Lecture 20: Frequency Response: Miller Effect

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Lecture Outline

- Frequency response of the CE as voltage amp
- The Miller approximation
- Frequency Response of Voltage Buffer
Last Time: CE Amp with Current Input

Calculate the short circuit current gain of device (BJT or MOS)
CS Short-Circuit Current Gain

MOS Case

\[ A_i(j\omega) = \frac{g_m \left(1 - j\omega C_{gd} / g_m\right)}{j\omega(C_{gs} + C_{gd})} \approx \frac{g_m}{j\omega(C_{gs} + C_{gd})} \]

Note: Zero occurs when all of “gm” current flows into Cgd:

\[ g_m v_{gs} = v_{gs} j\omega C_{gd} \]
Common-Emitter Voltage Amplifier

Small-signal model: omit $C_{cs}$ to avoid complicated analysis

Can solve problem directly by phasor analysis or using 2-port models of transistor

OK if circuit is “small” (1-2 nodes)
CE Voltage Amp Small-Signal Model

Two Nodes! Easy

\[ R_{\text{in}}' = R_S \| r_\pi \]

\[ R_{\text{out}}' = r_o \| r_{oc} \| R_L \]

Same circuit works for CS with \( r_\pi \to \infty \)
KCL at input and output nodes; analysis is made complicated due to $Z_\mu$ branch $\rightarrow$ see H&S pp. 639-640.

$$\frac{V_{out}}{V_{in}} = -g_m \left( \frac{r_\pi}{r_\pi + R_S} \right) \left[ r_o \parallel r_{oc} \parallel R_L \right] \left( 1 - j\omega / \omega_z \right) \left( 1 + j\omega / \omega_{p1} \right) \left( 1 + j\omega / \omega_{p2} \right)$$

**Low-frequency gain:**

$$\frac{V_{out}}{V_{in}} = -g_m \left( \frac{r_\pi}{r_\pi + R_S} \right) \left[ r_o \parallel r_{oc} \parallel R_L \right] (1 - j0) \rightarrow -g_m \left( \frac{r_\pi}{r_\pi + R_S} \right) \left[ r_o \parallel r_{oc} \parallel R_L \right]$$

**Zero:** $\omega_z > \omega_T = \frac{g_m}{C_\pi + C_\mu}$

Note: Zero occurs due to feed-forward current cancellation as before
These poles are calculated after doing some algebraic manipulations on the circuit. It’s hard to get any intuition from the above expressions.
Miller Approximation

Results of complete analysis: not exact and little insight

Look at how $Z_\mu$ affects the transfer function: find $Z_{in}$
Input Impedance $Z_{in}(j\omega)$

\[ I_t = \frac{(V_t - V_{out})}{Z_\mu} \]

At output node:

\[ V_{out} = (-g_m V_t - I_t)R'_{out} \approx -g_m V_t R'_{out} \quad \text{Why?} \]

\[ I_t = \frac{(V_t - A_{vC\mu} V_t)}{Z_\mu} \]

\[ Z_{in} = \frac{Z_\mu}{1 - A_{vC\mu}} \]
Miller Capacitance $C_M$

Effective input capacitance:

$$Z_{in} = \frac{1}{j\omega C_M} = \left( \frac{1}{1 - A_{vC\mu}} \right) \left( \frac{1}{j\omega C\mu} \right) = \frac{1}{j\omega \left[ (1 - A_{vC\mu})C\mu \right]}$$
Some Examples

Common emitter/source amplifier:

\[ A_{vC_\mu} = \text{Negative, large number (-100)} \]

\[ C_M = (1 - A_{vC_\mu})C_\mu \approx 100C_\mu \]

Miller Multiplied Cap has Detrimental Impact on bandwidth

Common collector/drain amplifier:

\[ A_{vC_\pi} = \text{Slightly less than 1} \]

\[ C_M = (1 - A_{vC_\pi})C_\pi \approx 0 \]

“Bootstrapped” cap has negligible impact on bandwidth!
CE Amplifier using Miller Approx.

Use Miller to transform $C_\mu$

$$CM = C_\mu(1 + g_m R'_\text{out})$$

Analysis is straightforward now … single pole!
Comparison with “Exact Analysis”

Miller result (calculate RC time constant of input pole):

\[
\omega_{p1}^{-1} = (R_S \parallel r_\pi) \left\{ C_\pi + (1 + g_m R'_\text{out}) C_\mu \right\}
\]

Exact result:

\[
\omega_{p1}^{-1} = (R_S \parallel r_\pi) \left\{ C_\pi + (1 + g_m R'_\text{out}) C_\mu \right\} + R'_\text{out} C_\mu
\]
Method of Open Circuit Time Constants

- This is a technique to find the dominant pole of a circuit (only valid if there really is a dominant pole!)
- For each capacitor in the circuit you calculate an equivalent resistor “seen” by capacitor and form the time constant $\tau_i = R_i C_i$
- The dominant pole then is the sum of these time constants in the circuit

$$\omega_{p,\text{dom}} = \frac{1}{\tau_1 + \tau_2 + \cdots}$$
Equivalent Resistance “Seen” by Capacitor

- For each “small” capacitor in the circuit:
  - Open-circuit all other “small” capacitors
  - Short circuit all “big” capacitors
  - Turn off all independent sources
  - Replace cap under question with current or voltage source
  - Find equivalent input impedance seen by cap
  - Form RC time constant

- This procedure is best illustrated with an example…
Example Calculation

- Consider the input capacitance $C_1 = C_\pi + C_M$
- Open all other “small” caps (get rid of output cap)
- Turn off all independent sources
- Insert a current source in place of cap and find impedance seen by source $R_M = r_\pi \ || \ R_S$

\[
\tau_1 = (R_S \ || \ r_\pi) \left\{ C_\pi + (1 + g_m R'_\text{out}) C_\mu \right\}
\]
Common-Collector Amplifier

Procedure:

1. Small-signal two-port model
2. Add device (and other) capacitors
Two-Port CC Model with Capacitors

Find Miller capacitor for $C_\pi$ -- note that the base-emitter capacitor is between the input and output.
Voltage Gain $A_{vC\pi}$ Across $C_\pi$

\[
A_{vC\pi} \approx \frac{R_{out}}{R_{out} + R_L} \sim 1 \quad R_{out} = \frac{1}{g_m}
\]

\[
g_m R_L >> 1
\]

Note: this voltage gain is neither the two-port gain nor the “loaded” voltage gain

\[
C_{in} = C_\mu + C_M = C_\mu + (1 - A_{vC\pi}) C_\pi
\]

\[
C_{in} = C_\mu + \frac{1}{1 + g_m R_L} C_\pi
\]

\[
C_{in} \approx C_\mu
\]
Bandwidth of CC Amplifier

Input low-pass filter’s $-3$ dB frequency:

$$\omega_p^{-1} = (R_S \parallel R_{in}) \left( C_\mu + \frac{C_\pi}{1 + g_m R_L} \right)$$

Substitute favorable values of $R_S$, $R_L$:

$$R_S \approx 1/ g_m \quad R_L >> 1/ g_m$$

Model not valid at these high frequencies