Common Source: Discrete Biasing

With ideal MOS

\[ A_{\text{vol}} = \frac{g_m R_L V_D}{R_2} = \frac{V_D}{R_2} \]

\[ A_{\text{hor}} = \frac{g_m R_L V_D}{R_2} = \frac{V_D}{R_2} \]
Where is the Pole?

\[ \frac{V_x}{V_S} = \frac{V_x}{V_S} = \frac{j \omega C_2 R_2}{1 + j \omega C_2 R_2} \cdot (V_{G5} V_{B5} R_L) \]

Common Source Amplifier: \( A(j \omega) \)

DC Bias is problematic: what sets \( V_{GS} \)?
CS Short-Circuit Current Gain

Transfer function:

\[ A_i(j\omega) = \frac{g_m(1 - j\omega C_{gd}/g_m)}{j\omega(C_{gs} + C_{gd})} \]

\[ I_{out} = g_m V_{gs} - V_{gs} j\omega C_{gd} \]
\[ I_s = j\omega (C_{gs} + C_{gd}) V_{gs} \]
\[ \Rightarrow \frac{I_{out}}{I_s} = A_i(j\omega) = \frac{g_m - j\omega C_{gd}}{j\omega (C_{gs} + C_{gd})} \]

Magnitude Bode Plot

Transition frequency:
Current gain = 1
MOS Unity Gain Frequency

- Since the zero occurs at a higher frequency than pole, assume it has negligible effect:

\[ A_i \approx \frac{g_m}{j\omega (C_{gr} + C_{gd})} = 1 \rightarrow \omega_r = \frac{g_m}{(C_{gr} + C_{gd})} \]

\[ \omega_r \approx \frac{g_m}{C_{gr}} = \frac{\mu C_{ox} W}{2L} (V_{GS} - V_T) \approx \frac{3\mu(V_{GS} - V_T)}{2L^2} \]

Performance improves with \( L^2 \) for long channel devices!

For short channel devices the dependence is \( \sim L^1 \)

\[ \omega_r \approx \frac{3}{2} \frac{\mu(V_{GS} - V_T)}{L^2} \sim \frac{\mu V_{GS} - V_T}{L} = \frac{\mu E_{eff}}{L} = \frac{v}{L} = \frac{1}{\tau_c} \]

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DECOUPLING LOAD (DISCRETE)

- Decoupling load for discrete components.

\[ \omega_2 = \frac{1}{\sqrt{L C_L}} \]

\[ \omega_p = \frac{1}{L C_L (R_L + V_0)} \]

\[ \text{where } R_L, C_L \text{ are load components.} \]
Common-Source Voltage Amplifier

Small-signal model: omit $C_{cs}$ due to avoid complicated analysis

\[ \frac{V_g - V_s}{2S} + V_g j\omega C_{gs} + (V_g - V_{out}) j\omega C_{gd} = 0 \]

\[ (V_{out} - V_g) j\omega C_{gd} + g_m V_g + V_{out} / R_{oc} || R_L = 0 \]

\[ A_v(j\omega) = \frac{V_{out}}{V_s} \]
Frequency Response

KCL at input and output nodes; analysis is made complicated due to $Z_{gd}$ branch → see H&S pp. 639-640 (for common emitter)

\[
\frac{V_{out}}{V_{in}} = \frac{-g_m r_c \parallel R_L \parallel (1-j\omega/\omega_z)}{(1+j\omega/\omega_1)(1+j\omega/\omega_2)}
\]

Low-frequency gain:

\[ A_v = -\frac{g_m v_o \parallel C_c \parallel R_L}{R_C}
\]

Zero: $\omega_z > \omega_T = \frac{g_m}{C_{gs} + C_{gd}}$

\[
-\frac{g_m}{C_{gd}}
\]

Poles

\[
\omega_{p1} \approx \frac{1}{R_S \left(C_{gs} + \left(1+g_m R'_{out}\right)C_{gd}\right) + R'_{out}C_{gd}}
\]

\[
\omega_{p2} \approx \frac{R'_{out}/R_S}{R_S \left(C_{gs} + \left(1+g_m R'_{out}\right)C_{gd}\right) + R'_{out}C_{gd}}
\]
Miller Impedance

- Consider the current flowing through an impedance $Z$ hooked up to a “black-box” where the voltage gain from one terminal to the other is fixed

$$A_v = \frac{v_2}{v_1}$$

$$I = \frac{v_1 - v_2}{Z} = v_1 - \frac{A_v v_1}{Z} = v_1 \frac{1 - A_v}{Z}$$

- Notice that the current flowing into $Z$ from terminal 1 looks like an equivalent current to ground where $Z$ is transformed down by the Miller factor:

$$I = v_1 \frac{1 - A_v}{Z} \rightarrow Z_{M,1} = \frac{Z}{1 - A_v}$$

- From terminal 2, the situation is reciprocal

$$-I = \frac{v_2 - v_1}{Z} = \frac{v_2 - A_v^{-1} v_1}{Z} = v_2 \frac{1 - A_v^{-1}}{Z}$$

$$Z_{M,2} = \frac{Z}{1 - A_v^{-1}}$$
Miller Equivalent Circuit

Note: \( Z_{M,1} + Z_{M,2} = Z \)

- We can decouple these terminals if we can calculate the gain \( A_v \) across the impedance \( Z \)
- Often the gain \( A_v \) is weakly dependent on \( Z \)
- The approximation is to ignore \( Z \), calculate \( A_v \), and then use the decoupled Miller impedances

CE Amplifier using Miller Approx.

Use Miller to transform \( C_{gd} \)

\[
A_v = \frac{1}{1 + j \omega C_{gs} R_S} \cdot (-g_m || R_{oc} || R_L) \approx -g_m || R_{oc} || R_L \ (\omega \text{ is small})
\]

\[
V_{out} = \frac{V_{gs}}{g_m V_{gs}} \frac{1}{\frac{1}{R_{oc} || R_L} = R_{out}}
\]
Comparison with “Exact Analysis”

Miller result:
\[
\omega_{p1}^{-1} = R_S \left( C_{gs} + (1 + g_m R'_{out}) C_{gd} \right)
\]

Exact result:
\[
\omega_{p1}^{-1} = R_S \left( C_{gs} + (1 + g_m R'_{out}) C_{gd} \right) + R'_{out} C_{gd}
\]
Miller Effect Examples

Common source amplifier:

\[ A_v C_{gd} = \text{negative, large number (-100)} \]

Miller multiplied cap has detrimental impact on bandwidth

Common drain amplifier:

\[ A_v C_{gd} = \text{slightly less than 1} \]

“Bootstrapped” cap has negligible impact on bandwidth!