UNIVERSITY OF CALIFORNIA AT BERKELEY
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EE105 Lab Experiments

Bode Plot Tutorial

Contents

1 Introduction

Although you should have learned about Bode plots in previous courses, this tutorial will give you a brief review of the material in case your memory is fuzzy.

2 Bode Plots Basics

Making the Bode plots for a transfer function involves drawing both the magnitude and phase plots. The magnitude is plotted in decibels (dB) while the phase is plotted in degrees (°). For both plots, the horizontal axis is either frequency (f) or angular frequency (ω), measured in Hz and rad/s respectively. In addition, the horizontal axis should be logarithmic (i.e. increasing by factors of 10).

Most of the transfer functions we will encounter in this lab manual can be rearranged into the general form:

$$H(j\omega) = A \cdot \frac{j\omega/\omega_{z1}\,(1 + j\omega/\omega_{z2})\,(1 + j\omega/\omega_{z3})\dots}{j\omega/\omega_{p1}\,(1 + j\omega/\omega_{p2})\,(1 + j\omega/\omega_{p3})\dots}\ , \tag{1}$$

where A is an arbitrary constant and j is $\sqrt{-1}$. Besides the exception of $j\omega/\omega_c$, the basic component of this transfer function is $1 + j\omega/\omega_c$, where ω_c is some numerical constant. Let us analyze this basic component first before we analyze the transfer function as a whole.

2.1 Magnitude

Recall the definition of magnitude (measured in dB):

$$|H(j\omega)|_{\mathrm{dB}} = 20\log|H(j\omega)| = 20\log\sqrt{(\Re\,[H(j\omega)])^2 + (\Im\,[H(j\omega)])^2} \tag{2}$$

Let us apply this definition to our basic component $(1 + j\omega/\omega_c)$, which is also called a **zero** when it appears in the numerator of the transfer function:

$$|1 + j\omega/\omega_c|_{\mathrm{dB}} = 20\log|1 + j\omega/\omega_c| = 20\log\sqrt{1 + (\omega/\omega_c)^2} \tag{3}$$

For small values of ω, we have $20\log|1 + j\omega/\omega_c| \approx 0$ dB. For large values of ω, $20\log|1 + j\omega/\omega_c| \to \infty$. When $\omega = \omega_c$, the magnitude of the transfer function is approximately 3 dB.

Since there is little change in the magnitude of the transfer function from $\omega = 0$ to $\omega = \omega_c$, we can approximate the magnitude expression as equal to 0 dB within this interval. As for the $\omega > \omega_c$ interval, the (ω/ω_c) term dominates in the expression; thus, we can approximate the magnitude as $20\log(\omega/\omega_c)$. Also, notice that, within the $\omega > \omega_c$ interval, the magnitude increases by 20 dB when ω increases by a factor of 10. Therefore, the overall Bode plot approximation for a zero is the following: 0 dB for $\omega < \omega_c$ and a 20 dB/decade line for $\omega > \omega_c$. Please see Figure 1 for an illustration of this approximation.

Figure 1 also shows the magnitude Bode plot for a **DC zero**, which has the form $j\omega/\omega_c$. Because the DC zero lacks the "1" term, its Bode plot approximation differs from the normal zero: instead of two different regions, the DC zero consists of only a 20 dB/decade line. Its intersection with the frequency axis is located at ω_c because $20\log(\omega_c/\omega_c) = 0$dB. Note that ω_c is 10^5 rad/s in this example.

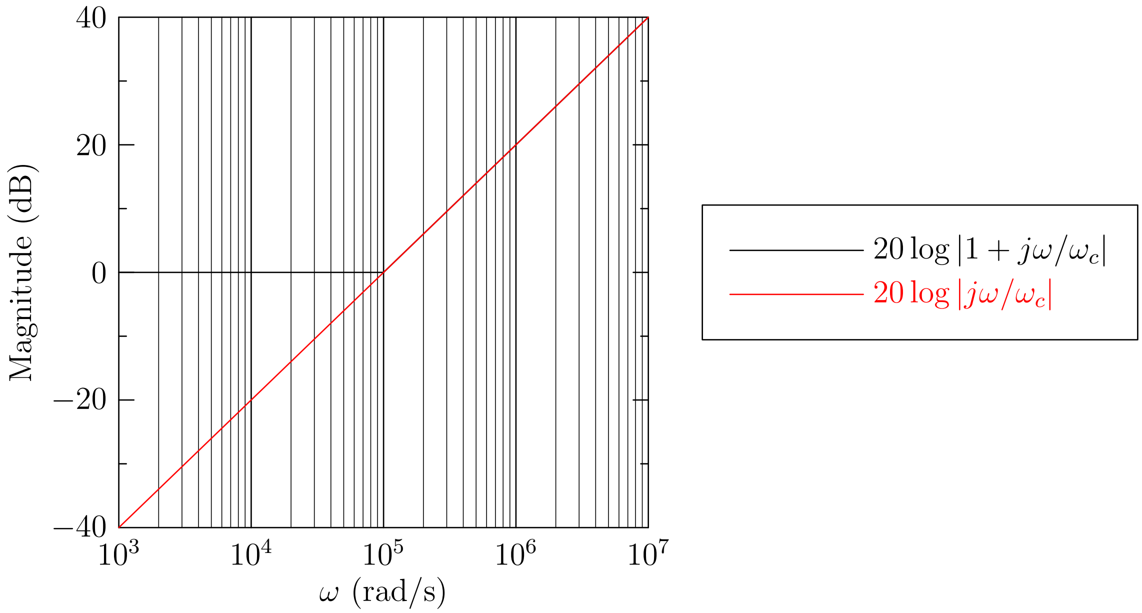

Figure 1: Magnitude Bode plots of a normal zero and a DC zero (with both having $\omega_c = 10^5$ rad/s). Notice the plots overlap for $\omega > \omega_c$.

The basic transfer function component, $1 + j\omega/\omega_c$, can also appear in the denominator (in which case it is called a **pole**). Even though this may seem like an entirely different problem, the same principles apply. Recall that we took the logarithm of our transfer function when we expressed our results in decibels. Also, recall that taking the logarithm of the inverse of a function simply gives the negated logarithm of the function. In other words, we simply have to negate the results of our zero analysis to get the appropriate expressions for poles. The same argument applies with **DC poles**, which has the form $j\omega/\omega_c$.

In general, a normal pole will have a constant 0 dB value for $\omega < \omega_c$ and will drop by 20 dB/decade for $\omega > \omega_c$. A DC pole will drop by 20 dB/decade for any ω and will intersect the frequency axis (0 dB) at $\omega = \omega_c$. The results are shown in Figure 2.

2.2 Phase

Now, let us take a look at the respective phases of a zero, DC zero, pole, and DC pole. Recall the definition of phase:

$$\text{Arg}\left(H(j\omega)\right) = \arctan\left(\frac{\Im[H(j\omega)]}{\Re[H(j\omega)]}\right) \tag{4}$$

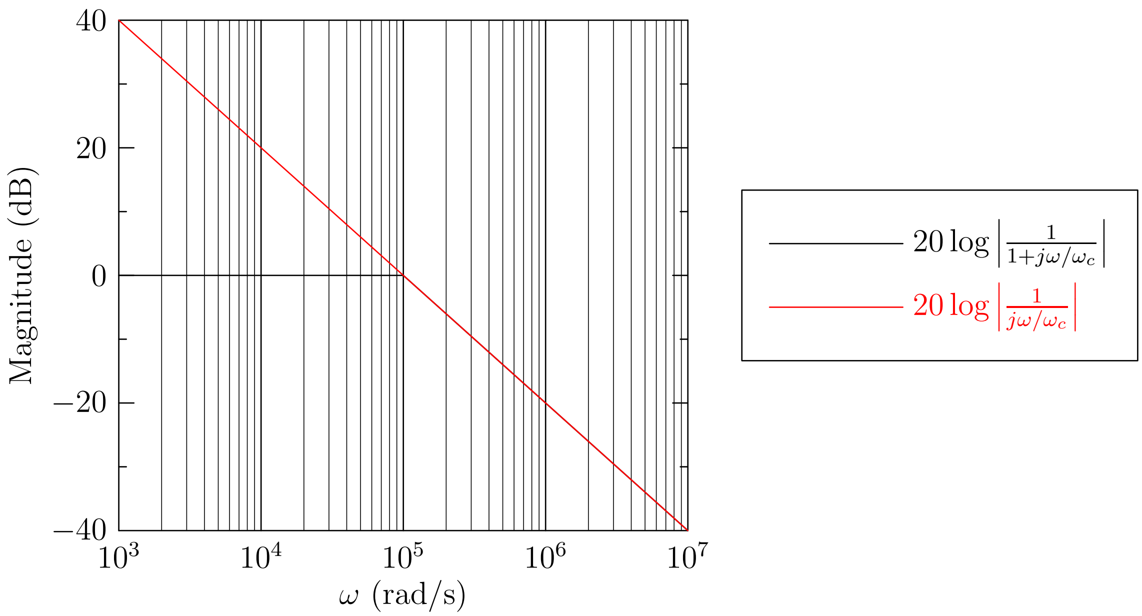

Figure 2: Magnitude Bode plots of a normal pole and a DC pole (with both having $\omega_c = 10^5$ rad/s). Notice the plots overlap for $\omega > \omega_c$.

Applying this definition to the normal zero:

$$\text{Arg}(1 + j\omega/\omega_c) = \arctan\left(\frac{\omega}{\omega_c}\right) \tag{5}$$

For $\omega = 0$, $\text{Arg}(1 + j\omega/\omega_c) = 0$. For $\omega \to \infty$, $\text{Arg}(1 + j\omega/\omega_c) \to 90°$. For $\omega = \omega_c$, $\text{Arg}(1 + j\omega/\omega_c) \to 45°$. Thus, our approximation for the phase of a zero is the following: $0°$ for $\omega < 0.1\omega_c$, $45°$ for $\omega = \omega_c$, and $90°$ for $\omega > 10\omega_c$. The phase Bode plot is then constructed using straight lines that join these regions together. As for the DC zero, its phase stays constant at $90°$. Please refer to Figure 3 for an illustration of these plots.

The phase analysis of poles and DC poles can be conducted in a similar fashion. Let us begin by applying the definition of phase to the pole and DC pole, respectively:

$$\text{Arg}\left(\frac{1}{1 + j\omega/\omega_c}\right) = \text{Arg}\,(1) - \text{Arg}\,(1 + j\omega/\omega_c) = 0 - \text{Arg}\,(1 + j\omega/\omega_c) = -\text{Arg}\,(1 + j\omega/\omega_c) \tag{6}$$

$$\text{Arg}\left(\frac{1}{j\omega/\omega_c}\right) = \text{Arg}\,(1) - \text{Arg}\,(j\omega/\omega_c) = 0 - \text{Arg}\,(j\omega/\omega_c) = -\text{Arg}\,(j\omega/\omega_c) \tag{7}$$

As you might notice, our phase plots for poles and DC poles will simply be the negated versions of the zero plots. Please see Figure 4 for an illustration.

3 Combining Poles and Zeroes

Generally, a transfer function may involve many poles and zeroes (as well as their DC counterparts). In order to simplify the task of drawing Bode plots, your first step should be to factor the transfer function into the canonical form as shown in Equation 1. This makes it easy to identify all of the poles and zeroes.

Then, you'll have to handle the constant coefficient, A (if it is present). The magnitude of A will affect your magnitude plot, and the sign of A will affect your phase plot. More specifically, your magnitude plot must be offset by $20\log|A|$. For example, if $A = 10$, then your magnitude plot must be shifted up by 20 dB.

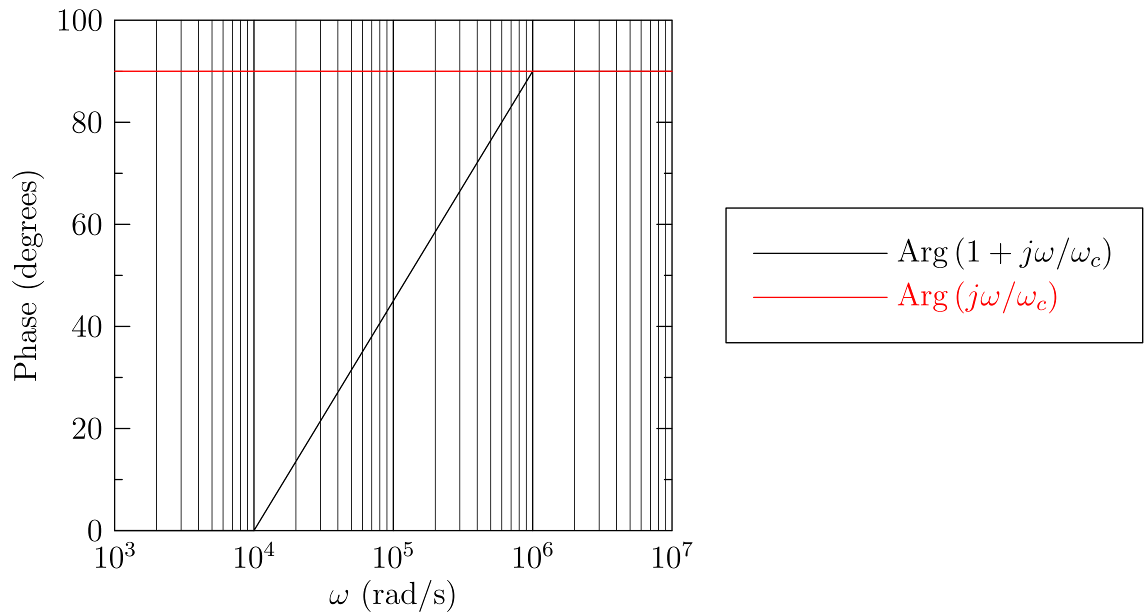

Figure 3: Phase Bode plots of a normal zero and a DC zero (with both having $\omega_c = 10^5$ rad/s). Notice the plots overlap for $\omega > 10\omega_c$.

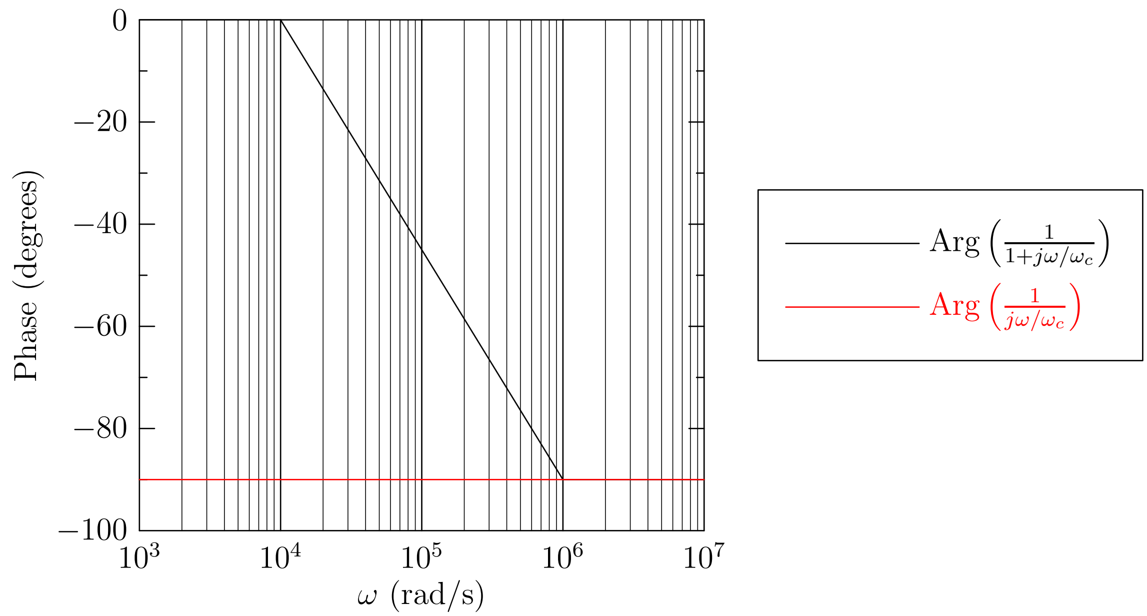

Figure 4: Phase Bode plots of a normal pole and a DC pole (with both having $\omega_c = 10^5$ rad/s). Notice the plots overlap for $\omega > 10\omega_c$.

Similarly, if $A = 1/10$, then your magnitude plot must be shifted down by 20 dB. If $A < 0$, then your phase plot must be shifted up (or down—it's the same in this case) by 180°. Next, you need to draw each pole and zero plot individually on the same graph (whether you're making a magnitude or phase plot). Finally, add together all the curves that you have drawn to obtain the final Bode plot. Remember to shift your plots accordingly based on the constant A as mentioned previously. This superposition principle is possible because of the decomposition of the transfer function into zeroes and poles.

When adding the poles and zeroes in the final plot, remember that in areas where two curves are constant, the result will just be the sum of the constant values. When one is a constant and the other is linear, the result will be a line that starts at that constant value and progress with a slope equal to the linear curve. Finally, when both are linear, the sum will be a line that has a slope equal to the sum of the slopes.