Announcements

- Homework 2, due next Tuesday, September 13 at 5pm
- All Lab sessions are still on (we will cut one for Lab 2)
- Lab 1 is next week
- Note that due to the Labor day, discussions are shifted:
  - Tuesday is the first one, Monday is the last one
- Reading: Chapter 3
In thermal equilibrium, there are no external fields and we thus expect the electron and hole current densities to be zero:

\[ J_n = 0 = qn_0\mu_n E_0 \mp qD_n \frac{dn_o}{dx} \]

\[ \frac{dn_o}{dx} = -\left( \frac{\mu_n}{D_n} \right) n_o E_0 = \left( \frac{q}{kT} \right) n_o \frac{d\phi_0}{dx} \]

\[ d\phi_0 = \left( \frac{kT}{q} \right) \frac{dn_o}{n_0} = V_{th} \frac{dn_o}{n_0} \]
Carrier Concentration and Potential (2)

- We have an equation relating the potential to the carrier concentration:

\[
d\phi_0 = \left( \frac{kT}{q} \right) \frac{dn_n}{n_0} = V_{th} \frac{dn_i}{n_0}
\]

- If we integrate the above equation we have:

\[
\phi_0(x) - \phi_0(x_0) = V_{th} \ln \frac{n_0(x)}{n_0(x_0)}
\]

- We define the potential reference to be intrinsic Si:

\[
\phi_0(x_0) = 0 \quad n_0(x_0) = n_i
\]

Carrier Concentration Versus Potential

- The carrier concentration is thus a function of potential:

\[
n_0(x) = n_i e^{\phi_0(x)/V_{th}}
\]

- Check that for zero potential, we have intrinsic carrier concentration (reference).

- If we do a similar calculation for holes, we arrive at a similar equation:

\[
p_0(x) = n_i e^{-\phi_0(x)/V_{th}}
\]

- Note that the law of mass action is upheld:

\[
n_0(x)p_0(x) = n_i^2 e^{-\phi_0(x)/V_{th}} e^{\phi_0(x)/V_{th}} = n_i^2
\]
The Doping Changes Potential

- Due to the log nature of the potential, the potential changes linearly for exponential increase in doping:

$$\phi_0(x) = V_m \ln \frac{n_0(x)}{n_i(x_0)} = 26\text{mV } \ln \frac{n_0(x)}{n_i(x_0)} \approx 26\text{mV } \ln 10 \log \frac{n_0(x)}{10^{10}}$$

$$\phi_0(x) \approx 60\text{mV } \log \frac{n_0(x)}{10^{10}}$$

$$\phi_0(x) \approx -60\text{mV } \log \frac{p_0(x)}{10^{10}}$$

- Quick calculation aid: For a p-type concentration of $10^{16}$ cm$^{-3}$, the potential is -360 mV

- N-type materials have a positive potential with respect to intrinsic Si

PN Junction: Overview

- p-type silicon
- Ionized acceptor (fixed)
- Hole (mobile)

- n-type silicon
- Ionized donor (fixed)
- Electron (mobile)
PN Junction: Overview

Present in most IC structures
**PN Junctions: Overview**

- The most important device is a junction between a p-type region and an n-type region.
- When the junction is first formed, due to the concentration gradient, mobile charges transfer near junction.
- Electrons leave n-type region and holes leave p-type region.
- These mobile carriers become minority carriers in new region (can’t penetrate far due to recombination).
- Due to charge transfer, a voltage difference occurs between regions.
- This creates a field at the junction that causes drift currents to oppose the diffusion current.
- In thermal equilibrium, drift current and diffusion must balance.

**PN Junction Currents**

- Consider the PN junction in thermal equilibrium.
- Again, the currents have to be zero, so we have

\[
J_n = 0 = qn_0 \mu_n E_0 + qD_n \frac{dn_o}{dx}
\]

\[
qn_0 \mu_n E_0 = -qD_n \frac{dn_o}{dx}
\]

\[
E_0 = -\frac{D_n}{n_0 \mu_n} \frac{dn_o}{dx} = -\frac{kT}{q} \frac{1}{n_0} \frac{dn_o}{dx}
\]

\[
E_0 = \frac{D_p}{n_0 \mu_p} \frac{dp_o}{dx} = -\frac{kT}{q} \frac{1}{p_0} \frac{dp_o}{dx}
\]
To solve for the electric fields, we need to write down the charge density in the transition region:

\[ \rho_0(x) = q(p_0 - n_0 + N_a - N_d) \]

In the p-side of the junction, there are very few electrons and only acceptors:

\[ \rho_0(x) \approx q(p_0 - N_a) \quad -x_p < x < 0 \]

Since the hole concentration is decreasing on the p-side, the net charge is negative:

\[ N_a > p_0 \quad \rho_0(x) < 0 \]
Charge on N-Side

- Analogous to the p-side, the charge on the n-side is given by:

\[ \rho_0(x) \approx q(-n_0 + N_d) \quad 0 < x < x_{n0} \]

- The net charge here is positive since:

\[ N_d > n_0 \quad \rho_0(x) > 0 \]

![Diagram showing charge on N-side](transition-region.png)

\[ n_b = n_0^2 \frac{N_d}{N_n} \]

“Exact” Solution for Fields

- Given the above approximations, we now have an expression for the charge density

\[ \rho_0(x) \approx \begin{cases} q(n_0 e^{-\phi_b(x)/V_n} - N_0) & -x_p < x < 0 \\ q(N_d - n_1 e^{\phi_b(x)/V_n}) & 0 < x < x_{n0} \end{cases} \]

- We also have the following result from electrostatics

\[ \frac{dE_0}{dx} = -\frac{d^2 \phi}{dx^2} = \frac{\rho_0(x)}{\varepsilon} \]

- Notice that the potential appears on both sides of the equation... difficult problem to solve

- A much simpler way to solve the problem...

![Diagram showing transition region](transition-region.png)
Depletion Approximation

Let’s assume that the transition region is completely depleted of free carriers (only immobile dopants exist).

Then the charge density is given by

\[
\rho_0(x) = \begin{cases} 
- qN_a & -x_{p0} < x < 0 \\
qN_d & 0 < x < x_{n0}
\end{cases}
\]

The solution for electric field is now easy

\[
\frac{dE_0}{dx} = \frac{\rho_0(x)}{\varepsilon_s}
\]

\[
E_0(x) = \int_{-x_{p0}}^x \frac{\rho_0(x')}{\varepsilon_s} dx' + E_0(-x_{p0})
\]

Field zero outside transition region

Depletion Approximation (2)

Since charge density is a constant

\[
E_0(x) = \int_{-x_{p0}}^x \frac{\rho_0(x')}{\varepsilon_s} dx' = -\frac{qN_a}{\varepsilon_s} (x + x_{p0})
\]

If we start from the n-side we get the following result

\[
E_0(x_{n0}) = \int_{x}^{x_{n0}} \frac{\rho_0(x')}{\varepsilon_s} dx' + E_0(x) = \frac{qN_d}{\varepsilon_s} (x_{n0} - x) + E_0(x)
\]

Field zero outside transition region

\[
E_0(x) = -\frac{qN_d}{\varepsilon_s} (x_{n0} - x)
\]