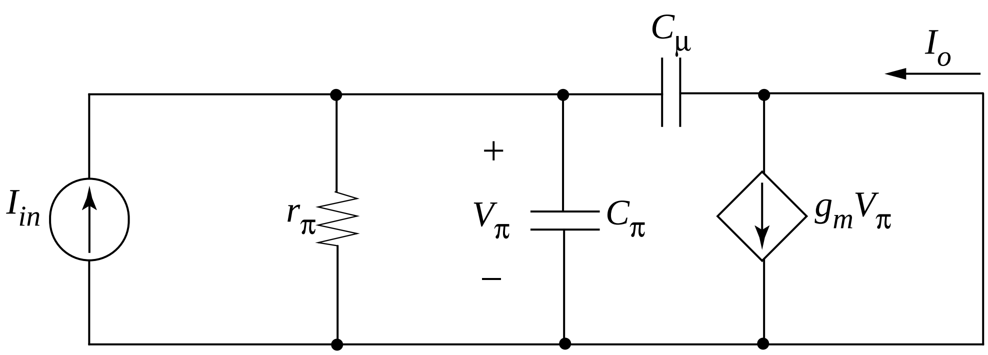
# Frequency Response of Transistor Amplifiers

- Simplest case: CE short-circuit current gain $A_i(j\omega)$ as a function of frequency




Kirchhoff's current law at the output node:

$$I_o = g_m V_\pi - V_\pi j\omega C_\mu$$

Kirchhoff's current law at the input node:

$$I_s = \frac{V_\pi}{Z_\pi} + V_\pi j\omega C_\mu \qquad \text{where} \qquad Z_\pi = r_\pi \| \left(\frac{1}{j\omega C_\pi}\right)$$

- Solving for $V_\pi$ at the input node:

$$V_\pi = \frac{I_s}{(1/Z_\pi) + j\omega C_\mu}$$

# Short-Circuit Gain Frequency Response

- Substituting $V_\pi$ into the output node equation--

$$\frac{I_o}{I_s} = \frac{g_m Z_\pi \left(1 - \dfrac{j\omega C_\mu}{g_m}\right)}{1 + j\omega C_\mu Z_\pi}$$

- Substituting for $Z_\pi$ and simplifying --

$$\frac{I_o}{I_s} = \frac{g_m r_\pi \left(1 - \dfrac{j\omega C_\mu}{g_m}\right)}{1 + j\omega r_\pi (C_\pi + C_\mu)} = \frac{\beta_o \left(1 - \dfrac{j\omega C_\mu}{g_m}\right)}{1 + j\omega r_\pi (C_\pi + C_\mu)} = \beta_o \left[\frac{1 - j\dfrac{\omega}{\omega_z}}{1 + j\dfrac{\omega}{\omega_p}}\right]$$

Current gain has one pole:

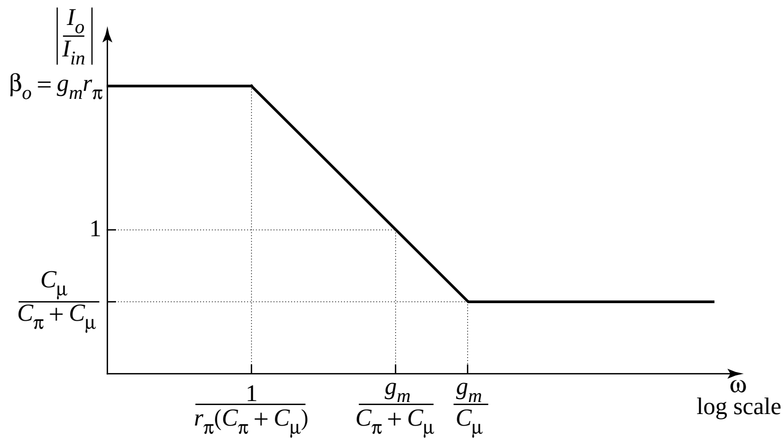
$$\omega_p = (r_\pi (C_\pi + C_\mu))^{-1}$$

and one zero

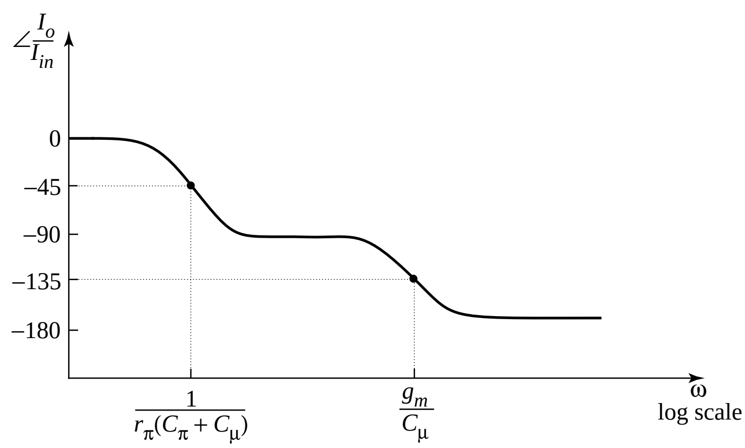
$$\omega_z = (g_m^{-1} C_\mu)^{-1} \gg \omega_p$$

# Bode Plot of Short-Circuit Current Gain

- Note low frequency magnitude of gain is $\beta_o$




(a)




(b)

- Frequency at which current gain is reduced to 0 dB is defined as the **transition frequency** $\omega_T$. Neglecting the zero,

$$\omega_T = \frac{g_m}{(C_\pi + C_\mu)}$$

# Transition Frequency of the Bipolar Transistor

- Dependence of transition time $\tau_T = \omega_T^{-1}$ on the bias collector current $I_C$:

$$\tau_T = \frac{1}{\omega_T} = \frac{C_\pi + C_\mu}{g_m} = \frac{g_m \tau_F + C_{jE} + C_\mu}{g_m}$$

$$\tau_T = \tau_F + \left(\frac{C_{jE} + C_\mu}{g_m}\right) = \tau_F + \frac{V_{th}}{I_C}(C_{jE} + C_\mu)$$

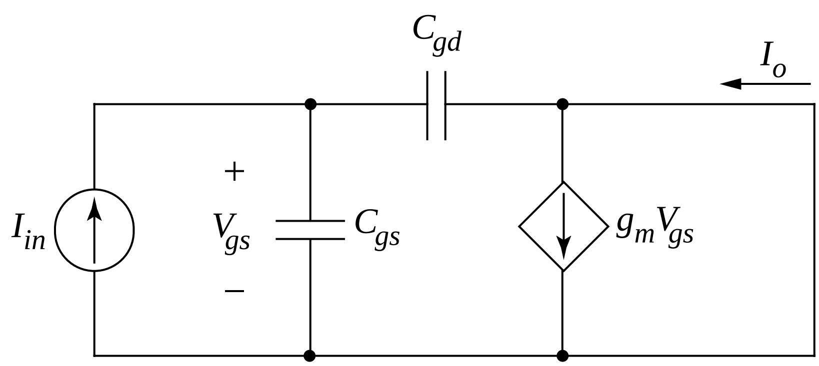
- If the collector current is increased enough to make the second term negligible, then the minimum $\tau_T$ is the base transit time, $\tau_F$.. In practice, the $\omega_T$ decreases at very high values of $I_C$ due to other effects and the minimum $\tau_T$ may not be achieved.

- Numerical values of $f_T = (1/2\pi)\omega_T$ range from 10 MHz for lateral pnp's to 10 GHz for oxide-isolated npn's

  Note that the small-signal model is not valid above $f_T$ (due to distributed effects in the base) and the zero in the current gain is not observed

# Common-Source Current Gain

- CS amplifier has a non-infinite input *impedance* for $\omega > 0$ and we can measure its small-signal current gain.




- Analysis is similar to CE case; result is

$$\frac{I_o}{I_{in}} = \frac{g_m\left(1 - \frac{j\omega C_{gd}}{g_m}\right)}{j\omega(C_{gs} + C_{gd})} \approx \frac{g_m}{\omega(C_{gs} + C_{gd})}$$

- Transition frequency for the MOSFET is

$$\boxed{\omega_T \approx \frac{g_m}{C_{gs} + C_{gd}}}$$

# Transition Frequency of the MOSFET

- Substitution of gate-source capacitance and transconductance:

$$C_{gs} = \frac{2}{3}WLC_{ox} \gg C_{gd} \text{ and } g_m = \frac{W}{L}\mu_n C_{ox}(V_{GS} - V_{Tn})$$

$$\omega_T \approx \frac{g_m}{C_{gs}} = \frac{\frac{W}{L}\mu_n C_{ox}(V_{GS} - V_{Tn})}{\frac{2}{3}WLC_{ox}} = \frac{3}{2}\mu_n\left[\frac{(V_{GS} - V_{Tn})}{L}\right]L$$

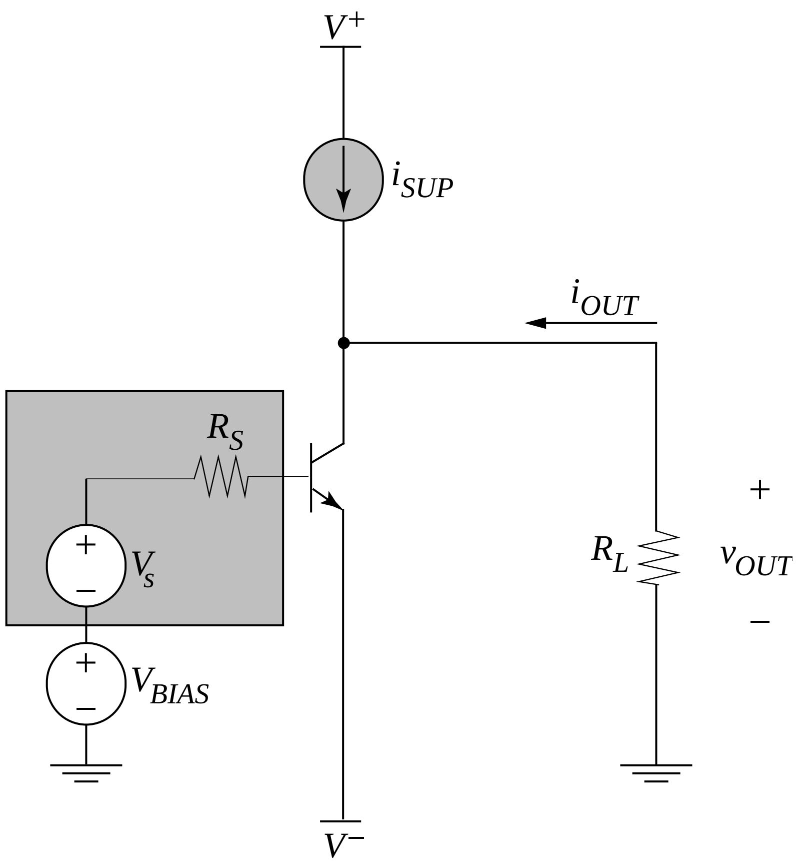
- The transition time is the inverse of $\omega_T$ and can be written as the average time for electrons to drift from source to drain

$$\tau_T = \frac{L}{\mu_n\left[\frac{2}{3}\frac{(V_{GS} - V_{Tn})}{L}\right]} = \frac{L}{|\overline{v_{dr}}|}$$

velocity saturation causes $\tau_T$ to decrease linearly with $L$; however,submicron MOSFETs have transition frequencies that are approaching those for oxide-isolated BJTs

# Frequency Response of Voltage Amplifiers

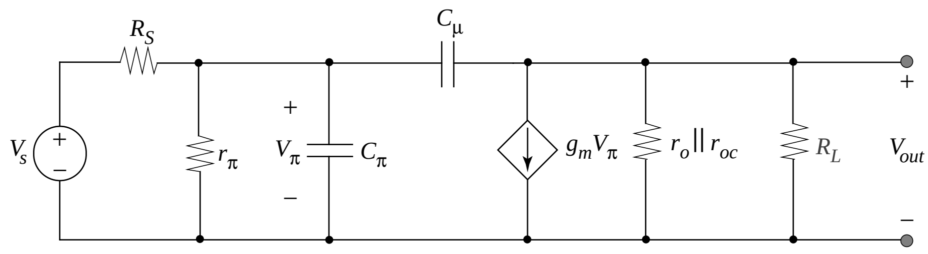
- Common-emitter amplifier:



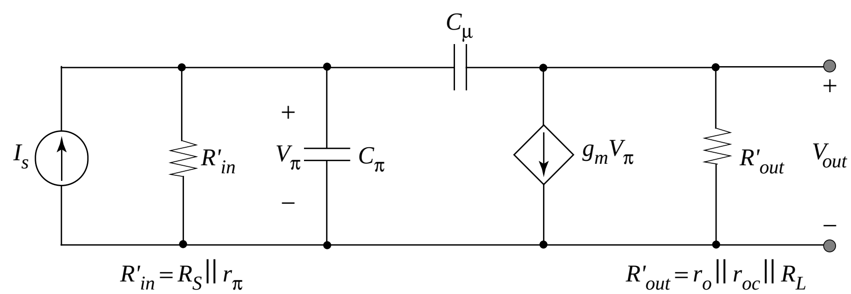

Procedure: substitute small-signal model and perform phasor analysis

# Brute Force Phasor Analysis

- "Exact" analysis: transform into Norton form at input to facilitate nodal analysis




(a)




(b)

Note that $C_{cs}$ is omitted, along with $r_b$

Details: see Section 10.4

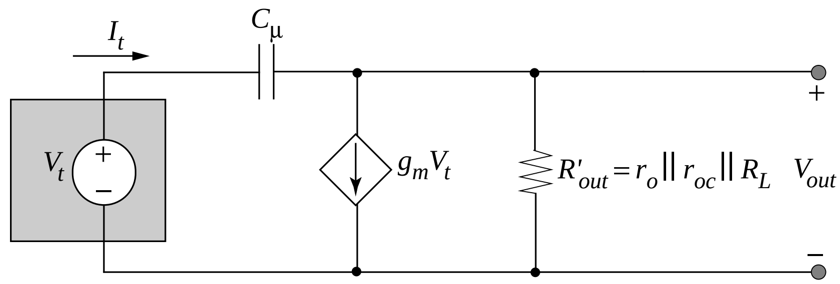
Factor (approximately!) into one high-frequency zero and two poles $\omega_1 << \omega_2$

$$\omega_1^{-1} = (r_\pi || R_S)(C_\pi + (1 + g_m r_o || r_{oc} || R_L) C_\mu) + (r_o || r_{oc} || R_L) C_\mu$$

$$\omega_2^{-1} = \frac{(r_o || r_{oc} || R_L)(r_\pi || R_S) C_\mu C_\pi}{(r_\pi || R_S)(C_\pi + (1 + g_m r_o || r_{oc} || R_L) C_\mu) + (r_o || r_{oc} || R_L) C_\mu}$$

# The Miller Approximation

- The "exact" analysis is not particularly helpful for gaining insight into the frequency response ... consider the effect of $C_\mu$ on the input only




neglect the feedforward current $I_\mu$ in comparison with $g_m\ V_\pi$ ... a good approximation

$I_t = (V_t - V_o) / Z_\mu$

$V_o = - g_m V_t\ R_L / (R_L + R_{out}') = A_{vC\mu}\ V_t$

where $A_{vC\mu}$ is the low frequency voltage gain across $C_\mu$

$I_t = V_t\,(1 - A_v) / Z_\mu$

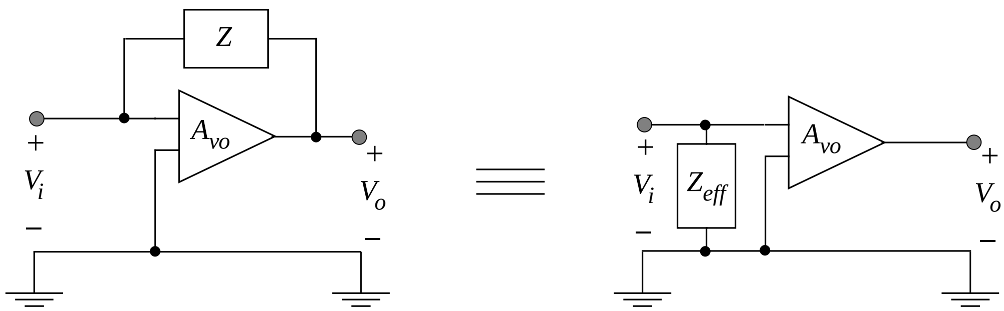
$Z_{eff} = V_t / I_t = Z_\mu / (1 - A_v)$

$$Z_{eff} = \frac{1}{j\omega C_\mu}\left(\frac{1}{1 - A_{vC_\mu}}\right) = \frac{1}{j\omega(C_\mu(1 - A_{vC_\mu}))} = \frac{1}{j\omega C_M}$$

$C_M = (1 - A_{vC_\mu})C_\mu$ is the **Miller capacitor**

# Generalized Miller Approximation

- An impedance $Z$ connected across an amplifier with voltage gain $A_{vZ}$ can be replaced by an impedance to ground ... multiplied by $(1\text{-}A_{vZ})$




$Z_{eff} = Z/(1 - A_{vo})$

- Common-emitter and common-source:

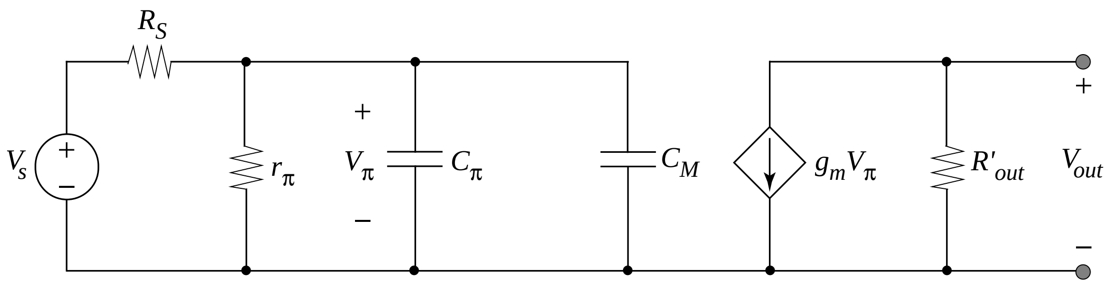
  $A_{vZ}$ = large and negative for $C_\mu$ or $C_{gd}$ --> capacitance at the input is **magnified**

- Common-collector and common-drain:

  $A_{vZ} \approx 1$ --> capacitance at the input due to $C_\pi$ or $C_{gs}$ is greatly **reduced**

# Voltage Gain vs. Frequency for CE Amplifier Using the Miller Approximation

- The Miller capacitance is lumped together with $C_\pi$, which results in a single-pole low-pass $R$C filter at the input




$C_M = C_\mu(1 + g_m R'_{out})$

Transfer function has one pole and no zero after Miller approximation:

$$\omega_{3dB}^{-1} = (r_\pi \| R_S)(C_\pi + C_M)$$

$$\omega_{3dB}^{-1} = (r_\pi \| R_S)[C_\pi + (1 + g_m r_o \| r_{oc} \| R_L)C_\mu]$$

$\omega_{3dB}^{-1} \approx \omega_1^{-1}$ from the exact analysis (final term $R_{out}{}'C_\mu$ is missing)