Lecture 19: Review, PN junctions, Fermi levels, forward bias

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Context

The first part of this lecture is a review of electrons and holes in silicon:

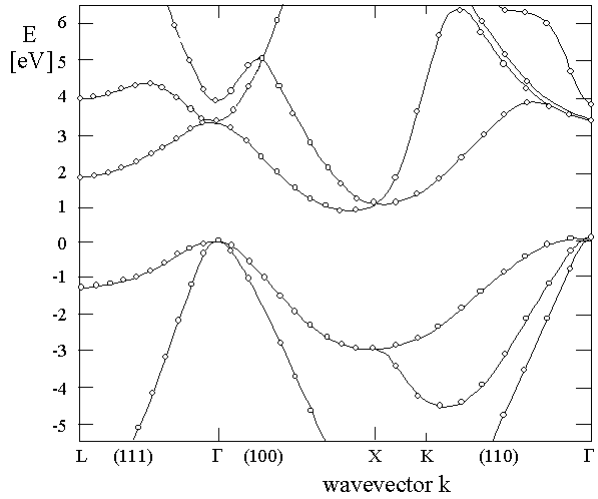
- Fermi levels and Quasi-Fermi levels
- Majority and minority carriers
- Drift
- Diffusion

And we will apply these to:

- Diode Currents in forward and reverse bias (chapter 6)
- BJT (Bipolar Junction Transistors) in the next lecture.

Electrons and Holes

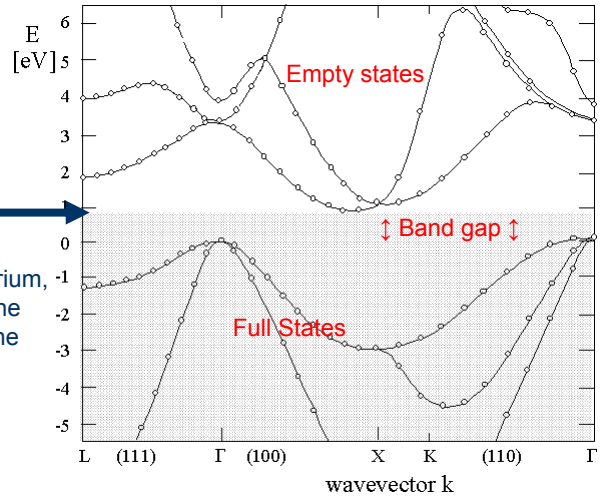
- Electrons in silicon can be in a number of different states:



EECS 105 Spring 2004, Lecture 19 *Prof. J. S. Smith*

Electrons and Holes

- Electrons in silicon can be in a number of different states:



In thermal equilibrium, at each location the electrons will fill the states up to a particular level

Fermi function

- In thermal equilibrium, the probability of occupancy of any state is given by the Fermi function:

$$F(E) = \frac{1}{1 + e^{\frac{E - E_f}{kT}}}$$

- At the energy $E=E_f$ the probability of occupancy is 1/2.
- At high energies, the probability of occupancy approaches zero exponentially
- At low energies, the probability of occupancy approaches 1

Exponential approximation (electrons)

- In semiconductors, the Fermi energy is usually in the band gap, far from either the conduction band or the valence band (compared to kT).
- For the conduction band, since the exponential is much larger than 1, we can use the approximation:

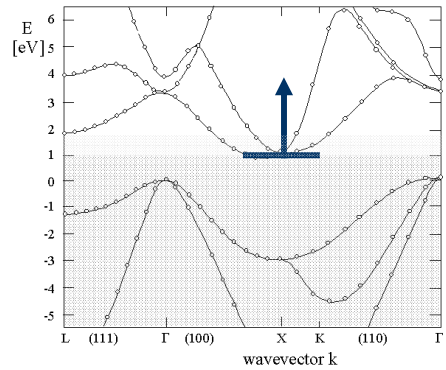
$$F(E) = \frac{1}{1 + e^{\frac{E - E_f}{kT}}} \approx \frac{1}{e^{\frac{E - E_f}{kT}}} = e^{-\left(\frac{E - E_f}{kT}\right)}$$

Electrons

- Under this approximation, we can integrate over the conduction band states, and we can write the result as:

$$n = N_c e^{-\frac{E-E_f}{kT}}$$

Where N_c is a number, called the effective density of states in the conduction band



Exponential approximation (holes)

- For the valence band band, since the exponential is much smaller than 1, we can use the approximation:

$$F(E) = \frac{1}{1+e^{\frac{E-E_f}{kT}}} \approx 1 - e^{\left(\frac{E-E_f}{kT}\right)}$$

since $\frac{1}{1+x} \approx 1 - x$ (for x small)

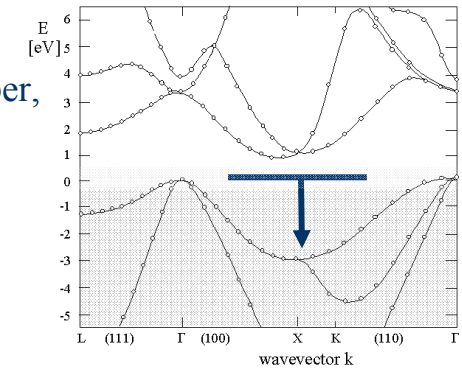
- Since we are counting holes as the absence of an electron, we have the probability of *not having an electron in a state:* $e^{\left(\frac{E-E_f}{kT}\right)}$

Holes

- Under this approximation, we can also integrate over the conduction band states, and we can write the result as:

$$p = N_V e^{\frac{E-E_f}{kT}}$$

- Where N_v is a number, called the effective density of states in the valence band



Intrinsic concentrations

- In thermal equilibrium, the Fermi energy must be the same everywhere, including the Fermi energy for the electrons and the holes, so:

$$pn = N_V e^{\frac{E-E_f}{kT}} N_c e^{-\frac{E-E_f}{kT}} = N_V N_c e^{-2\frac{E_f}{kT}} = n_i^2(T)$$

- We call this constant $n_i^2(T)$ because in a neutral, undoped semiconductor

$$p = n = n_i$$

Non Neutral, Non Equilibrium

Our devices:

- won't be in thermal equilibrium (or they wouldn't do anything interesting
- They mostly won't be undoped
- They might not be neutral (such as in a depletion zone)

But from these intrinsic, equilibrium, neutral results develop many useful approximations.

Rapid thermalization

- Even if a semiconductor is not in thermal equilibrium, electrons and holes can exchange energy with each other and the lattice so quickly that they mostly remain in a thermal distribution at a temperature T
- If they don't, its called a *Hot Carrier Effect*
- In the absence of hot carrier effects, both the electron and hole occupancies will be given by the Fermi function, but the distributions for the electrons and the holes may *not be given by the same Fermi energy!*

Minority Carriers

- This is because electrons take a relatively long time to recombine with holes, and that electrons in the valence band to jump to the conduction band and form an electron hole pair, so the relative number of electrons and holes can diverge far from equilibrium
- In an N type material, for example, holes can be *injected* raising $p > \frac{n_i^2}{n}$
  - The carrier type which there are fewer of are called *minority carriers,* in this case the holes.
  - Strangely enough, as we will see, the minority carriers often dominate the transport through a device.
  - Minority carriers aren't so important for FET's, so they are called *majority carrier devices*
  - The np product can be reduced below n_i^2, as in a reverse biased junction

φ_n and φ_p

In neutral silicon which is doped with an acceptor at a density far above the intrinsic carrier concentration:

$$n \approx N_d = N_c e^{-\frac{E-E_f}{kT}}$$

Which we can rewrite as:

$$n \approx N_d = n_i e^{-\frac{\phi_{n0}}{kT}}$$

And similarly:

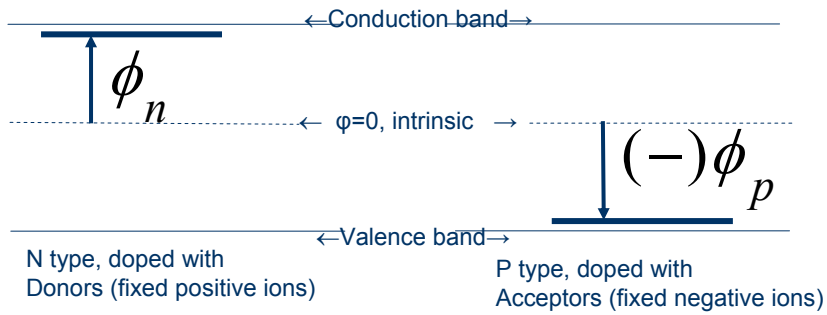
$$p \approx N_a = n_i e^{\frac{q\phi_{p0}}{kT}}$$

Since electrons are negative, potentials come out to be the negative of energy: $\text{Energy} = qV = (-e)\phi$

Reference: intrinsic

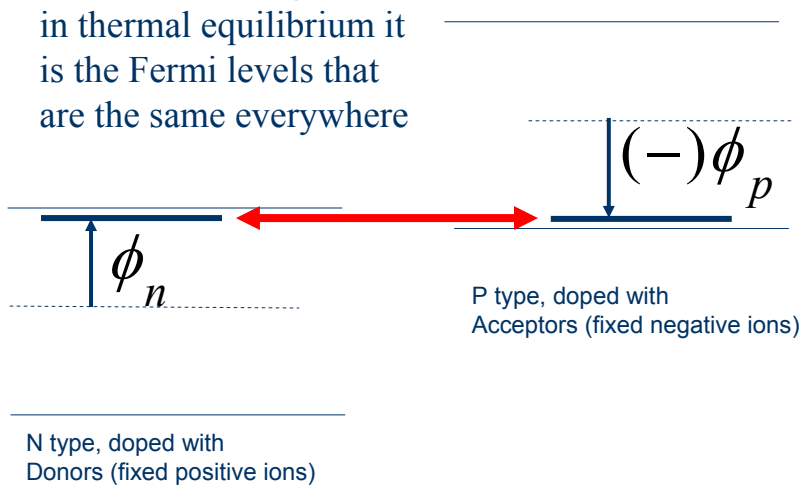
These equations use intrinsic silicon as a reference so the point where $\varphi_n=0$ and $\varphi_p=0\rightarrow$

$$n = n_i e^{-\frac{q(\phi_n=0)}{kT}} = n_i \quad \text{and} \quad p = n_i e^{\frac{q(\phi_p=0)}{kT}} = n_i$$



P and N regions in thermal Eq.

- Remember, though, that in thermal equilibrium it is the Fermi levels that are the same everywhere



Thermal equilibrium again

- In thermal equilibrium, the Fermi energy is the same everywhere, and the same for electrons and holes, so at a particular point,

$$\phi_n = \phi_p = \phi(x, y, z)$$

$$n = n_i e^{-\left(\frac{q\phi(x,y,z)}{kT}\right)}$$

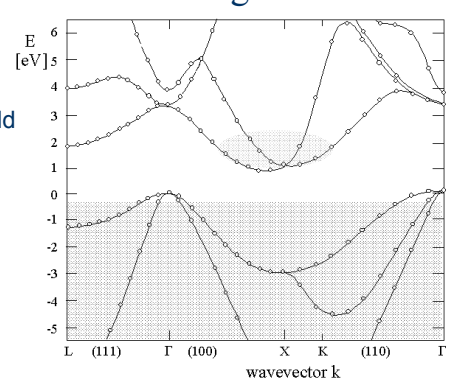
$$p = n_i e^{\frac{q\phi(x,y,z)}{kT}}$$

- And of course we get back $np = n_i^2$ everywhere which is always the case in equilibrium

Quasi-Fermi levels

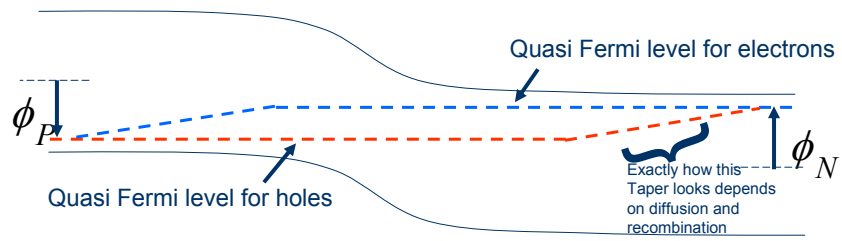
- Since when we are not in thermal equilibrium we can still use the Fermi function for electrons and holes, but with different Fermi energies, we call these new energies quasi-Fermi energies

On the band diagram, it would look like this:



Quasi-Fermi levels: Band edge diagram

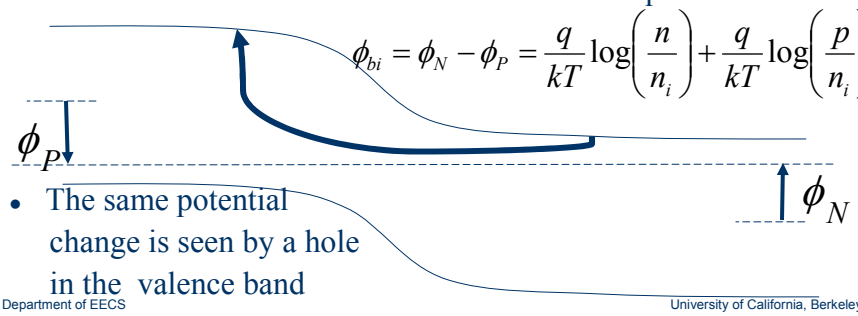
- When we draw a band edge diagram out of equilibrium, we need to draw a different Fermi level (quasi-Fermi level) for the electrons and holes
- This, for example, is what the band edge diagram would look like for a forward biased PN diode



Built-in field

- In thermal equilibrium, the PN diode has a potential difference for electrons, and a potential difference for holes, and an electric field that both see, with zero voltage appearing at the contacts, because the contacts are at the voltage of the Fermi level, not the conduction band on both sides or the valence band on both sides.
- The built in potential which is the energy change that an electron in the conduction band would see is equal to

$$\phi_{bi} = \phi_N - \phi_P = \frac{q}{kT}\log\left(\frac{n}{n_i}\right) + \frac{q}{kT}\log\left(\frac{p}{n_i}\right)$$



- The same potential change is seen by a hole in the valence band

Transport

- The net motion of electrons and holes through silicon was calculated to be the sum of drift and diffusion for each:

$$J_n = J_n^{dr} + J_n^{diff} = qn\mu_n E + qD_n \frac{dn}{dx}$$

$$J_p = J_p^{dr} + J_p^{diff} = qn\mu_p E + qD_p \frac{dp}{dx}$$

- Where $\frac{D}{\mu} = \frac{kT}{q}$ for each carrier type

Sidebar: Net current comes from the slope in E_f

- It is interesting to note that $\boxed{J_n = qn\mu_n E + q\mu_n \frac{kT}{q}\frac{dn}{dx}}$

$$n = n_i e^{-\left(\frac{\phi_n(x,y,z)}{kT}\right)} \rightarrow \frac{dn}{dx} = -n\frac{q}{kT}\frac{d\phi_n}{dx}$$

$$J_n = qn\mu_n E + q\mu_n \frac{kT}{q}\left(-n\frac{q}{kT}\frac{d\phi_n}{dx}\right) =$$

$$qn\mu_n\left(E - \frac{d\phi_n}{dx}\right) = qn\mu_n\left(\frac{dV}{dx} - \frac{d\phi_n}{dx}\right) \boxed{= -n\mu_n\left(\frac{dE_{fn}}{dx}\right)}$$

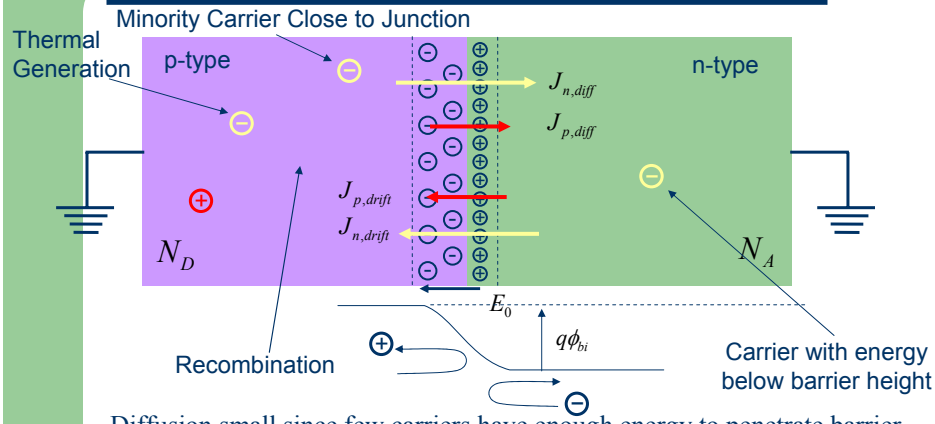
If the Fermi level is flat, there is no current

The result is the same for holes, but with a positive sign:

$$J_p = p\mu_p\left(\frac{dE_{fp}}{dx}\right)$$

→The Einstein relation comes from fundamentals!

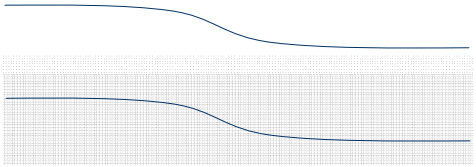
Diode under Thermal Equilibrium



- Diffusion small since few carriers have enough energy to penetrate barrier
- Drift current is small since minority carriers are few and far between: Only minority carriers generated within a diffusion length can contribute current
- **Important Point: Minority drift current is independent of barrier!**
- **Diffusion current strong (exponential) function of barrier**

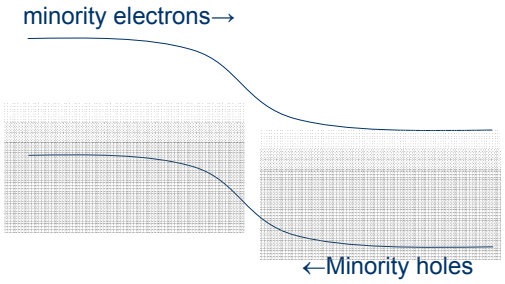
equilibrium

- The carriers which can diffuse across the barrier are those which have energies above the barrier height.
- The probability of having an energy higher that a certain amount is exponentially decreasing
- The minority carriers get swept across the depletion region, but there are few of them.
- In equilibrium, they balance. (carriers wander back and forth, with *no net current*)



Reverse bias

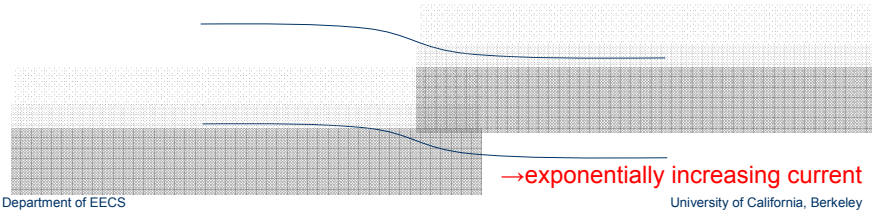
- In reverse bias, any minority carriers on either side can zip across the junction, but there aren't very many.
- The majority carriers are kept from crossing by the raised potential barrier



→small, constant reverse current

Forward Bias

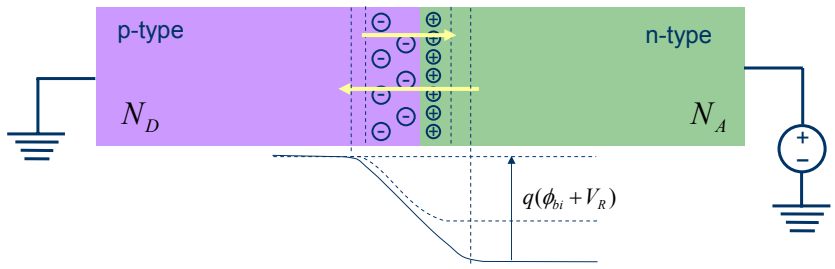
- In forward bias, the majority carriers see a reduced barrier, so the number of them that have high enough energies to cross the barrier increase exponentially.
- The minority carriers are still pulled across the depletion region, but there are still few of them, and so their contribution doesn't change much



→exponentially increasing current

Forward Bias

- Forward bias causes an exponential increase in the number of carriers with sufficient energy to penetrate barrier
- Diffusion current ***increases*** exponentially



- Drift current does not change
- Net result: Large forward current

First cut at PN junction currents

- Since the diffusion current will increase exponentially with the decrease in the height, we can model it:

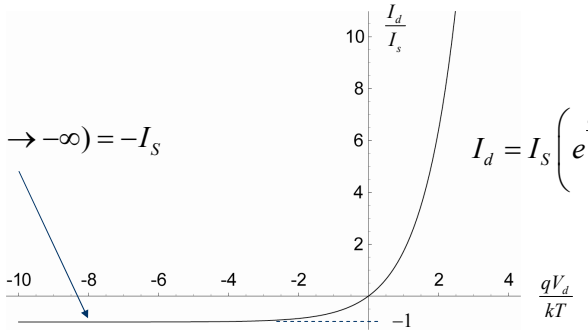
$$I_d = I_s e^{\frac{qV}{kT}}$$

- And since the total current must be zero at zero bias (thermal equilibrium), and the drift current doesn't change much with bias, we get a first take on the diode, the so-called "ideal diode" equation

$$I_d = I_s (e^{\frac{qV}{kT}} - 1)$$

Diode I-V Curve

$$I_d(V_d \to -\infty) = -I_S$$



$$I_d = I_S\left(e^{\frac{qV_d}{kT}} - 1\right)$$

- Diode IV relation is an exponential function
- For reverse bias the current saturates to a reverse leakage current due to the drift current of minority carriers

Minority Carrier injection

- The ideal diode equation presumes that the minority carriers go away after they are injected across the junction
- Since there is no field outside the depletion region, they move away by diffusion, and recombine with the majority carriers.
- The leakage current also depends on generation and diffusion, because the number of minority carriers near the edges of the depletion region get there by diffusion and generation, and this makes real diodes have IV characteristics which don't follow the ideal diode law, in particular they have higher reverse leakage than would be predicted from their forward characteristic.

Next time…

- In silicon, the recombination rate is very slow, so the injected minority carriers can go quite a distance, creating havoc in other devices sometimes
- The majority of the injection is into the lighter doped region, so it is possible to efficiently produce injected minority carriers in a region
- Minority carriers injected across one junction can cause a large leak across another reverse biased junction.
- This forms the basis for the BJT
- In the next lecture, we will study what happens to injected minority carriers further.