} = 100 \, \mu \text{A}/\text{V}^2\)

- **BJT:**
  \[ g_m = \frac{qI_C}{kT} = \frac{I_C}{V_{th}} \]
  \[ g_m = \frac{I_C}{V_{th}} = \frac{100\mu}{25m} = 4 \text{mS} \]

- **MOSFET:**
  \[ g_m = \frac{2I_D}{V_{GS}-V_T} \]
  \[ g_m = \frac{2I_D}{V_{GS}-V_T} = \sqrt{2\mu C_{ox} \frac{W}{L} I_D} = \sqrt{2 \times 100 \mu \times 8 \times 100 \mu} = 400 \mu \text{S} \]
BJT Base Currents

Unlike MOSFET, there is a DC current into the base terminal of a bipolar transistor:

$$I_B = I_C / \beta_S = (I_S / \beta_S) e^{\gamma_{BE} / kT} \left(1 + V_{CE} / V_A\right)$$

To find the change in base current due to change in base-emitter voltage:

$$\frac{1}{\sqrt{\pi}} = \left| \frac{\partial i_B}{\partial V_{BE}} \right|_{Q} = \left| \frac{\partial i_B}{\partial i_C} \right|_{Q} \frac{\partial i_C}{\partial V_{BE}} \right|_{Q} = \frac{1}{\beta} g_m$$

$$\frac{g_m}{g_m} = 4 \mu_m S$$

$$\beta = 100$$

$$r_{k}\beta \frac{g_m}{g_m} = 25 k$$

---

Small Signal Current Gain

The diagram illustrates the relationship between the collector current ($I_C$) and the base current ($I_B$) with varying base-emitter voltage ($V_{BE}$). The current gain $\beta$ is defined as

$$\beta = \frac{\Delta I_C}{\Delta I_B} = \beta_S$$
Input Resistance $r_{\text{IP}}$

\[
(r_{\pi})^{-1} = \left. \frac{\partial i_B}{\partial v_{BE}} \right|_{i_Q} = \frac{1}{\beta} \left. \frac{\partial i_C}{\partial v_{BE}} \right|_{i_Q} = \frac{g_m}{\beta}
\]

In practice, the DC current gain $\beta_F$ and the small-signal current gain $\beta_o$ are both highly variable (+/- 25%)

Typical bias point: DC collector current = 100 $\mu$A

---

Output Resistance $r_o$

Why does current increase slightly with increasing $v_{CE}$?

Model: introduce the Early voltage

\[
i_C = I_S e^{v_{BE}/V_{th}} (1 + v_{CE}/V_A)
\]
Graphical Interpretation of $r_o$

\[ V_o = \left( \frac{2 \beta I_c}{r_{ce}} \right)^{-1} \approx 2 \left( \frac{I_s e^\frac{v_{be}}{V_T} (1 + \frac{v_{ce}}{V_b})}{2 v_{ce}} \right)^{-1} = \left( \frac{I_C}{V_H} \right)^{-1} \]

BJT Small-Signal Model

\[ g_m i_b + \beta v_r = \beta i_b \]

\[ v_r = \beta \]

\[ i_c = i_b r_f \]

\[ v_{ce} = i_b r_f \]

\[ v_{be} = r_{pi} i_b \]
### BJT Capacitances

Base-charging capacitance $C_b$: due to minority carrier charge storage (mostly electrons in the base)

$$C_b = g_m \tau_F$$

Base-emitter depletion capacitance: $C_{jE} = 1.4 \; C_{jE0}$

Total B-E capacitance: $C_{\pi} = C_{jE} + C_b$