Announcements

- Homework 9 due next Tuesday
- Lab 7 this week
- Lab 8 next week (please read Chapter 9)
- Reading: Chapter 10 (10.2, 10.3.2, 10.4.3-5)
Lecture Material

- Last lecture
  - Second-order circuits
  - Started frequency response of amplifiers
- This lecture
  - Common source amplifier – frequency response
  - Miller effect
  - Zero-order time constants

Common Source Amplifier: $A_i(j\omega)$

DC Bias is problematic: what sets $V_{GS}$?
CS Short-Circuit Current Gain

Transfer function: 
\[ A_i(j\omega) = \frac{g_m (1 - j\omega C_{gd} / g_m)}{j\omega (C_{gs} + C_{gd})} \]

Magnitude Bode Plot

Transition frequency: 
Current gain = 1
MOS Unity Gain Frequency

Since the zero occurs at a higher frequency than pole, assume it has negligible effect:

\[ A_1 \approx \frac{g_m}{j\omega(C_{gs} + C_{gd})} = 1 \quad \Rightarrow \quad \omega_T = \frac{g_m}{(C_{gs} + C_{gd})} \]

\[ \omega_T \approx \frac{g_m}{C_{gs}} \left( \frac{\mu C_{ox} W}{L} (V_{GS} - V_T) \right) = \frac{3}{2} \frac{\mu (V_{GS} - V_T)}{L^2} \]

Performance improves with \( L^2 \) for long channel devices!
For short channel devices the dependence is \( \sim L^1 \)

\[ \omega_T \approx \frac{3}{2} \frac{\mu (V_{GS} - V_T)}{L^2} \sim \frac{\mu E_{eff}}{L} = \frac{\mu}{L} \tau_L \]

Time to cross channel

Common-Source Voltage Amplifier

Small-signal model: omit \( C_{CS} \) due to avoid complicated analysis
CS Voltage Amp Small-Signal Model

Frequency Response

KCL at input and output nodes; analysis is made complicated due to $Z_{gd}$ branch $\rightarrow$ see H&S pp. 639-640 (for common emitter)

\[
\frac{V_{out}}{V_{in}} = \frac{-g_m \left[ r_o \| r_{oc} \| R_L \right] \left(1 - j\omega / \omega_z \right)}{(1 + j\omega / \omega_p1)(1 + j\omega / \omega_p2)}
\]

Low-frequency gain:

Zero: $\omega_z > \omega_T = \frac{g_m}{C_{gs} + C_{gd}}$
### Poles

\[
\omega_p \approx \frac{1}{R_S \left( C_{gs} + \left( 1 + g_m R_{out} \right) C_{gd} \right) + R_{out}' C_{gd}}
\]

\[
\omega_p \approx \frac{R_{out}' / R_S}{R_S \left( C_{gs} + \left( 1 + g_m R_{out}' \right) C_{gd} \right) + R_{out}' C_{gd}}
\]

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### Miller Impedance

Consider the current flowing through an impedance \( Z \) hooked up to a “black-box” where the voltage gain from one terminal to the other is fixed.

\[
A_v = \frac{v_2}{v_1}
\]

\[
I = \frac{v_1 - v_2}{Z} = \frac{v_1 - A_v v_1}{Z} = v_1 \left( 1 - \frac{A_v}{Z} \right)
\]
Miller Impedance

- Notice that the current flowing into $Z$ from terminal 1 looks like an equivalent current to ground where $Z$ is transformed down by the Miller factor:

$$I = v_1 \frac{1 - A_v}{Z} \rightarrow Z_{M,1} = \frac{Z}{1 - A_v}$$

- From terminal 2, the situation is reciprocal

$$-I = \frac{v_2 - v_1}{Z} = \frac{v_2 - A_v^{-1}v_2}{Z} = v_2 \frac{1 - A_v^{-1}}{Z}$$

$$Z_{M,2} = \frac{Z}{1 - A_v^{-1}}$$

Miller Equivalent Circuit

\[ Z_{M,1} + Z_{M,2} = Z \]

- We can decouple these terminals if we can calculate the gain $A_v$ across the impedance $Z$
- Often the gain $A_v$ is weakly dependent on $Z$
- The approximation is to ignore $Z$, calculate $A_v$, and then use the decoupled Miller impedances
CE Amplifier using Miller Approx.

Use Miller to transform $C_{gd}$

Miller result:

$$\omega_{p1}^{-1} =$$

Exact result:

$$\omega_{p1}^{-1} = \left( R_S \parallel r_\pi \right) \left\{ C_\pi + \left( 1 + g_m R_{\text{out}}' \right) C_\mu \right\} + R_{\text{out}}' C_\mu$$
Some Examples

Common source amplifier:

\[ A_{V_{C_{gd}}} = \text{negative, large number (-100)} \]

Miller multiplied cap has detrimental impact on bandwidth

Common drain amplifier:

\[ A_{V_{C_{gd}}} = \text{slightly less than 1} \]

“Bootstrapped” cap has negligible impact on bandwidth!

Method of Open Circuit Time Constants

- This is a technique to find the dominant pole of a circuit (only valid if there really is a dominant pole!)
- For each capacitor in the circuit you calculate an equivalent resistor “seen” by capacitor and form the time constant \( \tau_i = R_i C_i \)
- The dominant pole then is the sum of these time constants in the circuit

\[ \omega_{p,\text{dom}} = \frac{1}{\tau_1 + \tau_2 + \cdots} \]
### Equivalent Resistance “Seen” by Capacitor

- For each “small” capacitor in the circuit:
  - Open-circuit all other “small” capacitors
  - Short circuit all “big” capacitors
  - Turn off all independent sources
  - Replace cap under question with current or voltage source
  - Find equivalent input impedance seen by cap
  - Form RC time constant
- This procedure is best illustrated with an example...

### Example Calculation

![Circuit Diagram]