1. **Diffusion**: the flow of charge due to density gradients.

a. Diffusion Current

The diffusion currents are expressed in terms of the electron or hole diffusion coefficient $D_n$ or $D_p$, as shown below:

$$ J_{n,\text{diff}} = -(q)D_n \frac{dn}{dx} = qD_n \frac{dn}{dx} $$

$$ J_{p,\text{diff}} = +(q)D_p \frac{dn}{dx} = -qD_p \frac{dp}{dx} $$

b. Einstein Relation:

$$ D_n = \frac{kT}{q} \mu_n $$

$$ D_p = \frac{kT}{q} \mu_p $$

2. **Total Current**: $J = J_{\text{diff}} + J_{\text{drift}}$

**EXAMPLE1**: We have a piece of uniformly doped silicon with a acceptor concentration of $N_a=2E16\text{cm}^{-3}$. By shedding light on one end ($x=0$) of the semiconductor, we create an electron concentration of $n(x)=1E13*\exp(-x/2\text{um})$. Calculate the electron diffusion current as a function of $x$.

**SOLUTION1**:  

$$ N_a = 2E16 \Rightarrow \mu_n = 1050\text{cm}^2/\text{Vs} $$

$$ D_n = \frac{kT}{q} \mu_n = 0.026 \times 1050 = 27.3\text{cm}^2/\text{s} $$

$$ J_{n,\text{diff}} = qD_n \frac{dn}{dx} = 1.6 \times 10^{-19} \times 27.3 \times \frac{d}{dx}\left(10^{13} e^{-x/(2\times10^{-4})}\right) $$

$$ = 0.22 e^{-x/2\mu_m} \text{A/cm}^2 $$

3. **Integrated Passives**

a. Resistors

- Diffusion Resistor: ion implantation/diffusion  
  100ohm/sq(unsilicided), 10ohm/sq(silicided)

- Poly Film Resistor: deposit a thin film of heavily doped poly-Si  
  10-100ohm/sq(unsilicided), 1ohm/sq(silicided)

b. Capacitors

- IC MIM Capacitor: by forming a Metal plate-Insulator(thin oxide)-Metal plate structure.
- Parallel Plate Capacitor:  
  $$ C = \frac{Ae}{d} $$

- Nonlinear capacitor: Q-V curve is not a straight line
Small signal capacitance: differential capacitance

\[ Q = Q_0 + q \approx f(V_s) + \frac{df(V)}{dV} \left. \right|_{V=V_s} V_s \]

\[ C \equiv \left. \frac{df(V)}{dV} \right|_{V=V_s} \]

4. PN Junction

a. Structure

b. Distribution of charge, electric field, and potential

-Depletion Approximation: The transition region is completely deplete of free carriers, the only charges exist are immobile dopants. This region is called “Depletion Region”

\[ \rho_0(x) = \begin{cases} 
-qN_a (-x_{po} < x < 0) \\
+qN_d (0 < x < x_{no}) 
\end{cases} \]

-Electric Field in Depletion Region:

\[ E_0(x) = \begin{cases} 
\frac{-qN_a}{\varepsilon_s} (x + x_{p0})(-x_{po} < x < 0) \\
\frac{-qN_d}{\varepsilon_s} (x_{n0} - x) (0 < x < x_{no}) 
\end{cases} \]
-Potential in Depletion Region
\[
\phi_p(x) = \phi_p + \frac{qN_a}{2\varepsilon_s}(x + x_{p0})^2
\]
\[
\phi_n(x) = \phi_n + \frac{qN_d}{2\varepsilon_s}(x - x_{n0})^2
\]

-Boundary Conditions:
Potential continuous:
\[
\phi_p(0) = \phi_p + \frac{qN_a}{2\varepsilon_s}(x_{p0})^2 = \phi_n(0) = \phi_n + \frac{qN_d}{2\varepsilon_s}(-x_{n0})^2
\]
Electrical Field continuous:
\[
qN_a x_{p0} = qN_d x_{n0}
\]

-Depletion Width & Built-in Potential
\[
X_{d0} = x_{n0} + x_{p0} = \sqrt{\frac{2\varepsilon_s\phi_{bi}}{q} \left( \frac{1}{N_a} + \frac{1}{N_d} \right)}
\]
\[
x_{n0} = \sqrt{\frac{2\varepsilon_s\phi_{bi}}{qN_d} \left( \frac{N_a}{N_a + N_d} \right)}
\]
\[
x_{p0} = \sqrt{\frac{2\varepsilon_s\phi_{bi}}{qN_a} \left( \frac{N_d}{N_a + N_d} \right)}
\]
\[
\phi_{bi} \equiv \phi_n - \phi_p = V_{bi} \left( \ln \left( \frac{N_d}{n_i} \right) + \ln \left( \frac{N_a}{n_i} \right) \right)
\]
\[
= V_{bi} \ln \frac{N_a N_d}{n_i^2} > 0
\]

c. PN Junction Capacitor
-PN Junction under Bias
\[ x_n(V_D) = \sqrt{\frac{2\varepsilon_s(\phi_{bi}-V_D)}{qN_d}} \left( \frac{N_a}{N_a+N_d} \right) = x_{n0} \sqrt{1 - \frac{V_D}{\phi_{bi}}} \]

\[ x_p(V_D) = \sqrt{\frac{2\varepsilon_s(\phi_{bi}-V_D)}{qN_d}} \left( \frac{N_d}{N_a+N_d} \right) = x_{p0} \sqrt{1 - \frac{V_D}{\phi_{bi}}} \]

\[ C_j = \frac{C_{j0}}{1 - \frac{V_D}{\phi_{bi}}} = \frac{\varepsilon_s}{X_{d0}} \]

\[ C_{j0} = \frac{\varepsilon_s N_a N_d}{2\phi_{bi} N_a + N_d} = \frac{\varepsilon_s}{X_d} \]

\[ C_j(V_D) = \frac{\varepsilon_s}{X_d(V_D)} \]

**-EXAMPLE2:**
Consider the following silicon P-N junction:

<table>
<thead>
<tr>
<th>( \text{P}^+ )</th>
<th>( \text{P} )</th>
<th>( \text{N} )</th>
<th>( \text{N}^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.2 ( \mu )m</td>
<td>0.4 ( \mu )m</td>
<td></td>
</tr>
</tbody>
</table>

(a) If \( N_a = 4 \times 10^{16} \text{cm}^{-3} \) in the P-region and \( N_d = 1 \times 10^{17} \text{cm}^{-3} \) in the N-region, under increasing reverse bias, which region (N or P) will become completely depleted first?
(b) What is the reverse bias at this condition?
(c) What is the small-signal capacitance (F/cm\(^2\)) at the bias condition?

**SOLUTION2:**
(a) Under reverse bias, the depletion region will expand. The side which has fewer dopants will fully deplete first.

\[ Na \times W_p = 4 \times 10^{16} \times 1.2 > Nd \times W_n = 1 \times 10^{17} \times 0.4 \]

So N side will deplete first

(b) \( \phi_{bi} = V_{bi} \ln \frac{N_a N_d}{n_i} = 0.81 \text{V} \)

\[ x_{n0} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{qN_d} \left( \frac{N_a}{N_a+N_d} \right)} = 0.055 \mu \text{m} \]

\[ 0.4 = x_n(V_D) = x_{n0} \sqrt{1 - \frac{V_D}{\phi_{bi}}} \]
So $V_D = -42\text{V}$

(c) The small signal capacitance is determined by the depletion region width.

\[ X_n = W_n = 0.4\text{um}, \quad X_p = (N_d/N_a) \cdot X_n = (1 \cdot 10^{17}/4 \cdot 10^{16}) \cdot 0.4 = 1\text{um} \]

\[ X_d = X_n + X_p = 0.4 + 1 = 1.4\text{um} \]

\[ C_d = \epsilon_s / X_d = 8.85 \cdot 10^{-14} \cdot 12 / (1.4 \cdot 10^{-4}) = 7.6 \cdot 10^{-9}\text{F/cm}^2 \]

5. PN Junction Diode

- Thermal Equilibrium:
  small diffusion current: high barrier
  small drift current: few minority carriers and wide depletion region
  net current: none

- Reverse Bias:
  drift current: remains small
  diffusion current: reduced exponentially with bias
  net current: small reverse current

- Forward Bias:
  drift current: remains small
  diffusion current: increase exponentially with bias
  net current: large forward current

- IV curve:

\[ I_e(V_e \rightarrow -\infty) = -I_s \]

\[ I_e = I_s \left( \frac{e^p}{e^{qF}} - 1 \right) \]

- Minority Carrier Concentration
  with recombination, long base diode
without recombination or short base diode:

- **Diode Current Densities**

\[ J_{\text{diff}} = J_{\text{diff}}^n + J_{\text{diff}}^p = qn_i^2 \left( \frac{D_p}{N_d W_n} + \frac{D_n}{N_a W_p} \right) \left( \frac{qV_{bi}}{e^{xT}} - 1 \right) \]

- **Small Signal Model**

\[ I_D + I_0 = I_S \left( e^{\frac{kT}{q(V_d + i_d)}} - 1 \right) \approx I_S e^{\frac{qV_d}{kT}} e^{\frac{qV_d}{kT}} \]

\[ i_D = \frac{qV_d}{kT} = g_d v_d \]

- **Charge Storage**
Consider an ideal, long-base, silicon PN junction diode with uniform cross section and constant doping on either side of the junction. P side is heavily doped and N side has a doping concentration of 1E16 cm\(^{-3}\).

For the n-side of the junction:

(a) Calculate the density of the minority carriers as a function of x (distance from the junction) when the applied voltage is 0.589V (which is 23kT/q).

(b) Find the minority carrier currents as functions of x (distance from the junction)

\[ C_d = \frac{1}{2} \frac{qI_d}{kT} \tau \]

\[-\text{EXAMPLE3} \]

\[ P(x) = \frac{n^2}{N_d} \left( e^{qV_T/kT} - 1 \right)^{-x/L_p} = 10^{14} e^{-x/L_p} \text{ cm}^{-3}. \]

(b) Minority current:

\[ J_p(x) = -qD_p \frac{dp(x)}{dx} = q \frac{n^2}{N_d L_p} D_p \left( e^{qV_T/kT} - 1 \right)^{-x/L_p} = 0.5e^{-x/L_p} \text{ A/cm}^2. \]