Lecture 9: PN Junctions

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Lecture Outline

- PN Junctions Thermal Equilibrium
- PN Junctions with Reverse Bias
PN Junctions: Overview

- The most important device is a junction between a p-type region and an n-type region.
- When the junction is first formed, due to the concentration gradient, mobile charges transfer near junction.
- Electrons leave n-type region and holes leave p-type region.
- These mobile carriers become minority carriers in new region (can’t penetrate far due to recombination).
- Due to charge transfer, a voltage difference occurs between regions.
- This creates a field at the junction that causes drift currents to oppose the diffusion current.
- In thermal equilibrium, drift current and diffusion must balance.
PN Junction Currents

- Consider the PN junction in thermal equilibrium
- Again, the currents have to be zero, so we have

\[ J_n = 0 = qn_0 \mu_n E_0 + qD_n \frac{dn_o}{dx} \]

\[ qn_0 \mu_n E_0 = -qD_n \frac{dn_o}{dx} \]

\[ E_0 = \frac{-D_n \frac{dn_o}{dx}}{n_0 \mu_n} = -\frac{kT}{q} \frac{1}{n_0} \frac{dn_o}{dx} \]

\[ E_0 = \frac{D_p \frac{dp_o}{dx}}{n_0 \mu_p} = -\frac{kT}{q} \frac{1}{p_0} \frac{dp_o}{dx} \]
**PN Junction Fields**

p-type

\( N_A \)

n-type

\( N_D \)

\[ P_0 = N_a \]

\[ P_0(x) \]

\[ E_0 \]

\[ J_{\text{diff}} \]

\[ J_{\text{diff}} \]

\[ n_0 = \frac{n_i^2}{N_a} \]

\[ n_0 = \frac{n_i^2}{N_d} \]

\[ x_{p0} \]

\[ x_{n0} \]

Transition Region
Total Charge in Transition Region

- To solve for the electric fields, we need to write down the charge density in the transition region:
  \[ \rho_0(x) = q(p_0 - n_0 + N_d - N_a) \]

- In the p-side of the junction, there are very few electrons and only acceptors:
  \[ \rho_0(x) \approx q(p_0 - N_a) \quad -x_{p0} < x < 0 \]

- Since the hole concentration is decreasing on the p-side, the net charge is negative:
  \[ N_a > p_0 \quad \rho_0(x) < 0 \]
Charge on N-Side

- Analogous to the p-side, the charge on the n-side is given by:
  \[ \rho_0(x) \approx q(-n_0 + N_d) \quad 0 < x < x_{n0} \]

- The net charge here is positive since:
  \[ N_d > n_0 \quad \rho_0(x) > 0 \]
“Exact” Solution for Fields

- Given the above approximations, we now have an expression for the charge density
  \[ \rho_0(x) \equiv \begin{cases} 
  q(n_i e^{-\phi_0(x)/V_{th}} - N_a) & -x_{po} < x < 0 \\
  q(N_d - n_i e^{\phi_0(x)/V_{th}}) & 0 < x < x_{n0} 
  \end{cases} \]

- We also have the following result from electrostatics
  \[ \frac{dE_0}{dx} = -\frac{d^2\phi}{dx^2} = \frac{\rho_0(x)}{\epsilon_s} \]

- Notice that the potential appears on both sides of the equation… difficult problem to solve

- A much simpler way to solve the problem…
Depletion Approximation

- Let’s assume that the transition region is completely depleted of free carriers (only immobile dopants exist).

- Then the charge density is given by:
  \[
  \rho_0(x) = \begin{cases} 
  -qN_a & -x_{po} < x < 0 \\
  +qN_d & 0 < x < x_{n0}
  \end{cases}
  \]

- The solution for electric field is now easy:
  \[
  E_0(x) = \int_{-x_{po}}^{x} \frac{\rho_0(x')}{\varepsilon_s} \, dx' + E_0(-x_{p0})
  \]
Depletion Approximation (2)

- Since charge density is a constant

\[ E_0(x) = \int_{x_{p0}}^x \frac{\rho_0(x')}{\varepsilon_s} dx' = -\frac{qN_a}{\varepsilon_s} (x + x_{p0}) \]

- If we start from the n-side we get the following result

\[ E_0(x_{n0}) = \int_{x}^{x_{n0}} \frac{\rho_0(x')}{\varepsilon_s} dx' + E_0(x) = \frac{qN_d}{\varepsilon_s} (x_{n0} - x) + E_0(x) \]

Field zero outside transition region

\[ E_0(x) = -\frac{qN_d}{\varepsilon_s} (x_{n0} - x) \]
Plot of Fields In Depletion Region

- E-Field zero outside of depletion region
- Note the asymmetrical depletion widths
- Which region has higher doping?
- Slope of E-Field larger in n-region. Why?
- Peak E-Field at junction. Why continuous?
Continuity of E-Field Across Junction

- Recall that E-Field diverges on charge. For a sheet charge at the interface, the E-field could be discontinuous.
- In our case, the depletion region is only populated by a background density of fixed charges so the E-Field is continuous.
- What does this imply?

\[ E_n^0(x = 0) = -\frac{qN_a}{\varepsilon_s} x_{po} = -\frac{qN_d}{\varepsilon_s} x_{no} = E_p^0(x = 0) \]

\[ qN_a x_{po} = qN_d x_{no} \]

- Total fixed charge in n-region equals fixed charge in p-region! Somewhat obvious result.
Potential Across Junction

- From our earlier calculation we know that the potential in the n-region is higher than p-region.
- The potential has to smoothly transition from high to low in crossing the junction.
- Physically, the potential difference is due to the charge transfer that occurs due to the concentration gradient.
- Let’s integrate the field to get the potential:

\[
\phi(x) = \phi(-x_{po}) + \int_{x_{po}}^{x} \frac{qN_a}{\varepsilon_s} (x' + x_{po}) \, dx'
\]

\[
\phi(x) = \phi_p + \frac{qN_a}{\varepsilon_s} \left( \frac{x'^2}{2} + x'x_{po} \right) \bigg|_{-x_{po}}^{x}
\]
Potential Across Junction

- We arrive at potential on p-side (parabolic)
  \[ \phi_p^p(x) = \phi_p + \frac{qN_a}{2\varepsilon_s} (x + x_{p0})^2 \]

- Do integral on n-side
  \[ \phi_n(x) = \phi_n - \frac{qN_d}{2\varepsilon_s} (x - x_{n0})^2 \]

- Potential *must* be continuous at interface (field finite at interface)
  \[ \phi_n(0) = \phi_n - \frac{qN_d}{2\varepsilon_s} x_{n0}^2 = \phi_p + \frac{qN_a}{2\varepsilon_s} x_{p0}^2 = \phi_p(0) \]
Solve for Depletion Lengths

- We have two equations and two unknowns. We are finally in a position to solve for the depletion depths

\[
\phi_n - \frac{qN_d}{2\varepsilon_s} x_{n0}^2 = \phi_p + \frac{qN_a}{2\varepsilon_s} x_{p0}^2 \quad (1)
\]

\[
qN_a x_{po} = qN_d x_{no} \quad (2)
\]

\[
x_{no} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{qN_d} \left( \frac{N_a}{N_a + N_d} \right)} \quad x_{po} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{qN_a} \left( \frac{N_d}{N_d + N_a} \right)}
\]

\[
\phi_{bi} \equiv \phi_n - \phi_p > 0
\]
Sanity Check

- Does the above equation make sense?
- Let’s say we dope one side very highly. Then physically we expect the depletion region width for the heavily doped side to approach zero:

\[
x_{n0} = \lim_{N_a \to \infty} \sqrt{\frac{2 \varepsilon_s \phi_{bi}}{qN_a}} \left( \frac{N_d}{N_d + N_a} \right) = 0 \quad \checkmark
\]

\[
x_{p0} = \lim_{N_a \to \infty} \sqrt{\frac{2 \varepsilon_s \phi_{bi}}{qN_a}} \left( \frac{N_d}{N_d + N_a} \right) = \sqrt{\frac{2 \varepsilon_s \phi_{bi}}{qN_a}}
\]

- Entire depletion width dropped across p-region
The sum of the depletion widths is the “space charge region”

\[ X_{d0} = x_{p0} + x_{n0} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{q} \left( \frac{1}{N_a} + \frac{1}{N_d} \right)} \]

This region is essentially depleted of all mobile charge

Due to high electric field, carriers move across region at velocity saturated speed

\[ X_{d0} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{q} \left( \frac{1}{10^{15}} \right)} \approx 1\mu \quad E_{pn} \approx \frac{1V}{1\mu} = 10^4 \frac{V}{cm} \]
Have we invented a battery?

- Can we harness the PN junction and turn it into a battery?

\[ \phi_{bi} \equiv \phi_n - \phi_p = V_{th} \left( \ln \frac{N_D}{n_i} + \ln \frac{N_A}{n_i} \right) = V_{th} \ln \frac{N_D N_A}{n_i^2} \]

- Numerical example:

\[ \phi_{bi} = 26 \text{mV} \ln \frac{N_D N_A}{n_i^2} = 60 \text{mV} \times \log \frac{10^{15} \times 10^{15}}{10^{20}} = 600 \text{mV} \]
Contact Potential

- The contact between a PN junction creates a potential difference.
- Likewise, the contact between two dissimilar metals creates a potential difference (proportional to the difference between the work functions).
- When a metal semiconductor junction is formed, a contact potential forms as well.
- If we short a PN junction, the sum of the voltages around the loop must be zero:

\[ 0 = \phi_{bi} + \phi_{pm} + \phi_{mn} \]

\[ \phi_{bi} = - (\phi_{pm} + \phi_{mn}) \]
PN Junction Capacitor

- Under thermal equilibrium, the PN junction does not draw any (much) current.
- But notice that a PN junction stores charge in the space charge region (transition region).
- Since the device is storing charge, it’s acting like a capacitor.
- Positive charge is stored in the n-region, and negative charge is in the p-region:

\[ qN_a x_{po} = qN_d x_{no} \]
Reverse Biased PN Junction

- What happens if we “reverse-bias” the PN junction?

\[ -\phi_{bi} + V_D \]

- Since no current is flowing, the entire reverse biased potential is dropped across the transition region.

- To accommodate the extra potential, the charge in these regions must increase.

- If no current is flowing, the only way for the charge to increase is to grow (shrink) the depletion regions.
Voltage Dependence of Depletion Width

Can redo the math but in the end we realize that the equations are the same except we replace the built-in potential with the effective reverse bias:

\[ x_n(V_D) = \sqrt{\frac{2\varepsilon_s (\phi_{bi} - V_D)}{qN_d}} \left( \frac{N_a}{N_a + N_d} \right) = x_{n0} \sqrt{1 - \frac{V_D}{\phi_{bi}}} \]

\[ x_p(V_D) = \sqrt{\frac{2\varepsilon_s (\phi_{bi} - V_D)}{qN_a}} \left( \frac{N_d}{N_a + N_d} \right) = x_{p0} \sqrt{1 - \frac{V_D}{\phi_{bi}}} \]

\[ X_d(V_D) = x_p(V_D) + x_n(V_D) = \sqrt{\frac{2\varepsilon_s (\phi_{bi} - V_D)}{q}} \left( \frac{1}{N_a} + \frac{1}{N_d} \right) \]

\[ X_d(V_D) = X_{d0} \sqrt{1 - \frac{V_D}{\phi_{bi}}} \]
Charge Versus Bias

- As we increase the reverse bias, the depletion region grows to accommodate more charge

\[ Q_J(V_D) = -qN_a x_p(V_D) = -qN_a \sqrt{1 - \frac{V_D}{\phi_{bi}}} \]

- Charge is not a linear function of voltage
- This is a non-linear capacitor
- We can define a small signal capacitance for small signals by breaking up the charge into two terms

\[ Q_J(V_D + v_D) = Q_J(V_D) + q(v_D) \]
Derivation of Small Signal Capacitance

- From last lecture we found

\[ Q_j(V_D + v_D) = Q_j(V_D) + \frac{dQ_D}{dV} \bigg|_{v_D} v_D + \cdots \]

\[ C_j = C_j(V_D) = \frac{dQ_j}{dV} \bigg|_{V=V_D} = \frac{d}{dV} \left( -qN_a x_{p0} \sqrt{1 - \frac{V}{\phi_{bi}}} \right) \bigg|_{V=V_R} \]

\[ C_j = \frac{qN_a x_{p0}}{2\phi_{bi} \sqrt{1 - \frac{V_D}{\phi_{bi}}}} = \frac{C_{j0}}{\sqrt{1 - \frac{V_D}{\phi_{bi}}}} \]

- Notice that

\[ C_{j0} = \frac{qN_a x_{p0}}{2\phi_{bi}} = \frac{qN_a}{2\phi_{bi}} \sqrt{\left( \frac{2\varepsilon_s}{qN_a} \right) \left( \frac{N_d}{N_a + N_d} \right)} = \sqrt{\frac{q\varepsilon_s}{2\phi_{bi}}} \frac{N_a N_d}{N_a + N_d} \]
Physical Interpretation of Depletion Cap

\[ C_{j0} = \sqrt{\frac{q\varepsilon_s}{2\phi_{bi}} \frac{N_a N_d}{N_a + N_d}} \]

- Notice that the expression on the right-hand-side is just the depletion width in thermal equilibrium

\[ C_{j0} = \varepsilon_s \sqrt{\frac{q}{2\varepsilon_s \phi_{bi}}} \left( \frac{1}{N_a} + \frac{1}{N_d} \right)^{-1} = \frac{\varepsilon_s}{X_{d0}} \]

- This looks like a parallel plate capacitor!

\[ C_j(V_D) = \frac{\varepsilon_s}{X_d(V_D)} \]
A Variable Capacitor (Varactor)

- Capacitance varies versus bias: \( \frac{C_j}{C_{j0}} \)

- Application: Radio Tuner
“Diffusion” Resistor

- Resistor is capacitively isolation from substrate
  - Must Reverse Bias PN Junction!