Problem Set 1
Due Monday September 1, 2003

1. In the following problems, perform the operating indicated and reduce the final answer to the form of $A + iB$
   
   (a) $\frac{1+i}{1-i}$
   
   (b) $\frac{i}{1-i} + \frac{1-i}{i}$
   
   (c) $\sqrt{i}$
   
   (d) $i^i$
   
   (e) $i^{570} - 4i^{29} - 5i$

2. Prove that for any number $z$,
   
   $Re(z) = \frac{1}{2}(z + \bar{z})$
   
   $Im(z) = \frac{1}{2i}(z - \bar{z})$

3. Prove that conjugation distributes over sums, differences, and quotients.

4. Write each of the following numbers in polar form.
   
   (a) $-1$
   
   (b) $3$
   
   (c) $-4i$
   
   (d) $2 - i$
   
   (e) $\sqrt{2}(1 + i)$

5. Find the six sixth roots of unity and plot them on the complex plane.

6. Prove that for any $z \neq 0$ and $w \neq 0$
   
   $\arg\left(\frac{z}{w}\right) = \arg(z) - \arg(w)$

7. Consider the high-pass filter shown below.

   ![High-pass filter diagram]
(a) Calculate the sinusoidal steady-state frequency response by (i) directly solving the differential equations and by (ii) the method of phasor analysis.

(b) Calculate the output voltage when the filter is driven by a signal $V_s(t) = 5V \cos(\omega t + 30^\circ)$ where $\omega = 2\pi \times 100$ kHz. Assume that the $RC$ time constant of the circuit is ten microseconds. Use phasors for your calculations. What’s the magnitude response in dB?

(c) Describe the operation of the circuit as $C \to \infty$. Can you think of an application for such a circuit?

8. A simple physical model for an inductor is shown in below. The series resistance models the loss and the shunt capacitance models the self-resonant frequency of the device. (a) Find the impedance of the circuit as a function of frequency (for this part only, use $L = 1$ nH, $R = 3\Omega$, and $C = 150$ fF). (b) What’s the magnitude of the impedance at DC? What’s the magnitude of the impedance at infinite frequency? (c) Are there any frequencies when the phase of the impedance is zero? (d) Plot the magnitude and phase response for the circuit.

\begin{figure}[h]
\centering
\includegraphics[width=0.2\textwidth]{circuit.png}
\caption{A simple physical model for an inductor.}
\end{figure}