EE105 – Fall 2014
Microelectronic Devices and Circuits

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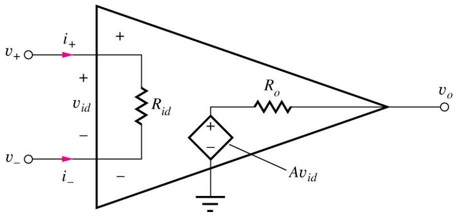
Ideal vs Non-ideal Op Amps



Ideal Op Amp

$$A = \infty$$
$$R_{id} = \infty$$
$$R_o = 0$$

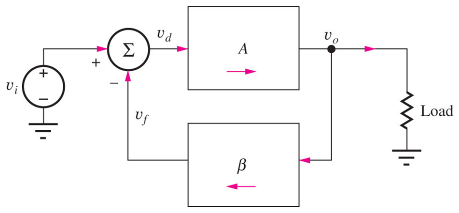
Non-ideal Op Amp

$$A < \infty$$
$$R_{id} < \infty$$
$$R_o > 0$$

Other non-ideal characteristics:

- **Finite common-mode rejection ratio (CMRR)**
  - **Output voltage is not just proportional to the difference input voltage but also their absolute values**
- **Common-mode input resistance**
- **dc error sources (offset voltage, input bias current, offset current)**
- **Output voltage, slew rate, maximum current limits**

Classic Feedback Systems



$$v_d = v_i - v_f$$
$$v_o = A v_d$$
$$v_f = \beta v_o$$

$$A_v = \frac{v_o}{v_i} = \frac{A}{1 + A\beta} = \frac{1}{\beta}\left(\frac{A\beta}{1 + A\beta}\right) = A_v^{Ideal}\left(\frac{T}{1+T}\right)$$

- **A = transfer function of open -loop amplifier or open-loop gain.**
- **β = transfer function of feedback network.**
- **T = loop gain = Aβ**
  - **For negative feedback: T(s) > 0**
  - **For positive feedback: T(s) < 0**

Gain Error and Fractional Gain Error

- **Gain Error:**
  - **GE = (ideal gain) - (actual gain)**
- **For non-inverting amplifier,**

$$GE = \frac{1}{\beta} - \frac{1}{\beta}\left(\frac{T}{1+T}\right) = \frac{1}{\beta}\left(\frac{1}{1+T}\right)$$

- **Fractional or percentage error:**

$$FGE = \frac{GE}{A_v^{Ideal}} = \frac{\frac{1}{\beta}\left(\frac{1}{1+T}\right)}{\frac{1}{\beta}} = \frac{1}{1+T} \cong \frac{1}{T} \quad \text{for} \quad T >> 1$$

Gain Error Example: Noninverting Amplifier

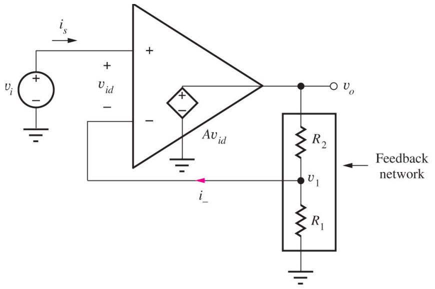
- **Problem: Find actual gain and gain error for an amplifier**
- **Given data: Ideal closed-loop gain of 200 (46 dB), open-loop gain of op amp is 10,000 (80 dB).**
- **Approach: Amplifier is designed to give ideal gain and deviations from ideal case are determined.**
- ***Note: R_1*** **and** ***R_2*** **are not normally designed to compensate for finite open-loop gain of amplifier.**
- **Analysis:**

$$A_v = A_v^{Ideal}\left(\frac{T}{1+T}\right) \quad \text{with} \quad T = A\beta = 10^4\left(\frac{1}{200}\right) = 50$$

$$A_v^{Ideal} = \frac{1}{\beta} = 200 \qquad A_v = 200\left(\frac{50}{1+50}\right) = 196 \qquad FGE = \frac{200-196}{200} = 0.02$$

$$\text{Note: } \; FGE \cong \frac{1}{T} = \frac{1}{50} = 0.02$$

Gain of Noninverting Amplifier



For $i_- = 0$, $\quad v_1 = \frac{R_1}{R_1+R_2}v_o = \beta v_o$

$$\beta = \frac{R_1}{R_1+R_2}$$ **Feedback factor**

$$v_o = Av_{id} = A\left(v_i - v_1\right) = A\left(v_i - \beta v_o\right)$$

$$A_v = \frac{v_o}{v_i} = \frac{A}{1+A\beta} = \frac{1}{\beta}\left(\frac{A\beta}{1+A\beta}\right) = \frac{1}{\beta}\left(\frac{T}{1+T}\right)$$

$T = A\beta$ is called the loop gain

For $T >> 1$

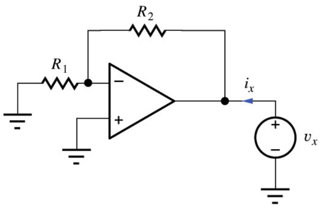
$$A_v \rightarrow A_v^{Ideal} = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

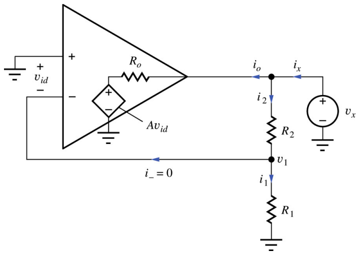
$$v_{id} = v_i - v_1 = v_i - \beta v_o$$

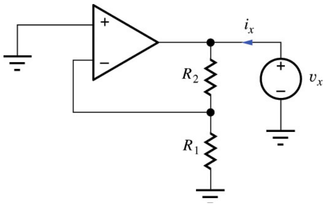
$$v_{id} = v_i - \frac{T}{1+T}v_i = \frac{v_i}{1+T}$$

Although no longer zero, v_{id} is small for large loop gain T.

Output Resistance (Both Noninverting and Inverting Amplifiers)







The output terminal is driven by test source v_x and current i_x is calculated to determine output resistance (all independent sources are turned off). Note that the equivalent circuit is same for both inverting and non-inverting amplifiers.

$$R_{out} = \frac{v_x}{i_x}$$

Output Resistance (cont'd)

Analysis: $i_x = i_o + i_2$

$$i_o = \frac{v_x - Av_{id}}{R_o} \quad \text{and} \quad i_2 = \frac{v_x}{R_2 + R_1}$$

$$\text{Also, } v_{id} = -v_1 \quad \text{and} \quad v_1 = \frac{R_1}{R_2 + R_1} v_x = \beta v_x$$

$$\frac{1}{R_{out}} = \frac{i_x}{v_x} = \frac{1 + A\beta}{R_o} + \frac{1}{R_2 + R_1}$$

$$R_{out} = \frac{R_o}{1 + T} \Big\| \left(R_2 + R_1\right)$$

Thus, shunt feedback reduces R_{out}.

Typically, $R_o/\left(1 + \mathrm{T}\right) << \left(R_2 + R_1\right)$ and $R_{out} \cong \frac{R_o}{1 + T}$. For infinite T, $R_{out} = 0$.

Nonideal Op Amps
Open-loop Gain: Design Example

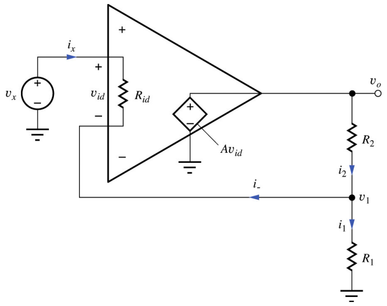
- **Problem: Design non-inverting amplifier and find the required open-loop gain**
- **Given Data: A_v = 35 dB, $R_{out} \le 0.2\ \Omega$, $R_o = 250\ \Omega$**
- **Analysis:**

$$\left|A_v^{Ideal}\right| = 10^{\frac{35dB}{20dB}} = 56.2 \quad \Big| \quad \beta = \frac{1}{A_v^{Ideal}} = \frac{1}{56.2}$$

$$R_{out} = \frac{R_o}{1 + A\beta} \le 0.2\ \Omega$$

$$\therefore \quad A \ge \frac{1}{\beta}\left(\frac{R_o}{R_{out}} - 1\right) \quad \Big| \quad A \ge 56.2\left(\frac{250}{0.2} - 1\right) = 7.02x10^4 \ \text{ or } \ 96.9\text{ dB}$$

Input Resistance of Noninverting Amplifier



Test voltage source v_x is applied to input and current i_x is calculated:

$$i_x = \frac{v_x - v_1}{R_{id}} \quad \Big| \quad v_1 = i_1 R_1 \cong i_2 R_1$$

Assuming $i_- << i_2$ gives $i_1 \cong i_2$

$$v_1 = \frac{R_1}{R_1 + R_2} v_o = \beta v_o$$

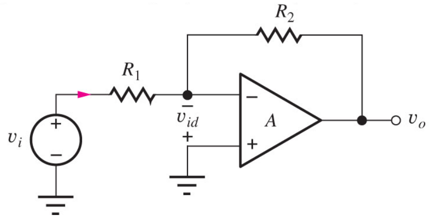
$$v_1 = \beta\left(Av_{id}\right) = A\beta\left(v_x - v_1\right)$$

$$v_1 = \frac{A\beta}{1 + A\beta} v_x = \frac{T}{1+T} v_x$$

$$\therefore \quad i_x = \frac{v_x - v_1}{R_{id}} = \frac{v_x - \dfrac{T}{1+T} v_x}{R_{id}} = \frac{v_x}{R_{id}\left(1+T\right)}$$

$$R_{in} = R_{id}\left(1+T\right)$$

Gain of Inverting Amplifier



$$i_1 = \frac{v_i - (-v_{id})}{R_1} \quad \text{and} \quad i_2 = \frac{(-v_{id}) - v_o}{R_2}$$

$$i_2 = i_1 \quad \text{and} \quad v_o = Av_{id}$$

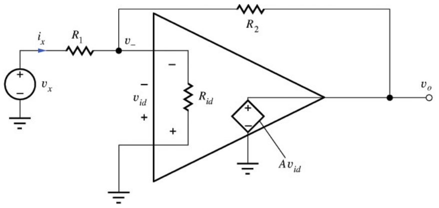
After some algebra,

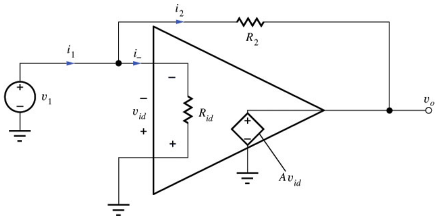
$$A_v = \frac{v_o}{v_i} = \left(-\frac{R_2}{R_1}\right)\frac{A\beta}{1 + A\beta} = A_v^{Ideal}\left(\frac{T}{1+T}\right)$$

$$\text{with} \quad \beta = \frac{R_1}{R_1 + R_2}$$

Let i_1 be the current going from the input source through R_1 and i_2 be the current going through R_2 into the amplifier output.

Input Resistance of Inverting Amplifier





R_1 removed

$$R_{in} = \frac{v_x}{i_x} = \frac{i_x R_1 + v_-}{i_x} = R_1 + \frac{v_-}{i_x}$$

$$i_1 = i_- + i_2 = \frac{v_1}{R_{id}} + \frac{v_1 - v_o}{R_2} = \frac{v_1}{R_{id}} + \frac{v_1 + Av_1}{R_2}$$

$$\therefore \quad G_1 = \frac{i_1}{v_1} = \frac{1}{R_{id}} + \frac{1+T}{R_2}$$

$$R_{in} = R_1 + R_{id} \parallel \left(\frac{R_2}{1+A}\right) \cong R_1 + \frac{R_2}{1+A}$$

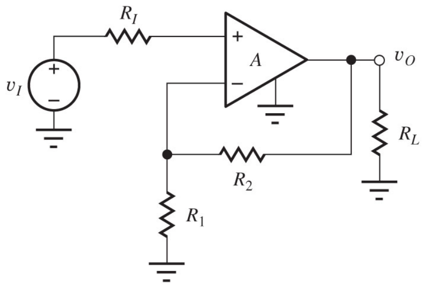
For large A, $R_{in} \cong R_1$

Nonideal Amplifier Summary
Inverting and Noninverting Amplifier

TABLE 11.1
Inverting and Noninverting Amplifier Summary

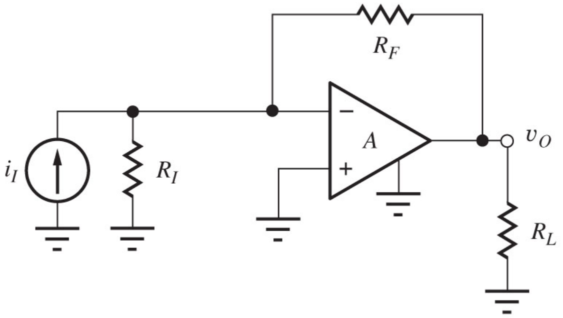
$\beta = \frac{R_1}{R_1 + R_2}$ and $T = A\beta$	INVERTING AMPLIFIER	NONINVERTING AMPLIFIER
Voltage gain A_v	$-\frac{R_2}{R_1}\left(\frac{T}{1+T}\right) \cong -\frac{R_2}{R_1}$	$\left(1+\frac{R_2}{R_1}\right)\left(\frac{T}{1+T}\right) \cong 1+\frac{R_2}{R_1}$
Input resistance R_{in}	$R_1 + \left(R_{id} \Big\| \frac{R_2}{1+A}\right) \cong R_1$	$R_{id}(1+T)$
Output resistance R_{out}	$\frac{R_O}{1+T}$	$\frac{R_O}{1+T}$
Fractional gain error (FGE)	$\frac{1}{1+T}$	$\frac{1}{1+T}$

Feedback Amplifier Categories
Voltage Amplifiers - Series-Shunt Feedback



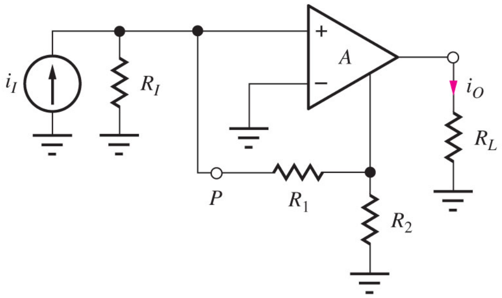
- A voltage amplifier should have a high input resistance to measure the desired voltage and a low output resistance to drive the external load. To achieve the desired behavior, the input ports of the amplifier and feedback network are connected in series, and the output ports are connected in parallel (shunt).

Feedback Amplifier Categories
Transresistance Amplifiers - Shunt-Shunt Feedback



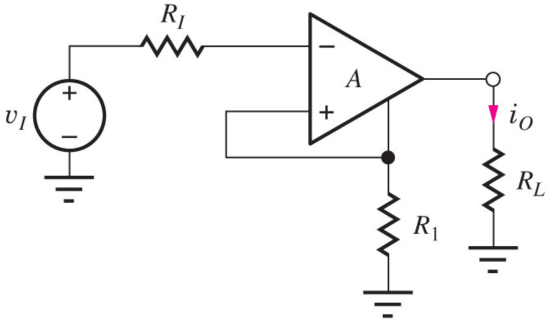
- A transresistance amplifier converts an input current to an output voltage. Thus it should have a low input resistance to sink the desired current and a low output resistance to drive the external load. To achieve the desired behavior, the input ports of the amplifier and feedback network are connected in parallel (shunt), and the output ports are connected in parallel (shunt).

Feedback Amplifier Categories
Current Amplifiers - Shunt-Series Feedback



- A current amplifier should provide a low resistance current sink at the input and a high resistance current source at its output. The input ports of the amplifier and feedback network are connected in parallel, and the output ports are connected in series.

Feedback Amplifier Categories
Transconductance Amplifiers - Series-Series Feedback



- The transconductance amplifier converts an input voltage to an output current. This amplifier should have a high input resistance and a high output resistance. The input ports and the output ports of the amplifier and feedback networks are connected in series.

Feedback Amplifier Category Summary

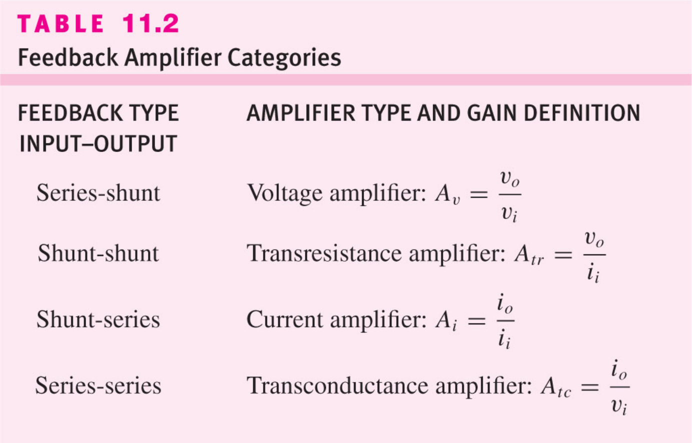


TABLE 11.2
Feedback Amplifier Categories

FEEDBACK TYPE INPUT–OUTPUT	AMPLIFIER TYPE AND GAIN DEFINITION
Series-shunt	Voltage amplifier: $A_v = \frac{v_o}{v_i}$
Shunt-shunt	Transresistance amplifier: $A_{tr} = \frac{v_o}{i_i}$
Shunt-series	Current amplifier: $A_i = \frac{i_o}{i_i}$
Series-series	Transconductance amplifier: $A_{tc} = \frac{i_o}{v_i}$