Announcements

- Homework 8 due tomorrow 12 noon
- Lab 7 next week
- Reading: Chapter 10 (10.2, 10.3.2)

Lecture Material

- Last lecture
  - Bode plots
  - Second order functions
- This lecture
  -Finish second-order circuits
  - Frequency response of amplifiers

Low-Noise Amplifier

- D. Shaeffer, T. Lee, ISSCC’97

Series LCR Step Response

- Consider the transient response of the following circuit when we apply a step at input
- Without inductor, the cap charges with RC time constant (EECS 40)
- Where does the inductor come from?
  - Intentional inductor placed in series
  - Every physical loop has inductance! (parasitic)
LCR Step Response: $L$ Small

- We know the steady-state response is a constant voltage of $V_{dd}$ across capacitor (inductor is short, cap is open)
- For the case of zero inductance, we know solution is of the following form:

\[
V_v(t) = V_{dd}(1-e^{-t/\tau})
\]

Initial Conditions

- For the solution of a second order circuit, we need to specify to initial conditions (IC):
  \[
  V_v(0) = V_C(0) = 0V \\
  i(0) = i_L(0) = 0V
  \]
- For $t > 0$, the source voltage is $V_{dd}$. Solve the following non-homogeneous equation subject to above IC:

\[
V_{dd} = V_C(t) + RC \frac{dv_C}{dt} + LC \frac{d^2v_C}{dt^2}
\]

- Steady state:
  \[
  \frac{dv}{dt} \rightarrow 0 \\
  V_{dd} = V_C(\infty)
  \]

Guess Solution!

- Let’s subtract out the steady-state solution:

\[
V_v(t) = V_{dd} + v(t)
\]

\[
\frac{dv}{dt} = \frac{dv_C}{dt} + \frac{RC}{L} \frac{dv_C}{dt} + \frac{L}{C} \frac{d^2v_C}{dt^2}
\]

- Guess solution is of the following form:

\[
v(t) = A e^{st}
\]

Again We’re Back to Algebra

- Our guess is valid if we can find values of “$s$” that satisfy this equation:

\[
0 = 1 + RCs + LCS^2 \\
1 + (st)^2 + (st)^2 = 0
\]

\[
Q = \frac{1}{2s} \\
\tau = \frac{1}{\omega_0}
\]

- The solutions are:

\[
st = -\zeta \pm \sqrt{\zeta^2 - 1}
\]

- This is the same equation we solved in the last lecture!

- There we found three interesting cases:

<table>
<thead>
<tr>
<th>$\zeta$</th>
<th>$s$</th>
<th>$t$</th>
<th>$Q$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; 1$</td>
<td>Undamped</td>
<td>(-\zeta + \sqrt{\zeta^2 - 1})</td>
<td>(-\omega_0)</td>
<td>(\frac{1}{\tau})</td>
</tr>
<tr>
<td>$= 1$</td>
<td>Critically damped</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$&gt; 1$</td>
<td>Overdamped</td>
<td>(-\zeta - \sqrt{\zeta^2 - 1})</td>
<td>(-\omega_0)</td>
<td>(\frac{1}{\tau})</td>
</tr>
</tbody>
</table>

General Case

- Solutions are real or complex conj depending on if $\zeta > 1$ or $\zeta < 1$

\[
s = \frac{1}{\tau}(-\zeta \pm \sqrt{\zeta^2 - 1})
\]

\[
v_C(t) = V_{dd} + A e^{st} + B e^{st} \quad v_C(0) = V_{dd} + A + B = 0
\]

\[
i(t) = C \frac{dv_C(t)}{dt} \quad \left|_{t=0} \right. = 0 \implies A s_2 e^{s_2 t} + B s_2 e^{s_2 t} \left|_{t=0} \right. = 0
\]

\[
A s_2 + B s_2 = 0
\]

\[
A + B = -V_{dd}
\]
Final Solution (General Case)

- Solve for $A$ and $B$:
  \[
  A = \frac{-V_{df}}{1 - \frac{s_1}{s_2}} \quad B = -V_{df} - A = \frac{-V_{df}(1 - \frac{s_1}{s_2})}{1 - \frac{s_1}{s_2}} + \frac{s_1V_{df}}{s_2} \quad \frac{\dot{V}_s}{\dot{s}} = \frac{s_1V_{df}}{s_2} \]

- \[v_C(t) = V_{df} + \frac{s_2V_{df}}{s_1} \exp(s_1t) + \frac{\dot{V}_s}{\dot{s}} \exp(s_1t) \]

- \[v_C(t) = V_{df} \left(1 - \frac{1}{1 - \frac{s_1}{s_2}} \exp\left(\frac{s_1}{s_2}t\right) - \frac{s_1}{s_2} \exp\left(-\frac{s_1}{s_2}t\right)\right) \]

Overdamped Case

- $\zeta > 1$: Time constants are real and negative

  \[s = \frac{1}{\tau} (-\zeta \pm \sqrt{\zeta^2 - 1}) = \frac{1}{\tau} \quad V_{df} = 1 \quad \zeta = \frac{s_2}{s_1} > 1 \]

Critically Damped

- $\zeta > 1$: Time constants are real and equal

  \[s = \frac{1}{\tau} (-\zeta \pm \sqrt{\zeta^2 - 1}) = \frac{1}{\tau} \quad \lim_{\zeta \to \infty} v_C(t) = V_{df} \left[1 - e^{-\tau t} - e^{-\tau t} \right] \]

  \[v_C(t) = V_{df} \left[1 - e^{-\tau t} + e^{-\tau t}\right] \quad \tau = \frac{s_1}{s_2} = 1 \quad \zeta = 1 \]

Underdamped

- Now the $s$ values are complex conjugate

  \[s_1 = a + jb \quad s_2 = a - jb \]

  \[v_C(t) = V_{df} + A \exp(at + jbt) + B \exp(at - jbt) \quad v_C(t) = V_{df} + e^{at} \left(A \exp(jbt) + B \exp(-jbt)\right) \]

  \[v_C(t) = V_{df} + e^{at} \left(A \exp(jbt) + A^* \exp(-jbt)\right) \]

Underdamped (cont)

- So we have:

  \[v_C(t) = V_{df} + e^{at} \left(A \exp(jbt) + A^* \exp(-jbt)\right) \quad v_C(t) = V_{df} + e^{at} \left[2 \Re(A \exp(jbt))\right] \]

  \[v_C(t) = V_{df} + e^{at} \left[2 \Re(A \exp(jbt))\right] \quad |A| = \frac{V_{df}}{1 + \frac{s_1}{s_2}} \quad \phi = \arctan\left(\frac{V_{df}}{1 + \frac{s_1}{s_2}}\right) \]

Underdamped Peaking

- For $\zeta < 1$, the step response overshoots:

  \[v_C(t) = V_{df} + e^{at} \left(A \exp(jbt) + A^* \exp(-jbt)\right) \]

  \[\tau = 1 \quad V_{df} = 1 \quad \zeta = 0.5 \]
Extremely Underdamped

\[ \tau = 1 \\
V_{ds} = 1 \\
\zeta = 0.01 \]

Common Source Amplifier: \( A_j(\omega) \)

DC Bias is problematic: what sets \( V_{ds} \)?

Common Source: Discrete Biasing

With ideal MOS

CS Short-Circuit Current Gain

Transfer function:

\[ A_j(\omega) = \frac{g_m(1 - j\omega C_{ps} / g_m)}{j\omega(C_{ps} + C_{gd})} \]

Magnitude Bode Plot

Transition frequency: Current gain = 1

MOS Unity Gain Frequency

\[ A = \frac{g_m}{j\omega(C_{ps} + C_{gd})} = 1 \quad \text{Current gain} = 1 \]

\[ \alpha_T = \frac{g_m}{(C_{ps} + C_{gd})} \]

\[ \alpha_T = \frac{g_m}{C_{ps}} = \frac{\mu C_{ox} W (V_{ds} - V_T)}{L} = \frac{3}{2} \frac{\mu (V_{ds} - V_T)}{L} \]

Performance improves with \( L^2 \) for long channel devices!

For short channel devices the dependence is \( L \)

\[ \alpha_T = \frac{3}{2} \frac{\mu (V_{ds} - V_T)}{L} = \frac{E}{L} = \frac{V}{L} = \tau / 2 \]