Announcements

- Homework 9 due next Tuesday
- Lab 7 this week
- Lab 8 next week (please read Chapter 9)
- Reading: Chapter 10 (10.2, 10.3.2, 10.4.3-5)
Lecture Material

- Last lecture
  - Second-order circuits
  - Started frequency response of amplifiers
- This lecture
  - Common source amplifier – frequency response
  - Miller effect
  - Zero-order time constants

Common Source: Discrete Biasing

With ideal MOS

\[ A_v = \frac{V_{DD}}{V_{DS} + V_{TH}} \]

\[ f_c = \frac{1}{2\pi R_1 C_S} \]

\[ \omega_c = \frac{1}{\sqrt{R_1 R_2}} \]

\[ \text{With ideal MOS} \]
Common Source Amplifier: $A_i(j\omega)$

DC Bias is problematic: what sets $V_{GS}$?
CS Short-Circuit Current Gain

Transfer function:

\[ A(j\omega) = \frac{g_m (1 - j\omega C_{gd}/g_m)}{j\omega(C_{gs} + C_{gd})} \]

\[ I_{in} = g_m V_{gs} - V_{gs} \cdot j\omega C_{gd} \]
\[ I_S = j\omega (C_{gs} + C_{gd}) V_{gs} \]
\[ \frac{I_{in}}{I_S} = A(j\omega) = \frac{g_m - j\omega C_{gd}}{j\omega(C_{gs} + C_{gd})} \]

Magnitude Bode Plot

Transition frequency:
Current gain = 1
MOS Unity Gain Frequency

Since the zero occurs at a higher frequency than pole, assume it has negligible effect:

\[ A_i \approx \frac{g_m}{j\omega(C_{gs} + C_{gd})} = 1 \quad \Rightarrow \quad \omega_r = \frac{g_m}{(C_{gs} + C_{gd})} \]

\[ \omega_r \approx \frac{g_m}{C_{gs}} + \frac{\mu C_{ox} W}{2L} (V_{GS} - V_T) = \frac{1}{2} \frac{\mu(V_{GS} - V_T)}{L^2} \]

Performance improves with \( L^2 \) for long channel devices!
For short channel devices the dependence is \( \sim L^1 \)

\[ \omega_r \approx \frac{3}{2} \frac{\mu(V_{GS} - V_T)}{L^2} \sim \frac{\mu V_{GS} - V_T}{L} \]

Time to cross channel

\[ \omega_r = \sigma \quad \omega_p = \frac{1}{\tau} \]

Decoupling Load (Discrete)
Common-Source Voltage Amplifier

Small-signal model: omit $C_{cs}$ due to avoid complicated analysis

CS Voltage Amp Small-Signal Model

\[
\frac{V_g - V_S}{R_S} + V_g j \omega C_{gs} + (V_g - V_{th}) j \omega C_{gd} = 0
\]

\[
(V_{out} - V_g) j \omega C_{gd} + g_m V_g + V_{out} / g_m r_{oc} R_L = 0
\]

\[
A_v(j\omega) = \frac{V_{out}}{V_S}
\]
Frequency Response

KCL at input and output nodes; analysis is made complicated due to $Z_{gd}$ branch $\rightarrow$ see H&S pp. 639-640 (for common emitter)

\[
\frac{V_{out}}{V_{in}} = -g_m \left[ \frac{r_o}{r_{oc}} || R_L \right] \frac{1 - j\omega/\omega_z}{(1 + j\omega/\omega_{p1})(1 + j\omega/\omega_{p2})}
\]

Low-frequency gain: $A_v = g_m V_0 || r_{oc} || R_L$

Zero: $\omega_z > \omega_T = \frac{g_m}{C_{gs} + C_{gd}}$

\[
-\frac{2}{C_{gd}A}
\]

Poles

\[
\omega_{p1} \approx \frac{1}{R_S \left( C_{gs} + (1 + g_m R'_{out}) C_{gd} \right) + R'_{out} C_{gd}}
\]

\[
\omega_{p2} \approx \frac{R'_{out}/R_S}{R_S \left( C_{gs} + (1 + g_m R'_{out}) C_{gd} \right) + R'_{out} C_{gd}}
\]

\[
|A| = \frac{\omega_{p1}}{\omega_{p2}}
\]

\[
\omega_{p1} \approx -\frac{2\pi f_c}{A}
\]

\[
\omega_{p2} \approx -\frac{4\pi f_c}{A}
\]
Miller Impedance

Consider the current flowing through an impedance $Z$ hooked up to a “black-box” where the voltage gain from one terminal to the other is fixed

$$A_v = \frac{v_2}{v_1}$$

$$I = \frac{v_1 - v_2}{Z} = \frac{v_1 - A_v v_1}{Z} = v_1 \frac{1 - A_v}{Z}$$

Miller Impedance

Notice that the current flowing into $Z$ from terminal 1 looks like an equivalent current to ground where $Z$ is transformed down by the Miller factor:

$$I = v_1 \frac{1 - A_v}{Z} \rightarrow Z_{M,1} = \frac{Z}{1 - A_v}$$

From terminal 2, the situation is reciprocal

$$-I = \frac{v_2 - v_1}{Z} = \frac{v_2 - A_v^{-1} v_2}{Z} = v_2 \frac{1 - A_v^{-1}}{Z}$$

$$Z_{M,2} = \frac{Z}{1 - A_v^{-1}}$$
**Miller Equivalent Circuit**

Note: $Z_{M,1} + Z_{M,2} = Z$

- We can decouple these terminals if we can calculate the gain $A_v$ across the impedance $Z$.
- Often the gain $A_v$ is weakly dependent on $Z$.
- The approximation is to ignore $Z$, calculate $A_v$, and then use the decoupled Miller impedances.

\[
Z_{M,1} = \frac{Z}{1 - A_v} \quad Z_{M,2} = \frac{Z}{1 - A_v^{-1}}
\]

**CE Amplifier using Miller Approx.**

Use Miller to transform $C_{gd}$.

\[
A_v = \frac{1}{1 + j\omega C_{gs} R_S} \cdot \left( -g_m V_{gs} \| R_L \right) \approx -g_m V_{gs} \| R_L \quad \text{(multiplier)}
\]

\[
V_S = \frac{g_m V_{gs}}{g_m(1 + \lambda) R_L} + \frac{1}{g_m(1 + \lambda)} V_L = R_{out}'
\]
Comparison with “Exact Analysis”

Miller result:

$$\omega_{p1}^{-1} = R_S \left( C_{gs} + \left(1 + g_m R_{out}' \right) C_{gd} \right)$$

Exact result:

$$\omega_{p1}^{-1} = R_S \left[ C_{gs} + \left(1 + g_m R_{out}' \right) C_{gd} \right] + R'_{out} C_{gd}$$