

Microelectronic Devices and Circuits- EECS105

Second Midterm Exam

Wednesday, November 13, 2002

Costas J. Spanos

University of California at Berkeley  
College of Engineering

Department of Electrical Engineering and Computer Sciences

Your Name: Official Solutions.  
(last) (first)

Your Signature: \_\_\_\_\_  
n = 69      $\hat{\mu} = 60, \hat{G} = 18$

1. Print and sign your name on this page before you start.
2. You are allowed two 8.5"x11" handwritten sheets with formulas. No books or notes!
3. Do everything on this exam, and make your methods as clear as possible.

Problem 1 \_\_\_\_\_ / 35  
Problem 2 \_\_\_\_\_ / 35  
Problem 3 \_\_\_\_\_ / 30  
  
TOTAL \_\_\_\_\_ / 100

**Problem 1 of 3 Answer each question briefly and clearly. Sketch a simple drawing if it helps you make your point. (35 points)**

What is channel-length modulation, and how does it impact the behavior of a MOSFET? (5pts)

As  $V_{DS}$  increase, the depletion region around the drain increases and the channel gets shorter. As a result,  $I_D$  increases.

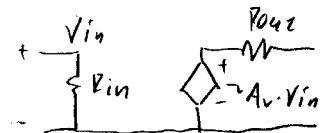
$$I_D = \frac{1}{2} \frac{W}{L} \mu n C_{ox} (V_{in} - V_{DS})^2 (1 + \lambda V_{DS})$$

↑  
this is due to channel length modulation

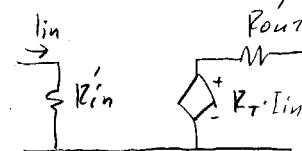
Circle the most likely value for each of the following small signal parameters of a typical, silicon-based BJT (5pts)

$r_o$	10 $\Omega$	10 k $\Omega$	<u>100 k<math>\Omega</math></u>	10 M $\Omega$
$r_{\pi}$	2.5 $\Omega$	250 $\Omega$	<u>25 k<math>\Omega</math></u>	2.5 M $\Omega$
$g_m$	1 $\mu$ S	<u>1 mS</u>	0.1 S	10 S

You are given a 2-port, which has  $R_{in} = 1k\Omega$ ,  $R_{out} = 100k\Omega$ , and an open circuit voltage gain  $A_v = -10$ . Please draw the equivalent transresistance 2-port and calculate its  $R_{in}$ ,  $R_{out}$ , and  $R_m$ . (5pts)



voltage Amp.



$$P_i \cdot I_{in}' = (A_v \cdot V_{in}) = A_v \cdot I_{in} \cdot R_{in} \Rightarrow$$

$$R_m = A_v \cdot R_{in} = -10 \cdot 1k\Omega = -10k\Omega$$

Voltage is proportional to input current (trans-resistance Amp)

$$R_{in}' = R_{in} = 1k\Omega \quad P_{out}' = P_{out} = 100k\Omega$$

What happens to the absolute, no-load ( $R_L$  infinity) voltage gain of a CS amplifier as you decrease the  $L$  of the transistor? Assume that the supply current stays the same. Provide a one sentence explanation or a formula to justify your answer (assuming that the bias current stays constant). (5 pts)

circle one: the absolute gain... increases decreases stays the same

and this is why:

$$g_m \propto \sqrt{\frac{W}{L}} \quad \text{so, if } L \downarrow \text{ then } g_m \uparrow \text{ by } \frac{1}{\sqrt{L}}$$

$$r_o = \frac{L}{\mu_n I_D} \quad \text{so, if } L \downarrow \text{ then } r_o \downarrow \text{ by } L \quad \left. \begin{array}{l} \text{gain} \propto |g_m r_o| \\ \text{so gain} \downarrow \text{ by } \frac{1}{L} \end{array} \right\}$$

What happens to the bandwidth of a CS amplifier as you decrease the  $L$  of the transistor? Provide a one sentence explanation or a formula to justify your answer (assuming that the bias current stays constant). (5 pts)

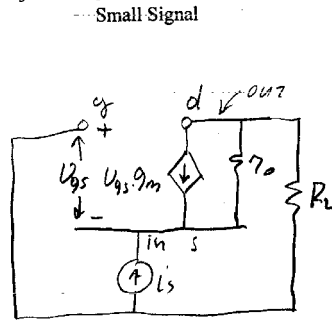
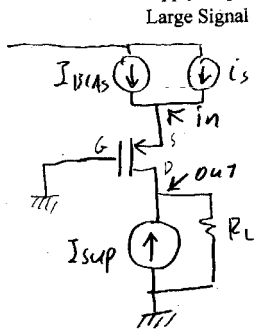
circle one: the bandwidth... increases decreases stays the same

and this is why:

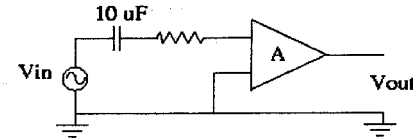
ultimately, bandwidth is limited by  $f_T \approx \frac{1}{2\pi T_F}$

$$T_F = \frac{L}{V_{sat}}, \text{ so as } L \downarrow \text{ } T_F \downarrow \text{ } f_T \uparrow$$

Sketch the large and small signal circuits of a CG amplifier utilizing a pmos transistor and an ideal current supply  $I_{sup}$ . Include an ideal current source  $i_s$  as the input, and a load  $R_L$ . (5 pts)



In the EE105 lab on amplifiers, a  $10 \mu F$  capacitor is connected to the small-signal input voltage  $v_{in}$ . (See the following figure for a generic amplifier circuit.) What is the purpose of this capacitor? Why do we choose the capacitance to be so large? (5 pts)



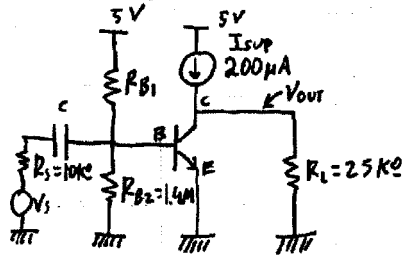
we use a huge  $C$  ( $10 \mu F$ ) to make sure that even low frequency signals go through.

We do not use a short because we want to block any DC voltage from going through.

**Problem 2 of 3 (35 points)**

For each of the following questions, make sure that you show the expressions before you plug in the specific values. A correct expression is worth 70% of the credit, even if the numerical calculation is incorrect!

Consider the following CE amplifier.



$\beta_0 = 100$   
 $V_{th} = 26mV$   
 Assume that  $V_A$  and  $r_{oc}$  are infinite  
 Assume that  $V_{BE\ on} = 0.7V$

$R_{B2} = 1.4M\Omega$   
 $R_s = 10k\Omega$   
 $R_L = 25k\Omega$   
 $I_{SUP} = 200\mu A$

a) Calculate the value of  $R_{B1}$  so that  $V_{OUT} = 2.5V$ . (15 points)

$$V_{OUT} = 2.5V \rightarrow I_{R_L} = 100\mu A$$

$$I_{SUP} = I_C + I_{R_L} \rightarrow I_C = 100\mu A$$

$$I_C = \beta I_B \rightarrow I_B = 1\mu A$$

$$V_{BE} = V_{R_{B2}} = 0.7V \rightarrow I_{R_{B2}} = 0.5\mu A$$

$$I_{R_{B1}} = I_{R_{B2}} + I_B = 1.5\mu A$$

$$V_{R_{B1}} = 5V - V_{BE} = 4.3V$$

$$R_{B1} = \frac{4.3V}{1.5\mu A} = 2.86 M\Omega$$

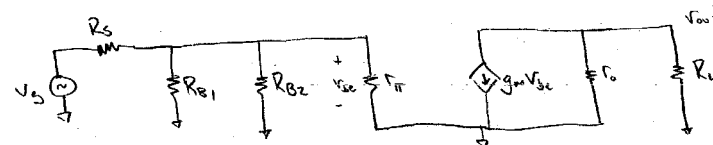
Final expression of  $R_{B1}$ :

$$R_{B1} = \frac{5V - V_{BE}}{\frac{V_{BE}}{R_{B2}} + \frac{I_{SUP} - \frac{V_{OUT}}{R_L}}{\beta}}$$

Calculated  $R_{B1}$  value:

$$2.86 M\Omega$$

b) Draw the small signal equivalent of this circuit, assuming that C is huge and therefore it can be replaced with a short circuit for small signal purposes. Ignore all other parasitic capacitors. Write the expression and calculate the small signal voltage gain  $v_{out}/v_s$ . (15 points)



$$v_{out} = g_m v_{be} (r_o \parallel R_L) \quad r_o = \infty$$

$$= g_m R_L \frac{v_{th}}{R_s + (r_{\pi} \parallel R_{B1} \parallel R_{B2})} = \frac{g_m R_L}{1 + \left(\frac{1}{r_{\pi}} + \frac{1}{R_{B1}} + \frac{1}{R_{B2}}\right) R_s} = \frac{\frac{I_C}{V_{th}} R_L}{1 + \left(\frac{I_C}{\beta V_{th}} + \frac{1}{R_{B1}} + \frac{1}{R_{B2}}\right) R_s}$$

Final Expression of Voltage Gain  $v_{out}/v_s$

$$\frac{-\frac{I_C}{V_{th}} R_L}{1 + \left(\frac{I_C}{\beta V_{th}} + \frac{1}{R_{B1}} + \frac{1}{R_{B2}}\right) R_s}$$

Calculated value of  $v_{out}/v_s$

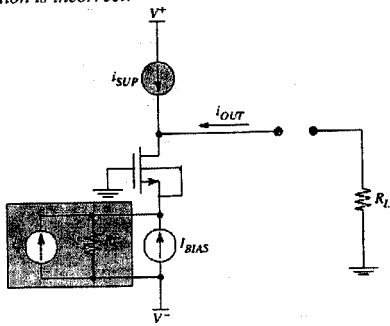
$$-70.8$$

c) There is something obviously inefficient with the way this amplifier is biased. Can you point to the problem and explain how you would fix it? (Here I expect a verbal explanation - not any quantitative analysis). (5 points)

Half of the supply current is constantly being burned in the load resistor. We could avoid this problem if we used a coupling capacitor on the output as well as on the input. That would allow the signal current to flow into the load but prevent DC current from flowing into it.

**Problem 3 of 3 (30 points)**

For each of the following questions, make sure that you show the expressions *before* you plug in the specific values. A correct expression is worth 70% of the credit, even if the numerical calculation is incorrect!

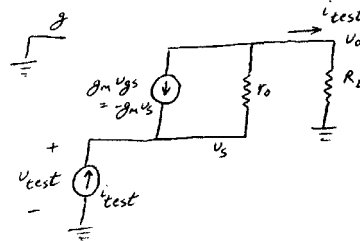


- $V_{Tn} = 1V$
- $\mu_n C_{ox} = 50 \mu A/V^2$
- $\lambda_n = 0.1 V^{-1} \mu m / L$  (where  $L$  in  $\mu m$ )
- $-I_{BIAS} = I_{SUP} = 100 \mu A$
- $r_{oc}$  is infinite (i.e.  $i_{SUP}$  is an ideal current source)
- $R_S = 1k\Omega$
- $R_L = 100k\Omega$

a) We need to design this amplifier so that  $R_{in, max} = 100\Omega$ , and  $R_{out, min} = 10M\Omega$ . Calculate the minimum  $W$  and the minimum  $L$  of this transistor. (Note that these specs are very aggressive, so this will not turn out to be a "typical" MOS transistor with "typical" values.) (20 pts)

$R_{in}$

$R_{in}$



$$v_o = i_{test} R_L \quad (1)$$

KCL @ input:

$$i_{test} = g_m v_s + \frac{v_s - v_o}{r_o}$$

Using (1) and noting  $v_s = v_{test}$

$$i_{test} = g_m v_{test} + \frac{v_{test} - i_{test} R_L}{r_o}$$

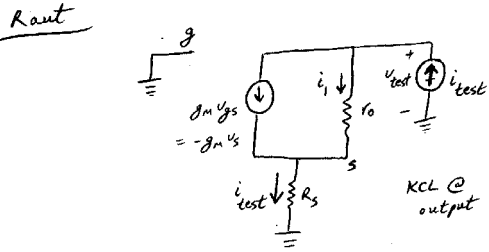
$$\therefore \frac{v_{test}}{i_{test}} = R_{in} = \frac{r_o + R_L}{g_m r_o + 1} \approx \frac{1}{g_m}$$

Now,  $r_o = \frac{L}{\lambda I_D}$  and  $g_m = \sqrt{2\mu C_{ox} \frac{W}{L} I_D}$

w/  $I_D = 100 \mu$ , and setting  $R_{in} = 100 \Omega$   
 $R_{out} = 10 M\Omega$

$$L \approx 9 \mu m$$

$$W \approx 91 \times 10^3 \mu m$$



$$v_s = i_{test} R_S$$

$$\therefore -g_m v_s = -g_m i_{test} R_S$$

$$i_1 = \frac{v_{test} - v_s}{r_o} = \frac{v_{test} - i_{test} R_S}{r_o}$$

KCL @ output:

$$i_{test} + g_m v_s = i_1$$

$$\therefore i_{test} + g_m i_{test} R_S = \frac{v_{test} - i_{test} R_S}{r_o}$$

$$\therefore \frac{v_{test}}{i_{test}} = R_{out} = \frac{r_o + R_S + g_m R_S r_o}{1} \approx r_o (1 + g_m R_S)$$

Value of min W/L

Write Expression for the minimum W/L here

$$\frac{W}{L} = \frac{1}{2 R_{in}^2 \mu C_{ox} I_D}$$

since  $R_{in} \approx \frac{1}{g_m}$   
 w/  $R_{in} = 100$

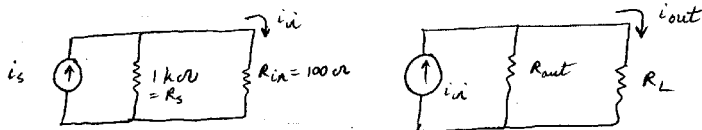
10,000

Write Expression for minimum L here

$$L = \frac{(10 \text{ M}\Omega) \cdot I_D}{(1 + g_m R_s)}$$

Value of min L

$$\approx 9 \mu\text{m}$$

b) Calculate the overall (loaded) current gain, (note that  $V_{b_s}=0\text{V}$ )  $i_{\text{out}}/i_s$ , (10pts)

$$i_{in} = i_s \frac{R_s}{R_s + R_{in}}$$

$$i_{out} = i_{in} \frac{R_{out}}{R_{out} + R_L}$$

$$\therefore A_i = \frac{i_{out}}{i_s} = \frac{i_{in}}{i_s} \cdot \frac{i_{out}}{i_{in}} = \frac{R_s}{R_s + R_{in}} \frac{R_{out}}{R_{out} + R_L}$$

Write Expression for  $i_{\text{out}}/i_s$  here

$$\frac{R_s}{R_s + R_{in}} \frac{R_{out}}{R_{out} + R_L}$$

Value of  $i_{\text{out}}/i_s$ 

$$\approx 0.9$$