Labor This Week
See the... to

Lab 3 get spice

Last time:

- Imaginary exponentials: simplify the math

Today:

- Phasor complex "prefactor" for \( e^{j\omega t} \)
- Complex number review
- Circuit analysis with phasors
Finding the "Real" Waveform

- How to connect the imaginary exponential solution to the measured waveform $v(t)$?

Conventionally, $v(t)$ is the real part of the imaginary exponential

$$\text{Re}(ve^{j(\omega t + \phi)}) = v \cos(\omega t + \phi)$$

$R \cdot x + jy = x$
Pushing This Idea Further …

There are two parameters needed to define a sinusoidal signal:
- magnitude
- phase

Why not work with a complex number as the signal and eliminate the imaginary exponential from the analysis (it cancelled out)?

Define the complex number consisting of the amplitude and phase a sinusoidal signal as a **phasor**

\[ v(t) = v \cos(\omega t + \phi) \iff v(t) = Ve^{j\omega t} \]

\[ V = ve^{j\phi} \]
Complex Number Summary

- Rectangular form: $z = x + jy$
- Magnitude $|z| = \sqrt{x^2 + y^2}$
- Phase $\angle z = \tan^{-1}(y/x)$
- Polar form: $z = |z|e^{j\theta}$

Useful results (easily shown in polar form):

$\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|}e^{j(\theta_1 - \theta_2)}$

$|z_1z_2| = |z_1||z_2|$

Question:

$|1 - j| = \sqrt{1^2 + (-1)^2}$
\[ \begin{align*} 
& \text{Go Polar!} \\
& \phi = 0 + \theta \quad \text{Polar Form} \\
& \text{Note: } \begin{cases} 
\text{with} \\
\text{there is:} \\
\end{cases} 
\end{align*} \]
Using Phasors: Capacitor Current

\[ i_c(t) = C \frac{dv_c}{dt} \]

\[ v_c(t) = V_c e^{j\omega t} \]

\[ i_c(t) = I_c e^{j\omega t} \]

Result:

\[ I_c = (j\omega C) \cdot V_c \]
Impedance of a Capacitor.

Definition: the impedance \( Z \) of a two-terminal circuit element is the ratio of the phasor voltage to the phasor current (positive reference convention)

\[
I_c \quad C \quad + \quad V_c
\]

\[
V_c = Z_c \cdot I_c
\]

Admittance:

\[
Y_c = \frac{1}{Z_c} = j\omega C
\]

\[
I_c = (g_{wc}) \cdot V_e
\]

\[
Z_c = \frac{V_e}{I_c}
\]

\[
Z_c = \frac{V_c}{g_{wc} \cdot V_e} = \frac{1}{j\omega C}
\]
Using Phasors: Inductor Voltage

\[ v_L(t) = L \frac{di_L}{dt} \]

\[ i_L(t) = I_0 e^{j\omega t} \]

\[ v_L(t) = V_L e^{j\omega t} \]

Result:

\[ v_L = V_L e^{j\omega t} \]

\[ I_L = I_0 e^{j\omega t} \]

\[ z_L = \frac{V_L}{I_0} \]

\[ z_L = \rho L \]
Inductor Impedance

\[ i(t) = \frac{V}{R} - \frac{1}{R} \int i(t) dt \]

\[ V(t) = I_E e^{\omega t} \]

Admittance: \[ Y_L = 1/Z_L = \frac{1}{\frac{1}{j\omega L}} = \frac{1}{\omega L} \]

\[ \frac{\partial L}{\partial t} = \frac{1}{\frac{1}{j\omega L}} = \frac{1}{\omega L} \]

\[ Z_L \]

\[ V_L \]

\[ L \]

\[ i(t) = \frac{V}{R} - \frac{1}{R} \int i(t) dt \]

\[ V(t) = I_E e^{\omega t} \]

\[ Y_L = 1/Z_L = \frac{1}{\frac{1}{j\omega L}} = \frac{1}{\omega L} \]

\[ \frac{\partial L}{\partial t} = \frac{1}{\frac{1}{j\omega L}} = \frac{1}{\omega L} \]
Kirchhoff's Current Law Example

At node $a$:

$$i_a(t) = i_{c_1}(t) + i_{c_2}(t)$$

$\text{rest of}$

$$\text{cnt.}$$

$I_s \propto e^{jwt}$

$$i_{c_1} = I_{c_1} e^{jwt} = C_1 \frac{d(v_{c_1})}{dt} = C_1 \frac{d(v_a - v_b)}{dt}$$

$$i_{c_2} = I_{c_2} e^{jwt} = C_2 \frac{d(v_{c_2})}{dt}$$

$$v_a(t) = V_a e^{jwt}$$

$$v_b(t) = V_b e^{jwt}$$

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Circuit Analysis with Phasors

Assumption: sources are sinusoidal, steady-state!

\[ v_s(t) = v_s \cos(\omega t + \phi) \]

\[ v_f(t) = 0 \]

Pick 0°.
\[ I_s = I_c_1 + I_c_2 \]

\[ I_s = \left( 2 \omega C_1 \right) V_s + (\omega C_2)(V_a - V_b) \]

KCL

\[ V_s = \frac{V_a - V_b}{Z_{in}} \]