

# Electrostatics: A Key to Understanding Electronic Devices

Physics approach: vector calculus, highly symmetrical problems

Gauss' Law:  $\nabla \cdot (\epsilon \vec{E}) = \rho$

Def. of Potential:  $\vec{E} = -\nabla \phi$

Poisson's Eqn.:  $\nabla \cdot (\epsilon (-\nabla \phi)) = -\epsilon \nabla^2 \phi = \rho$

## *Device physics*

Real problems (not symmetrical, complicated boundary conditions)

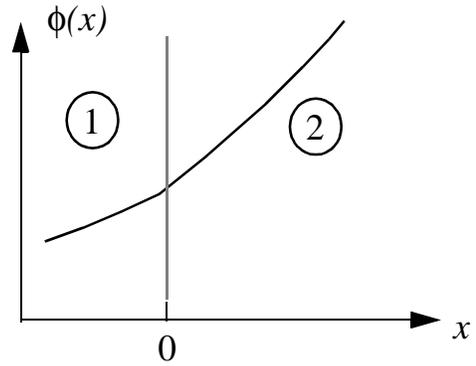
Gauss's Law:  $\frac{d(\epsilon E)}{dx} = \rho$

Definition of Potential:  $E = -\frac{d\phi}{dx}$

Poisson's Equation:  $\frac{d}{dx} \left( \epsilon \left( -\frac{d\phi}{dx} \right) \right) = -\epsilon \frac{d^2 \phi}{dx^2} = \rho$

# Boundary Conditions

1. Potential:  $\phi(x=0^-) = \phi(x=0^+)$



2. Electric Field:  $\epsilon_1 E(x=0^-) + Q = \epsilon_2 E(x=0^+)$

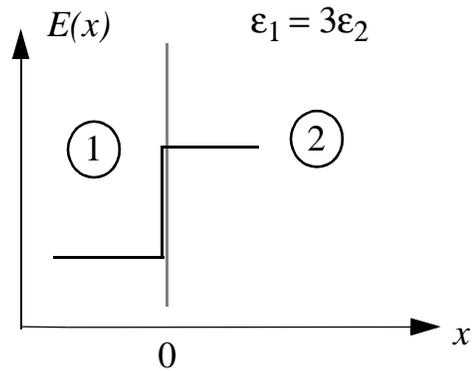
where  $Q$  is a surface charge (units,  $C/cm^2$ ) located at the interface

for the case where  $Q = 0$ :

common materials:

silicon,  $\epsilon_s = 11.7 \epsilon_0$

silicon dioxide ( $SiO_2$ ),  $\epsilon_{ox} = 3.9 \epsilon_0$

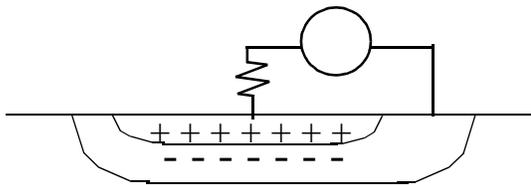


# Intuition for Electrostatics

## Sketching the answer BEFORE doing the math:

- \* Electric field points from positive to negative charge
- \* Electric field points “downhill” on a plot of potential
- \* Electric field is confined to a narrow charged region, in which the positive charge is balanced by an equal and opposite negative charge

why?



charge-storage structures with  
conducting paths between the  
+ and - electrodes ...

- \* The integral form of Gauss’s law is what should be used

Why? It relates the electric field at the *edges of a region* to the charge inside

The electric field on one side is often known to be zero

## Practical Electrostatics (cont.)

- \* Use boundary conditions on the potential or electric field to “patch” together solutions from regions having different material properties
- \* The voltage drop across charged regions is the second integral of  $\frac{-\rho}{\epsilon}$

**Charge density function  $\rho(x)$ : only two cases needed for basic device physics**

$$\rho = 0 \quad \Rightarrow \quad E \text{ constant} \quad \Rightarrow \quad \phi \text{ linear}$$

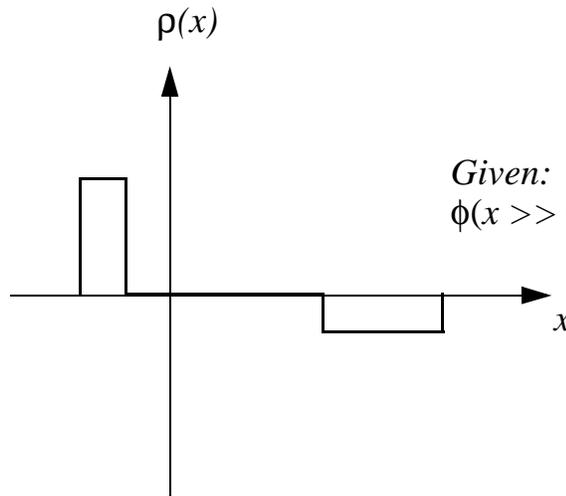
$$\rho = \rho_o = \text{constant} \quad \Rightarrow \quad E \text{ linear} \quad \Rightarrow \quad \phi \text{ quadratic}$$

- \* surface or sheet charge  $Q$  is sometimes present at the boundaries  
use Gauss's Law to determine its effect

# Example I: Applied Electrostatics

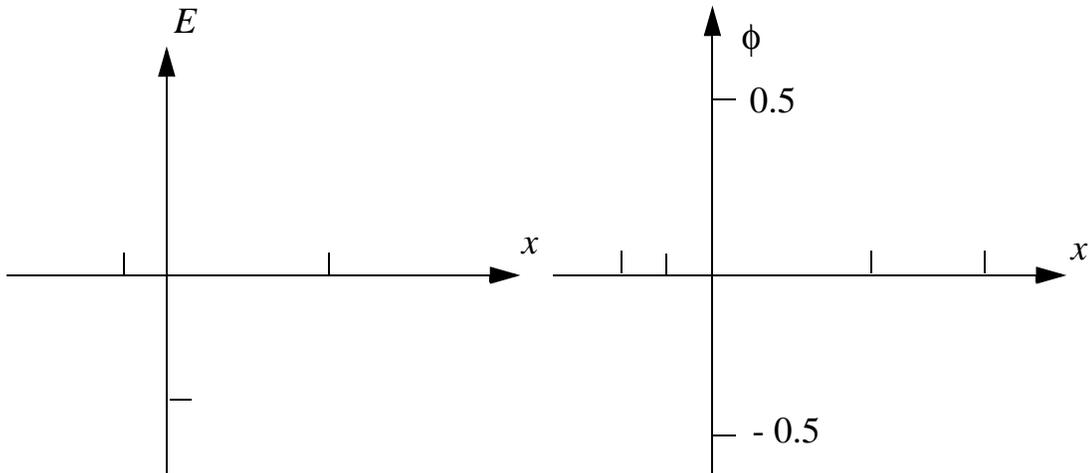
Given:  
charge distribution  
 $\rho(x)$

Given:  
 $\phi(x \ll 0) = 0.5 \text{ V}$



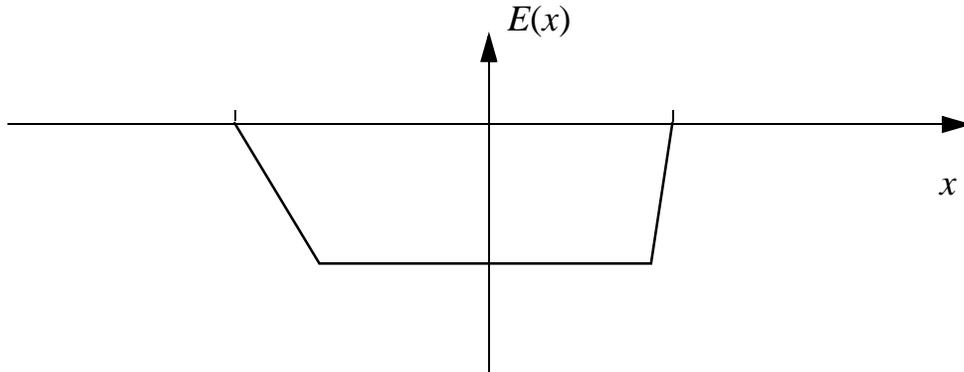
Given:  
 $\phi(x \gg 0) = -0.4 \text{ V}$

Sketch the electric field and the charge

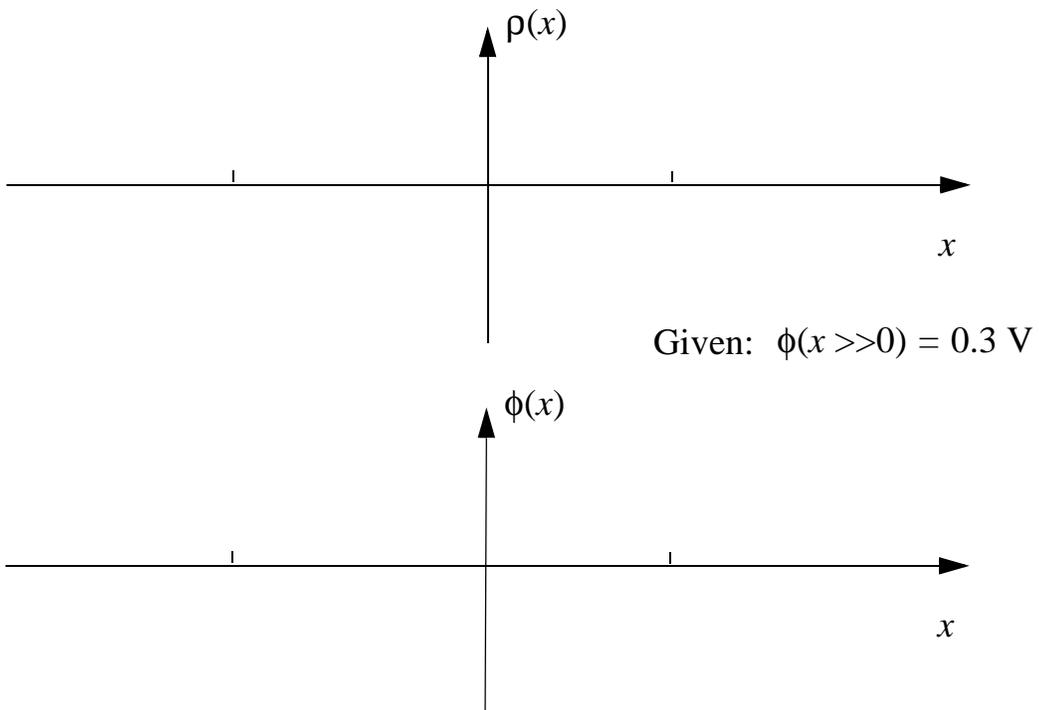


## Example II: Applied Electrostatics

\* Given the electric field,



Sketch the charge density and the potential



## Application of Gauss's Law

- \* At a point  $x$ , the electric field can be found as the charge enclosed, divided by the permittivity of the material...

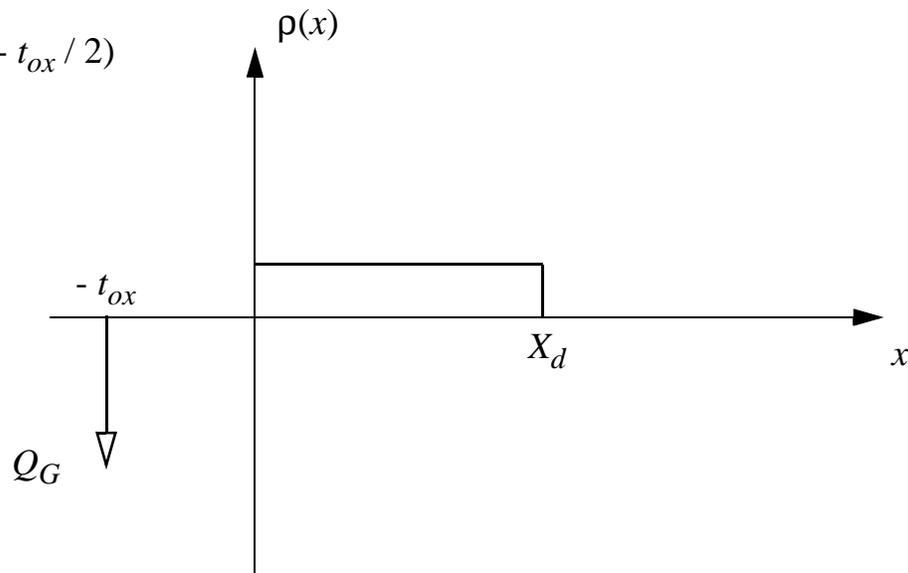
*caveats* (warnings):

(i) the field must be zero at the other side of the charged region

(ii) the sign of the field can be found by keeping track of the  $+x$  direction and the one-dimensional equivalent of the "outward normal;" however, the best approach is to know the sign of the field from the distribution of charge in the problem

- \* Example: metal-oxide-silicon structure

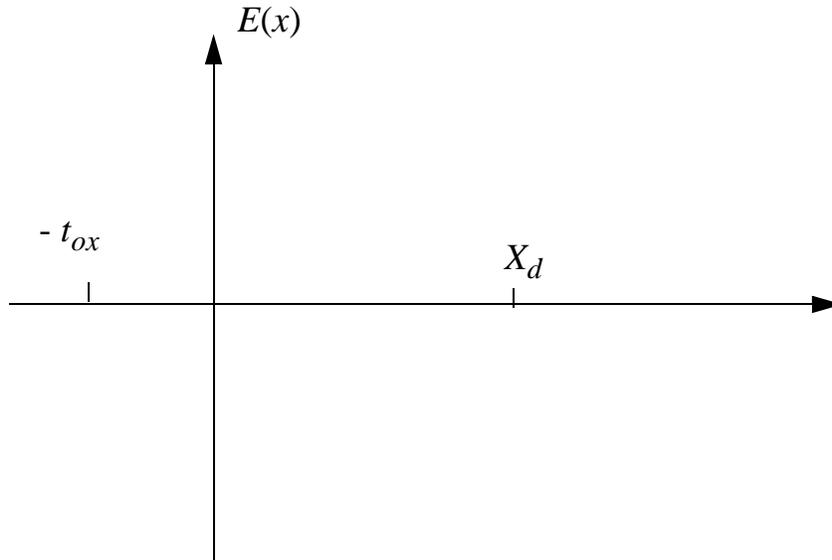
Find  $E(x = -t_{ox} / 2)$



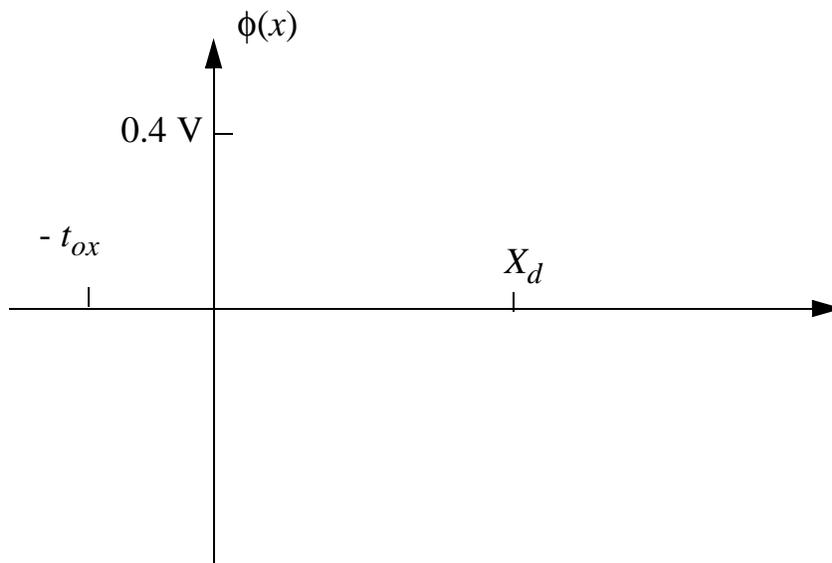
- \* Find  $E(x = 0^+) \dots$  just inside the silicon

## Boundary Condition on $E$ (cont.)

- \* Sketch  $E(x)$  from  $x = -t_{ox}$  to  $x = X_d$



Sketch  $\phi(x)$  through the structure, given that  $\phi(X_d) = 400$  mV

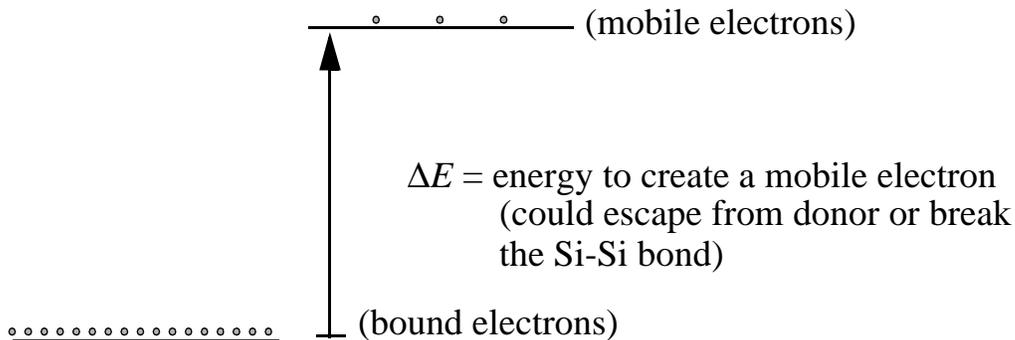


# Potential and Carrier Concentration in Silicon

\* Question

In thermal equilibrium, can there be potential differences inside silicon?

Surprising answer is, **yes!**



$$n_o = Ke^{-(\Delta E/kT)} = Ke^{-((-q\phi)/kT)} = Ke^{q\phi/kT}$$

Therefore, in thermal equilibrium the potential  $\phi$  changes whenever the electron concentration varies from position to position. (e.g., variable doping)

Pick the zero reference for  $\phi$  where the electron concentration is  $n_i$

$$n_i = Ke^{q(0)/(kT)} = K \quad \text{therefore} \quad n_o = n_i e^{q(\phi/kT)}$$

## Carrier Concentration and Potential (cont.)

The relationship can be inverted by taking the  $\ln = \log_e$  of both sides

$$\ln \left[ \frac{n_o}{n_i} \right] = \frac{q\phi}{kT} = \frac{\phi}{V_{th}} \quad \text{therefore} \quad \phi = V_{th} \ln \left[ \frac{n_o}{n_i} \right]$$

Note that we use the symbol  $n_o$ , to remind us that we are only dealing with the electron concentration in thermal equilibrium.

It would be nice to be able to find the potential in terms of the electron concentration instantaneously, without a calculator ...

$$\phi = V_{th} \ln \left[ \frac{n_o}{n_i} \right] = (26\text{mV})(\ln(10)) \log \left( \frac{n_o}{10^{10}} \right) = (60\text{mV}) \log \left[ \frac{n_o}{10^{10}} \right]$$

This expression is called “the 60 mV rule”

Donor concentrations from  $10^{13}$  to  $10^{19} \text{ cm}^{-3}$  therefore correspond to potentials of

$$\phi = (60 \text{ mV}) \times 3 = 180 \text{ mV to}$$

$$\phi = (60 \text{ mV}) \times 9 = 540 \text{ mV (at room temperature)}$$

## The 60 mV Rule

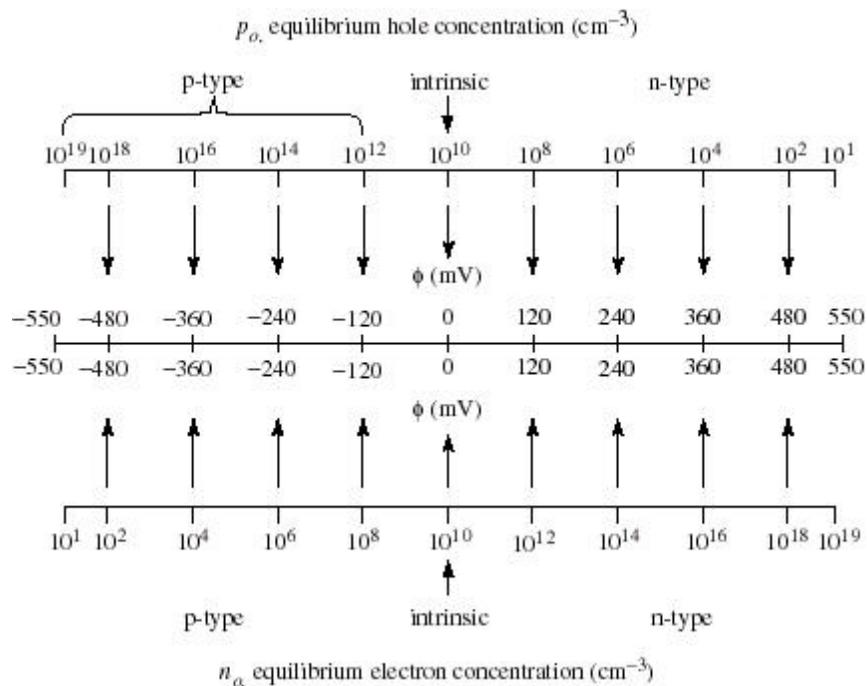
The hole concentration can also be related to the potential, by substituting

$$p_o = n_i^2 / n_o$$

into the 60 mV rule for electrons. The result is:

$$\phi = (26\text{mV})\ln(10)\log\left(\frac{10^{20}}{p_o 10^{10}}\right) = (-60\text{mV})\log\left(\frac{p_o}{10^{10}}\right)$$

The potential in p-type silicon (where  $p_o > 10^{10} \text{ cm}^{-3}$ ) is negative ... with respect to n-type or intrinsic silicon



## “Built-in Voltages”

We see that in thermal equilibrium, doping variations lead to potential variations. We call the difference in potential from such doping variations the “built-in” voltage.

Example: The what is the built in voltage across a region in which the doping changes from p type,  $p_o = 10^{17}/\text{cm}^3$  to n-type,  $n_o = 10^{17}/\text{cm}^3$  ?

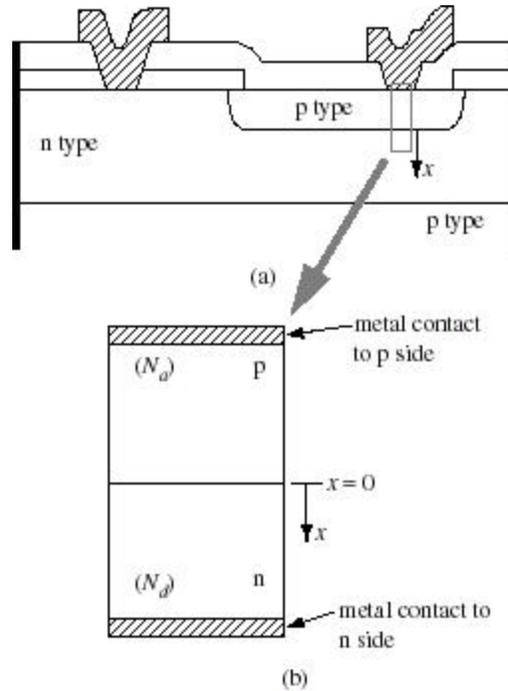
Answer:

Won't such a built in voltage lead to a current (in thermal equilibrium), violating the second law of thermodynamics?

Answer:

# pn Junctions

- \* ubiquitous IC structure -- pn junctions are everywhere!

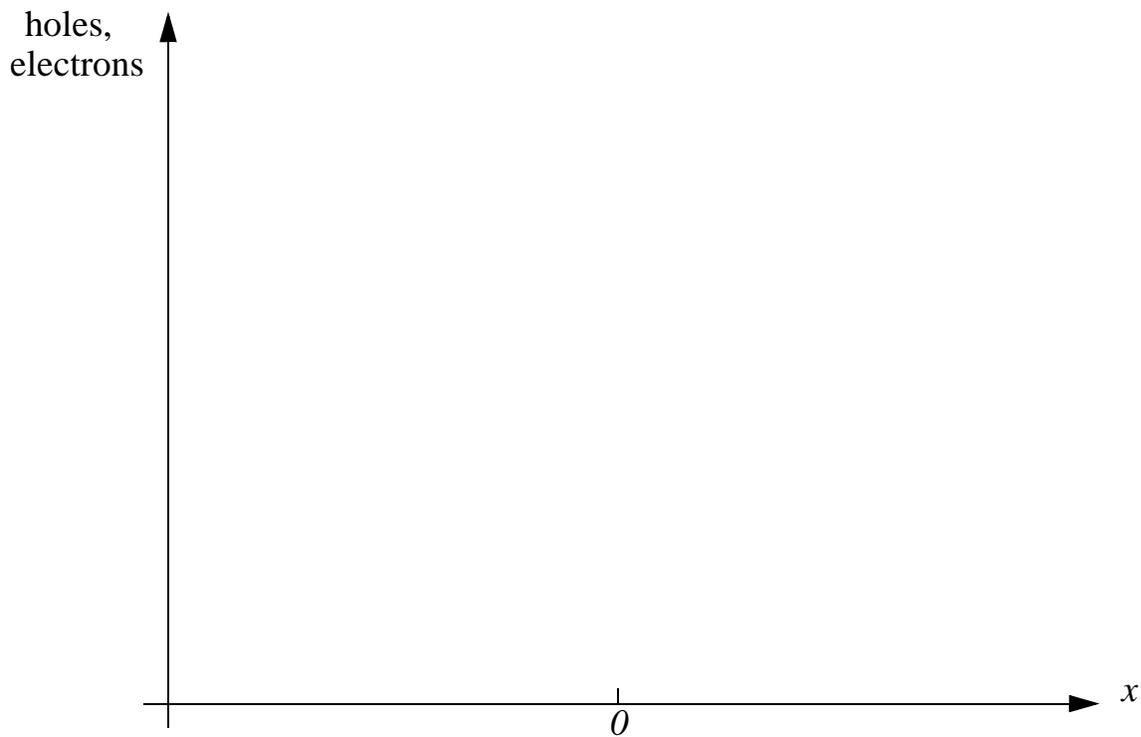


- \* thermal equilibrium: no hole current, no electron current ... no voltage applied between metal interconnects (could short them together)

# pn Junction in Thermal Equilibrium

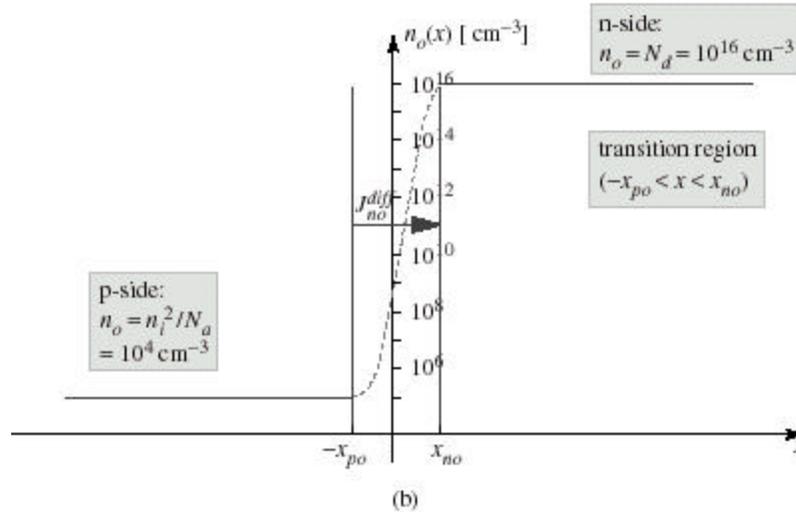
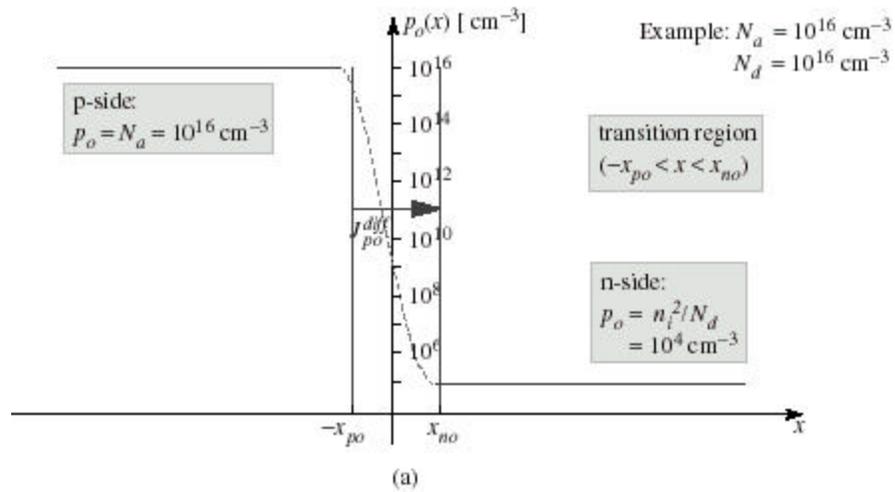
Basic observations:

1. total current density is zero
2. total electron and total hole current densities are separately zero
3. BUT electron and hole diffusion currents are HUGE near the junction
  - ... there must be cancelling drift currents
  - ... where do the electric fields come from that drive the drift currents?



# Diffusion Currents in Thermal Equilibrium

- \* Assume a transition region between  $-x_{po}$  and  $+x_{no}$  (don't know how wide yet)



*note:* we don't know how wide the transition region is (yet)

# Drift and Diffusion in the Transition Region

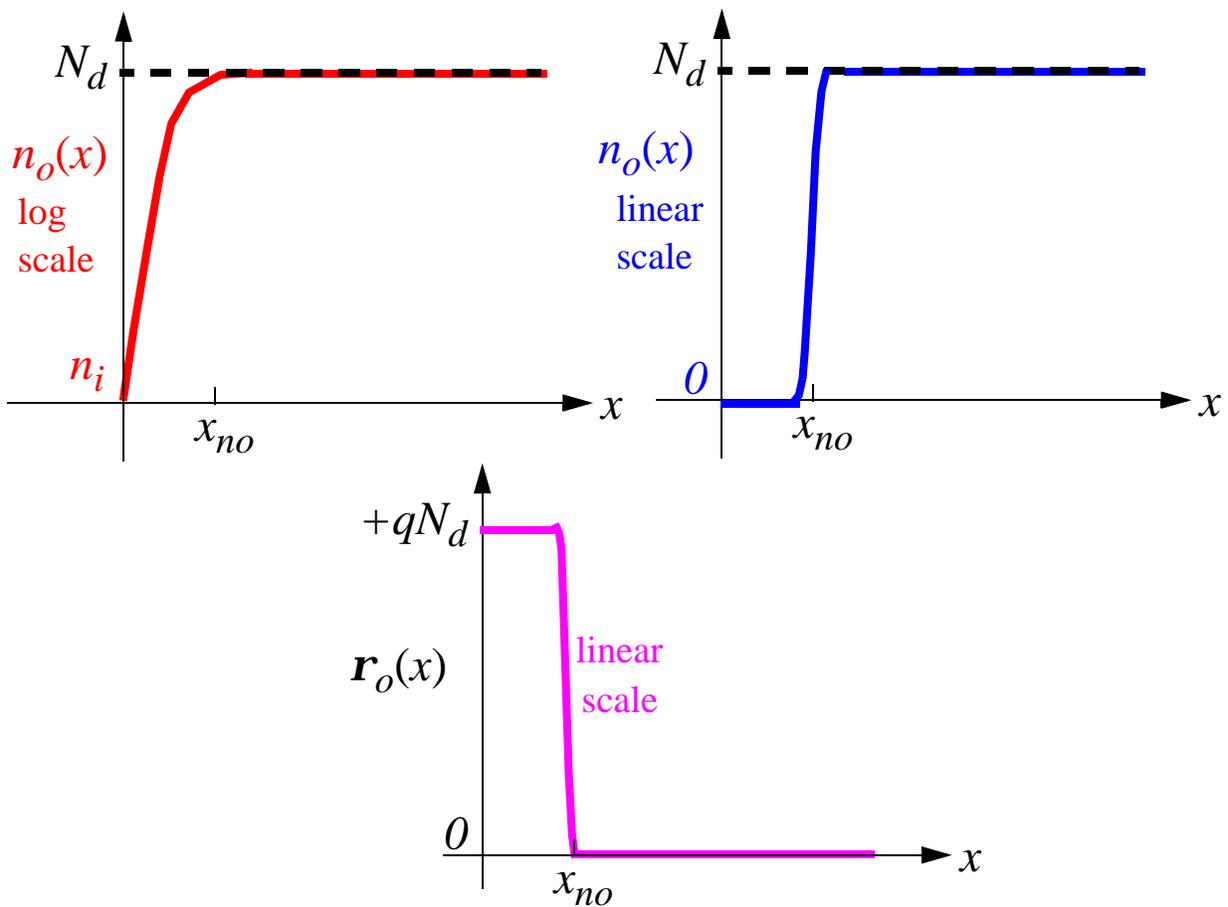
\*  $J_{no} = 0$  and  $J_{po} = 0$  due to equilibrium

\* negative electric field in the transition region is needed ...

where do + and - charges come from?

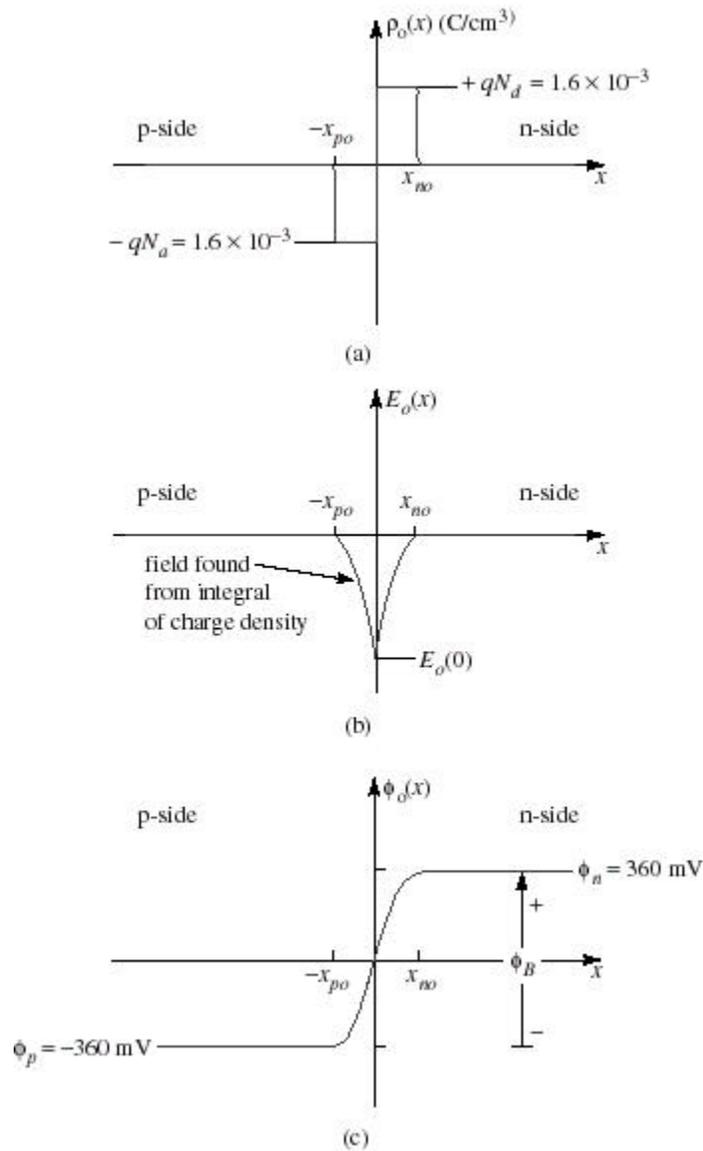
\* *Answer:* the roll-off in electron concentration between  $x = 0$  and  $x_{no}$  means that

$$\rho_o(x) = q(-n_o(x) + N_d) > 0$$



# Qualitative Electrostatics in Equilibrium

- \* The p-side of the transition region has a negative charge density that is opposite to the charge on the n-side
- \* From the charge density, we can find the electric field and the potential



# Quantitative pn Junction in Thermal Equilibrium

## The Depletion Approximation

- \* In the bulk regions far away from the junction, we can approximate

$$\rho_o = 0$$

- \* Near the junction, the charge density is non-zero. For example, on the n-side of the junction in the transition region,  $0 < x < x_{no}$ :

$$\rho_o = q (p_o + N_d - n_o - N_a) = q (N_d - n_o)$$

since there are no acceptors on this side ( $N_a = 0$ ) and the hole concentration is negligible

The maximum positive value for charge density on the n-side is when there are *no electrons present in equilibrium* -- that is, when the silicon in the transition region is *depleted* of electrons.

For hand calculations, we will assume that

$$\rho_o = \rho_{o,max} = q N_d \quad (0 < x < x_{no})$$

$$\rho_o = -q N_a \quad (-x_p < x < 0)$$

and proceed to find the width of the transition region, which we will rename the *depletion region*. The charge density is assumed to fall off *abruptly* from these values to zero in the bulk regions, where  $x < -x_{po}$  and  $x > x_{no}$

## One more time ...

- \* Bulk silicon is *NEUTRAL*, to a good approximation

>> region 1 is *bulk*:

$$\rho_o = q(N_d + p_o - N_a - n_o) \cong 0 \quad p_o \cong N_a$$

>> region 4 is *bulk*:

$$\rho_o = q(N_d + p_o - N_a - n_o) \cong 0 \quad n_o \cong N_d$$

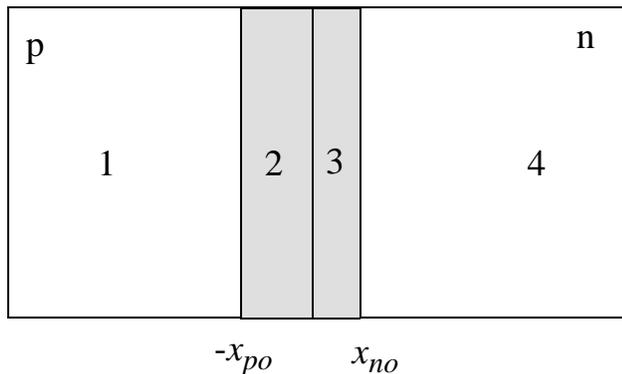
- \* Near the junction, the silicon is *DEPLETED* of mobile carriers:

>> region 2 is *depleted*:

$$\rho_o = q(N_d + p_o - N_a - n_o) \cong -qN_a$$

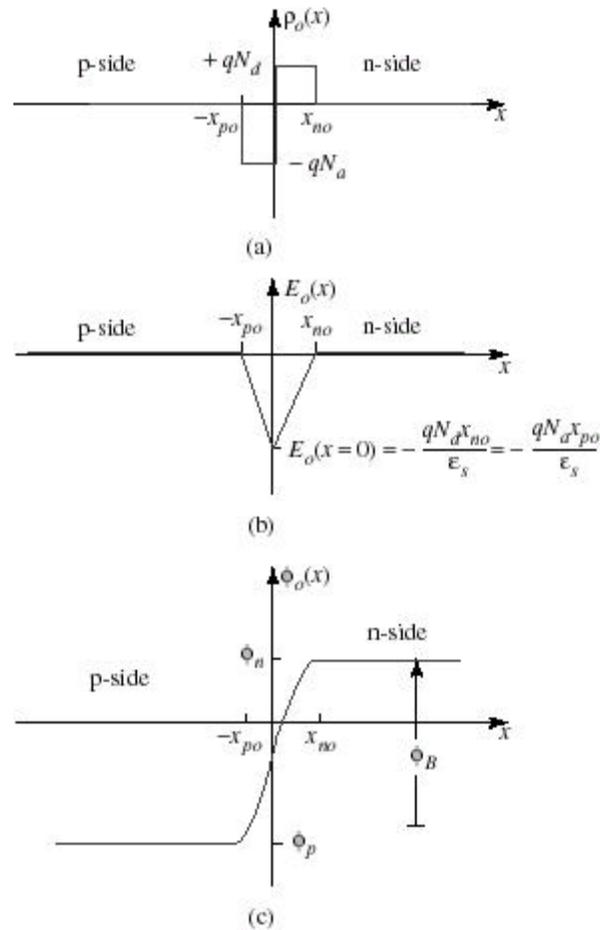
>> region 3 is *depleted*:

$$\rho_o = q(N_d + p_o - N_a - n_o) \cong qN_d$$



$x_{no}$ ,  $x_{po}$  are not known yet -- use boundary conditions to find them

## pn Junction in Thermal Equilibrium: Using the Depletion Approximation



- \* For detailed calculations, see H&S Section 3.4. Analysis is straightforward, but involved. Use the fact that:
  - > Charge in depletion region must sum to zero (why?)
  - > Electrostatic potential is continuous

## Depletion Widths in Thermal Equilibrium

$$x_{po} = \sqrt{\left(\frac{2\epsilon_s\phi_B}{qN_a}\right)\left(\frac{N_d}{N_d+N_a}\right)}$$

$$x_{no} = \sqrt{\left(\frac{2\epsilon_s\phi_B}{qN_d}\right)\left(\frac{N_a}{N_d+N_a}\right)}$$

$$X_{do} = x_{no} + x_{po} = \sqrt{\left(\frac{2\epsilon_s\phi_B}{q}\right)\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$$

- \* The barrier voltage

$$\phi_B = \phi_n - \phi_p$$

- \* Asymmetric junctions: i.e.,  $N_a \gg N_d$  or  $N_d \gg N_a$ .

>> most of depletion width is on the side with the *lower* doping, since

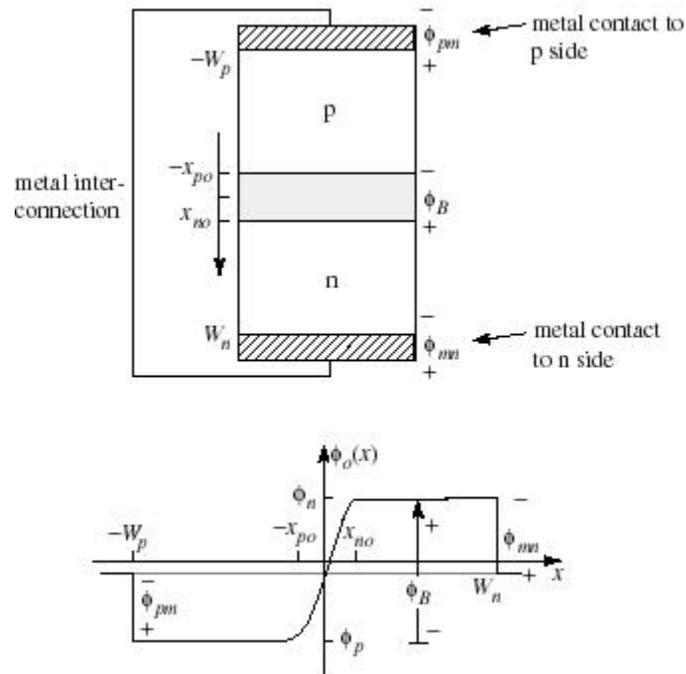
$$\frac{1}{N_a} + \frac{1}{N_d} \approx \frac{1}{N_d} \quad (N_a \gg N_d)$$

$$\frac{1}{N_a} + \frac{1}{N_d} \approx \frac{1}{N_a} \quad (N_d \gg N_a)$$

>> most IC pn junctions are highly asymmetric

## pn Junction under Reverse Bias

- \* First, we must understand the *complete* structure of the pn junction in thermal equilibrium:



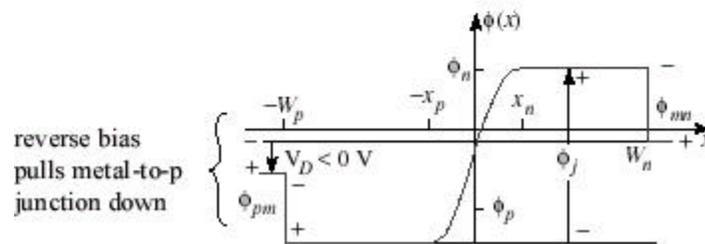
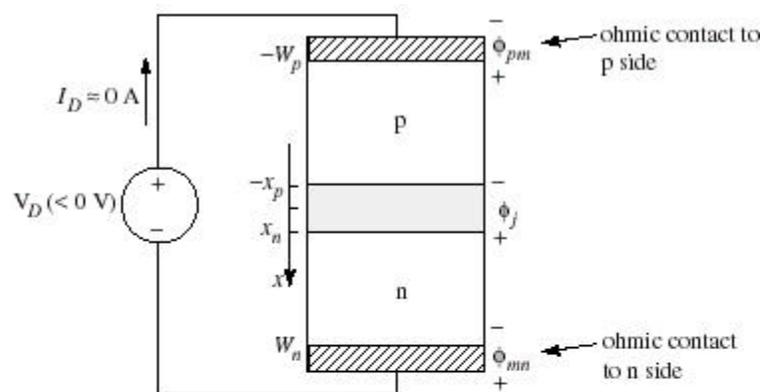
- \* How can  $V_D = 0$  and the built-in potential barrier be  $\phi_B = 1$  V (approx.)?  
 Answer: look at the *complete* circuit ... including the potential barriers at the p-type silicon-to-metal ( $\phi_{pm}$ ) and the metal-to-n-type silicon ( $\phi_{nn}$ ) junctions.
- \* Kirchhoff's Voltage Law:

$$-\phi_{pm} - \phi_B - \phi_{nn} = 0 \dots \text{NOT an accident!}$$

$$\phi_B = \phi_{pm} + \phi_{nn}$$

## Potential Plot through pn Junction

- \* We will define the applied voltage  $V_D$  as positive to p-side  
 $V_D > 0$  corresponds to forward with  $V_D < 0$  meaning reverse bias
- \* Add a battery  $V_D$  ... with  $V_D < 0$  (reverse bias now, forward bias in Chap. 6)



## pn Junction under Reverse Bias (cont.)

- \* Potential plot under reverse bias: contact potentials don't change ... they are *ohmic* contacts. Only place for change is at the pn junction
- \* The new potential barrier is called  $\phi_j$ . Find it using KVL again

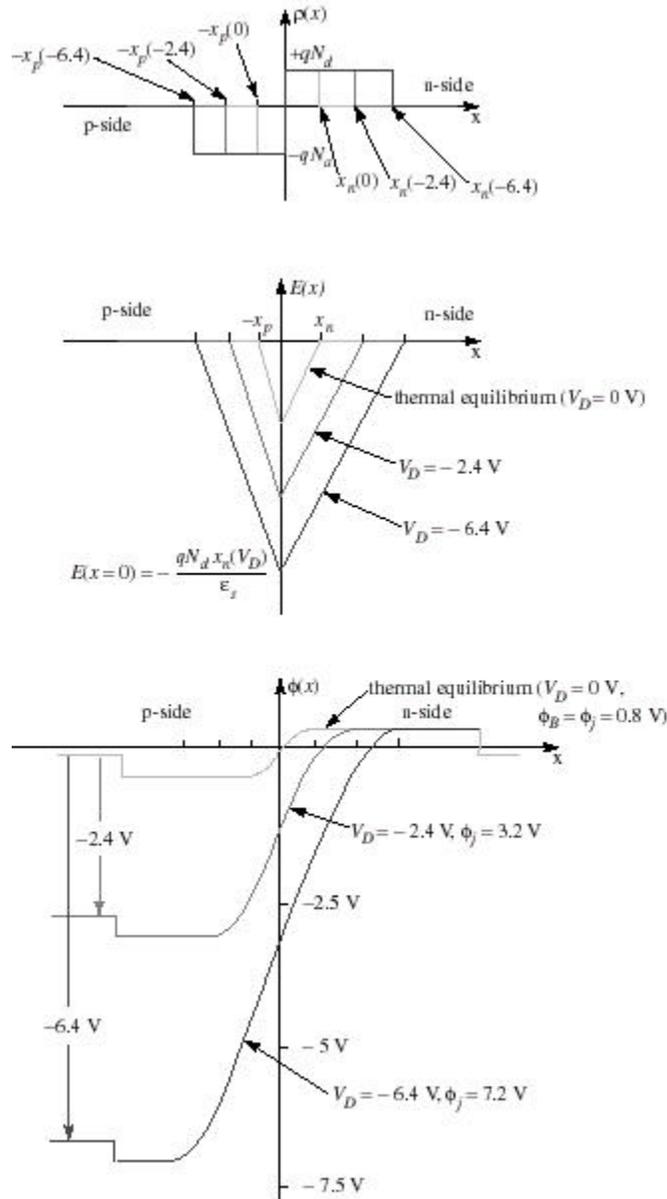
$$-V_D - \phi_{pm} - \phi_j - \phi_{mn} = 0$$

since the sum of the contact potentials is the built-in barrier

$$\phi_j = -V_D + \phi_{pm} + \phi_{mn} = \phi_B - V_D$$

- \* The potential barrier is *increased* over the built-in barrier by the reverse bias ... which widens the depletion region ( $x_n > x_{no}$ ,  $x_p > x_{po}$ ), but
- \* The solution using the depletion approximation is exactly the same; just replace  $\phi_B$  by  $(\phi_B - V_D)$

# Quantitative Reverse Bias Electrostatics



## Quantitative Results

Substitute  $\phi_j$  for  $\phi_B$  in the equilibrium depletion width and we find the depletion width under reverse bias (the math is the same):

$$x_p(V_D) = \sqrt{\left(\frac{2\epsilon_s(\phi_B - V_D)}{qN_a}\right)\left(\frac{N_d}{N_d + N_a}\right)} = x_{po}\sqrt{1 - (V_D/\phi_B)}$$

$$x_n(V_D) = \sqrt{\left(\frac{2\epsilon_s(\phi_B - V_D)}{qN_d}\right)\left(\frac{N_a}{N_d + N_a}\right)} = x_{no}\sqrt{1 - (V_D/\phi_B)}$$

$$X_d(V_D) = \sqrt{\left(\frac{2\epsilon_s(\phi_B - V_D)}{q}\right)\left(\frac{1}{N_a} + \frac{1}{N_d}\right)} = X_{do}\sqrt{1 - (V_D/\phi_B)}$$

\* Note  $x_{po}$ ,  $x_{no}$ , and  $X_{do}$  are the widths in thermal equilibrium

n+/p step junction: simplify general results

$$x_d =$$