Lecture 31

- Last time:
  - Short-circuit current gain of CE and CS amps
  - Unity-gain frequency \( f_0 \)

- Today:
  - Frequency response of the CE as voltage amp
  - The Miller approximation
Magnitude Bode Plot

$\beta_0$

20db (mV)

$40\text{db}

100$

$20\text{db} \text{ decada}$

$\gamma$

100

$\beta_0$

$\omega_p$

10$\omega_p$

100$\omega_p$

$\omega_0$

BW

$-20\text{db}$

$W_2 = \frac{b_0}{c_p}$

$A_i(j\omega) \approx \frac{b_0}{1+j\omega/\omega_p}$

$\omega_0$

$10^6$

$\omega_T$

Transition Frequency

Indenanz

$W_T$

$w_p$

$w_0$

$w_0 > w_p$?
Transition Frequency $\omega_T$

$$\omega_T = \beta_0 \omega_p = \frac{g_m}{C_m + C_t}$$

Dependence on DC collector current:

$$I_C = I_i + \left( \frac{I_e}{C_{je} + (I_e / V_{th}) Y_{th}} \right) (I_p + I_h)$$

Limiting case:

$$f_T = \frac{\omega_T}{2 \pi} \rightarrow \frac{1}{2 \pi f_F}$$

Current record: 40.6 GHz
Common Source Amplifier: $A_i(j\omega)$

DC Bias is problematic: what sets $V_{gs}$?
CS Short-Circuit Current Gain

Transfer function: \[ A_i(j\omega) = \frac{g_m (1 - j\omega C_{gd} / g_m)}{j\omega (C_{gs} + C_{gd})} \]

\[ V_{in} = \frac{V_{in}}{C_{gd}} \]
\[ A_i(j\omega) = 1 \]

\[ \frac{9\alpha}{j\omega \left( C_{gs} + C_{gd} \right)} = 1 \]

\[ W_T = \frac{9\alpha}{C_{gs} + C_{gd}} \]

\[ W_2 = \frac{9\alpha}{C_{gd}} \]

\[ C_{gs} + C_{gd} > C_{gd} \]
\[ W_T = \frac{q_m}{C_{gs} \cdot L_{yd}} \]

\[ \frac{1}{W_T} = \frac{L_{yd}}{q_m} = \frac{C_{gs} \cdot L_{yd}}{q_m} \quad C_{yd} < K \cdot C_{gs} \]

\[ C_{gs} \approx \frac{2}{3} \cdot C_0 \cdot W \cdot L \quad (C_0 = \sigma_0) \]

\[ q_m \approx \mu_n C_0 \left( \frac{W}{L} \right) (V_{gs} - V_{th}) \]

\[ L \leq \text{Length} \]

\[ T_T = \frac{L}{\left( \frac{3}{2} \right) \mu_n (V_{gs} - V_{th}) \cdot \frac{V_{gs} - V_{th}}{L}} \]

\[ \text{mobility } \mu_n \]

\[ \text{velocity } v \]
Common-Emitter Voltage Amplifier

Small-signal model: omit $C_{cs}$ due to avoid complicated analysis
CE Voltage Amp Small-Signal Model

switch to Norton equivalent at input

$R'_{in} = R_s \parallel r_{\pi}$

$R'_{out} = r_o \parallel r_{oc} \parallel R_L$
Frequency Response

KCL at input and output nodes; analysis is made complicated due to $Z_\mu$ branch → see H&S pp. 639-640.

$$\frac{V_{out}}{V_{in}} = \frac{-g_m \left( \frac{r_\pi}{r_\pi + R_S} \right) [r_o \parallel r_{oc} \parallel R_L] (1 - j\omega/\omega_z)}{(1 + j\omega/\omega_{p1}) (1 + j\omega/\omega_{p2})}$$

Low-frequency gain:

$$-g_m \frac{2n}{2n+8} \cdot \left[ R_0 \parallel \left( \frac{2n}{2n+8} \right) \right]$$

Zero: $\omega_z > \omega_T = \frac{g_m}{C_\pi + C_\mu}$

Very high $\omega_T$ our models do not apply there
Poles

\[\begin{align*}
\omega_{p1} & \approx \frac{1}{(R_S \parallel r_\pi)\{C_\pi + (1 + g_m R'_\text{out})C_\mu\} + R'_\text{out}C_\mu} \\
\omega_{p2} & \approx \frac{R'_\text{out}((R_S \parallel r_\pi))}{(R_S \parallel r_\pi)\{C_\pi + (1 + g_m R'_\text{out})C_\mu\} + R'_\text{out}C_\mu}
\end{align*}\]

\(R'_\text{out} = \text{set by load at output}\)

\(\text{Pout} = 20\text{dBm} \pm 1\text{dB}\)
1st pole much lower frequency!

2nd pole neglected (pole frequency is very high!)