Lecture 18:
Bipolar Single Stage Amplifiers

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Lecture Outline

- BJT Amps
- BJT Biasing
- Common Emitter Amp
- Common Base Amp
- Common Collector Amp  
  - AKA Emitter Follower
- $\beta$ Multiplier Concept
- Emitter Degeneration
Bipolar Amplifiers

Common-emitter amplifier:

Biasing: adjust $V_{BIAS} = V_{BE}$ so that $I_C = I_{SUP}$.
Small-Signal Two-Port Model

Parameters: \((I_C = 1 \text{ mA}, \beta = 100, V_A = 3\text{ V})\)

\[
R_{in} = r_\pi = \frac{\beta}{g_m} = 100 \frac{25\text{ mV}}{1\text{ mA}} = 2.5\text{k}\Omega
\]

\[
R_{out} = r_o \parallel r_{oc} = \frac{3\text{ V}}{1\text{ mA}} \parallel r_{oc} = 3\text{k} \parallel r_{oc} \approx 3\text{k}
\]

\[
G_m = g_m = \frac{1\text{ mA}}{25\text{ mV}} = \frac{1}{25}\text{ S} = 40\text{ mS}
\]
Common-Base Amplifier

To find $I_{BIAS}$, note that

$$I_{BIAS} = I_E = -(1/\alpha_F)I_C$$

Common-base current gain $A_i = -\alpha_F$
CB Input Resistance

Summing currents at the input node:

\[ i_t + \frac{v_\pi}{r_\pi} + g_m v_\pi + (v_o - v_t) g_o = 0 \quad \rightarrow \quad i_t = \left( \frac{1}{r_\pi} + g_m \right) v_t \]

\[ R_{in} = \frac{v_t}{i_t} = \left( \frac{1}{r_\pi} + g_m \right)^{-1} = \left( \frac{g_m}{\beta} + g_m \right)^{-1} \approx \frac{1}{g_m} = \frac{25mV}{1mA} = 25\Omega \]
CB Output Resistance

Same topology as CG amplifier, but with $r_\pi \parallel R_S$ rather than $R_S$

$$R_{out} = r_{oc} \parallel \left[ r_o \left( 1 + g_m (r_\pi \parallel R_S) \right) \right]$$

$R_S \gg r_\pi$  \quad $R_{out} = r_{oc} \parallel \left[ r_o \left( 1 + g_m r_\pi \right) \right]$  \quad $R_{out} = r_{oc} \parallel r_o \left( 1 + \beta \right)$
Output Impedance Details

- First draw small signal equivalent circuit with transistor and simplify as much as possible
- Then (if needed) add the small signal equivalent circuit
- If frequency is low, get rid of caps!
Output Impedance Calculation

\[ i_t = g_m v_{\pi} + \frac{v_t - (-v_{\pi})}{r_o} \]

\[ v_{\pi} = -i_t \left( R_s \parallel r_{\pi} \right) \]

\[ i_t = -g_m i_t \left( R_s \parallel r_{\pi} \right) + \frac{v_t}{r_o} + \frac{-R_s \parallel r_{\pi}}{r_o} i_t \]

\[ i_t \left( 1 + g_m \left( R_s \parallel r_{\pi} \right) + \frac{R_s \parallel r_{\pi}}{r_o} \right) = \frac{v_t}{r_o} \]
Why did we consider it a current amp?

Current Amp:
- Unity Current Gain (-1)
- Small Input Impedance
- Large (huge!) Output Impedance
Common-Collector Amplifier

DC Bias: output is one “$V_{BE}$ drop” down from input

“Emitter Follower”
Common-Collector Input Resistance

\[ v_t = i_t r_\pi + (\beta + 1)i_t (R_L \parallel r_o \parallel r_{oc}) \]

\[ R_{in} = r_\pi + (\beta + 1)(R_L \parallel r_o \parallel r_{oc}) \]

\[ R_{in} \approx r_\pi + (\beta + 1)R_L \]
Common-Collector Output Resistance

Divider between $v_t$ and $v_\pi$:

$$ v_\pi = \frac{r_\pi}{r_\pi + R_S} v_t $$

$$ i_t + g_m v_\pi + v_\pi r_\pi^{-1} - v_t (r_o \parallel r_{oc})^{-1} = 0 $$

$$ i_t = v_t \left( g_m + r_\pi^{-1} \right) \left( \frac{r_\pi}{r_\pi + R_S} \right) + v_t (r_o \parallel r_{oc})^{-1} $$
Common-Collector Output Res. (cont)

\[ i_t = v_t \left( g_m + r_{\pi}^{-1} \right) \left( \frac{r_\pi}{r_\pi + R_S} \right) + v_t \left( r_o \parallel r_{oc} \right)^{-1} \]

\[ i_t = v_t \left( g_m r_\pi + 1 \right) \left( \frac{1}{r_\pi + R_S} \right) \]

\[ R_{out} = \frac{v_t}{i_t} = \frac{r_\pi + R_S}{\beta + 1} \]

- Looking into base of emitter follower: load impedance larger by factor \( \beta+1 \)
- Looking into emitter of follower: “source” impedance smaller by factor \( \beta+1 \)
Common-Collector Voltage Gain

KCL at the output node: note $v_\pi = v_t - v_{out}$

$$v_{out} \left( r_{oc} \parallel r_o \right)^{-1} = g_m v_\pi + v_\pi r_\pi^{-1}$$

$$v_{out} \left( r_{oc} \parallel r_o \right)^{-1} = \left( g_m + r_\pi^{-1} \right) (v_t - v_{out})$$

$$v_{out} \left( \left( r_{oc} \parallel r_o \right)^{-1} + g_m + r_\pi^{-1} \right) = \left( g_m + r_\pi^{-1} \right) v_t$$

$v_{out} = v_t$
Common-Collector Two-Port Model

Voltage Amp:
- Unity Voltage Gain (+1)
- Large Input Impedance
- Small Output Impedance
# Summary of Two-Port Parameters

<table>
<thead>
<tr>
<th>Amplifier Type</th>
<th>Controlled Source</th>
<th>Input Resistance $R_{in}$</th>
<th>Output Resistance $R_{out}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common Emitter</td>
<td>$G_m = g_m$</td>
<td>$r_n$</td>
<td>$r_o \parallel r_{oc}$</td>
</tr>
<tr>
<td>Common Source</td>
<td>$G_m = g_m$</td>
<td>infinity</td>
<td>$r_o \parallel r_{oc}$</td>
</tr>
<tr>
<td>Common Base</td>
<td>$A_t = -1$</td>
<td>$1 / g_m$</td>
<td>$r_{oc} \parallel [(1 + g_m (r_n \parallel R_S)) r_o]$, for $g_m r_o &gt;&gt; 1$</td>
</tr>
<tr>
<td>Common Gate</td>
<td>$A_t = -1$</td>
<td>$1 / g_m$, $v_{sb} = 0$</td>
<td>$r_{oc} \parallel [(1 + g_m R_S) r_o]$, $v_{sb} = 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-otherwise-</td>
<td>-otherwise-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$1 / (g_m + g_{mb})$</td>
<td>$r_{oc} \parallel [(1 + (g_m + g_{mb}) R_S) r_o]$ both for $r_o &gt;&gt; R_S$</td>
</tr>
<tr>
<td>Common Collector</td>
<td>$A_v = 1$</td>
<td>$r_n + \beta_o (r_o \parallel r_{oc} \parallel R_L)$</td>
<td>$(1 / g_m) + R_S / \beta_o$</td>
</tr>
<tr>
<td>Common Drain</td>
<td>$A_v = 1$ if $v_{sb} = 0$, -otherwise- $g_m / (g_m + g_{mb})$</td>
<td>infinity</td>
<td>$1 / g_m$ if $v_{sb} = 0$, -otherwise- $1 / (g_m + g_{mb})$</td>
</tr>
</tbody>
</table>
Typical “Discrete” Biasing

- A good biasing scheme must be relatively insensitive to transistor parameters (vary with process and temperature).
- In this scheme, the base current is given by:

\[ V_B = V_{CC} \frac{R_1}{R_1 + R_2} \]

- The emitter current:

\[ I_E \approx \left( V_{CC} \frac{R_1}{R_1 + R_2} - V_{BE,on} \right) / R_E \]
Gain for “Discrete” Design

- Let’s derive it by intuition
- Input impedance can be made large enough by design
- Device acts like follower, emitter=base
- This signal generates a collector current

\[
R_{\text{in}} \approx (\beta + 1)(r_\pi + R_E) \\
\approx (\beta + 1)R_E
\]

\[
R_{\text{in2}} \approx (\beta + 1)R_E \parallel R_1 \parallel R_2
\]

Can be made large to couple All of source to input (even with \(R_S\))