Lecture 4

- Last time:
  - Circuit analysis with phasors: impedances

- Today:
  - Bode plot of low-pass filter (start)
  - Bode plot sketching for first-order transfer functions
  - Low-pass and high-pass filters

- Monday 1-2 Discussion

- 2/14: Cory Hinkle
- 2/21, and later: S. Evans Hines

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Redrawing the Circuit with Impedances

Note: this is not a "real" circuit that could be built and tested!
Transfer Function

Ratio of output to input phasor is called the transfer function of the circuit:

\[
H = \frac{V_c}{V_s} = \frac{1}{1 + j \omega RC}
\]

\[
|H|_{\text{dB}} = 20 \log_{10} \left| \frac{1}{1 + j \omega RC} \right|
\]

\[
\angle H = \angle \frac{1}{1 + j \omega RC} = \angle 1 - \tan^{-1} \left( \frac{\omega RC}{1} \right)
\]

\[
U_c(t) \text{ given } U_s(t)
\]
Bode Plots for Low-Pass Filter

1. Plot magnitude $|H|$ in dB vs. $\omega$ (log scale)

$$|H| \text{ dB} = 20 \log \left| \frac{1}{1 + j\omega \tau} \right|$$

2. Plot phase $\angle H$ in degrees vs. $\omega$ (log scale)

$$\angle H = -\tan^{-1}\left( \frac{\omega}{\tau} \right)$$

$$H(f) = \frac{1}{1 + j2\pi f \tau}$$

Why?
The Break Frequency \( \omega_{-3dB} = \frac{1}{\tau} \)

\[ |H|_{dB} = \frac{V_c}{V_s} \Rightarrow 0 \text{ dB} \]

\[ |H|_{dB} = \frac{V_c}{V_s} \Rightarrow 0 \text{ dB} \Rightarrow V_c = V_s \]

<table>
<thead>
<tr>
<th>( \omega )</th>
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<tbody>
<tr>
<td>0.10</td>
</tr>
<tr>
<td>0.01</td>
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<tr>
<td>0.001</td>
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\[ |H(j\omega)|_{dB} = 20 \log_{10} \left[ \sqrt{1 + (\omega \tau)^2} \right] \]

\[ \approx -20 \log_{10} (\omega \tau) = +20 \log_{10} \left( \frac{\omega}{\omega \tau} \right) \]
Sketching the Magnitude Plot

\[ |H|_{dB} = 20 \log \left( \frac{1}{|1 + j \omega \tau|} \right)_{dB} = 20 \log \left( \frac{1}{\sqrt{1 + (\omega \tau)^2}} \right) \]

- Low-frequency (\(\omega \tau \ll 1\)) asymptote:
  \[|H(\infty)| = 1; \quad |H(\infty)|_{dB} = 0 \text{ dB}\]

- High-frequency (\(\omega \tau \gg 1\)) asymptote:
  \[\frac{1}{\sqrt{1 + (\omega \tau)^2}} \approx \frac{1}{\omega \tau} = \frac{1}{10} \implies H(8 \cdot \left(\frac{1}{10}\right)) \approx -20 \text{ dB}\]

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Finding the Phase Plot

\[ \angle (H) = \angle \left[ \frac{1}{1 + j\omega \tau} \right] = 0 - \arctan(\omega \tau) \]

Why?

- **Theorem!**

  - Low-frequency asymptote
    \[ \omega T < 1 \quad \arctan(\omega 0) = 0 \]
    \[ \angle H \propto 0. \]

  - High-frequency asymptote
    \[ \omega T > 1 \quad \angle H = 0 - \tan^{-1}\left\{ \frac{B}{1}\right\} = 0 - 90^\circ \]
    \[ = -90^\circ. \]

Approx. linear with \( \omega \) for \( 1/(10\tau) < \omega < 10/\tau \)
Rapidly Sketching the Phase Plot

\[ \tan^{-1}\left(\frac{f(0)}{10}\right) = 6^\circ \]
\[ \tan^{-1}\left(\frac{f(10)}{10}\right) = 86^\circ \]
Average Power and Phasors

Integrate $P(t)$ over one period:

$$P = \int_0^T i(t)v(t)dt = \int_0^T I I^* \cos(\omega t + \angle I) V V^* \cos(\omega t + \angle V) dt$$

Result:

$$\langle P \rangle = \frac{1}{2} \left| V \right|^2 \cos(\angle I - \angle V) + \frac{1}{2} \left| I \right|^2 \left( \cos(\angle V + \sin(\angle I \sin V) \right)$$

$$V = \frac{\left| I \right| V^*}{\left| I \right|}$$
The High-Pass Filter

\[ V_s(t) = v_i \cos \omega t \]

\[ v_f(t) = v_r \cos [(\omega t + \phi_f)] \]
Impedance Divider

Insight:

\[ |Z_L| \text{ m. a.} \quad \frac{V_L}{V_s} = \frac{V_t}{V_s} \]

\[ H = \frac{V_t}{V_s} \]

High Q

\[ V_s \]

\[ \text{Seemingly R} \quad \text{vs} \]

\[ \text{vs} \]

\[ \phi \rightarrow \theta \]

\[ V_t - V_L \]

\[ \text{Low Q} \]

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\[ H = \frac{V_r}{V_c} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC}. \]

\[ \omega \to 0 \quad H \to \frac{j\omega \tau}{j\omega \tau} = 1 \]

\[ \omega \to \infty \quad H \to \frac{0}{1 + 0} = 0 \]
Graphical Addition of Magnitudes