Lecture 19

• Last time:
  – Phasor analysis techniques
  – Rapid sketching techniques for Bode Plots

• Today:
  – 2nd order circuits in the frequency domain
  – MOSFET models
thin-Film Bulk Acoustic Resonator (FBAR)

- Agilent Technologies
  *IEEE ISSCC 2001*

  2 GHz resonator

- $N > 1000$

- Brian Otis, Jan Rabaey (BWRC): low-noise oscillator

- Equivalent Circuit:
  
  ![Equivalent Circuit Diagram](image)
Phasor Analysis of 2\textsuperscript{nd} Order Circuit

Impedance divider:

\[ V_c = V_s \left[ \frac{1/j\omega C}{(1/j\omega C) + R + (j\omega L)} \right] \]
Transfer Function

Simplifying:

\[
H(j\omega) = \frac{1}{1 + j\omega RC - \omega^2 LC}
\]

Define parameters: \(\omega_o = 1/\sqrt{LC}\) \(\tau = RC\)

\[
H(j\omega) = \frac{1}{1 - (\omega/\omega_o)^2 + j\omega\tau}
\]
Limiting Cases: Magnitude and Phase

Low frequency: \( \omega << \omega_o \)

High frequency: \( \omega >> \omega_o \)

Resonant frequency: \( \omega = \omega_o \)
Inductor-Capacitor “Tuning”

At resonance, the impedance of the capacitor cancels the impedance of the inductor \( \Rightarrow \) phasor current is maximum and capacitor voltage peaks

How “sharp” or “narrow” is the resonance?

Define the quality factor

\[
Q = \frac{\omega_o}{\Delta \omega} \quad \text{\( \longrightarrow \)} \quad Q = \frac{1}{\omega_o \tau}
\]
Magnitude Bode Plot

$|H|_{dB}$

$\omega$

$\omega_o - \Delta \omega$  $\omega_o$  $\omega_o + \Delta \omega$
Phase Bode Plot

Phase$(H)$

-45

-90

-135

-180

$\omega_0 - \Delta \omega$

$\omega_0 + \Delta \omega$

$\omega$

$\omega_0$
Common Source Amplifier: $A_i(j\omega)$

DC Bias is problematic: what sets $V_{GS}$?
CS Short-Circuit Current Gain

Transfer function: \[ A_i(j\omega) = \frac{g_m \left(1 - j\omega C_{gd} / g_m \right)}{j\omega (C_{gs} + C_{gd})} \]
Magnitude Bode Plot

Transition frequency:
Current gain $\to 1$
MOS Unity Gain Frequency

- Since the zero occurs at a higher frequency than pole, assume it has negligible effect:

\[ A_i \approx \frac{g_m}{j\omega(C_{gs} + C_{gd})} = 1 \quad \rightarrow \quad \omega_T = \frac{g_m}{(C_{gs} + C_{gd})} \]

\[ \omega_T \approx \frac{g_m}{C_{gs}} \cdot \frac{\mu C_{ox} \frac{W}{L}(V_{GS} - V_T)}{2WLC_{ox}} = 3 \frac{\mu(V_{GS} - V_T)}{2L^2} \]

Performance improves like \( L^2 \) for long channel devices!
For short channel devices the dependence is like \( \sim L^1 \)

\[ \omega_T \approx \frac{3}{2} \frac{\mu(V_{GS} - V_T)}{L^2} \sim \frac{\mu E_{eff}}{L} = \frac{\nu}{L} = \tau_L \]

Time to cross channel