

## CE Frequency Response

- The "exact" analysis is worked out on pp. 639-641 of H&S.

$$\frac{V_{out}}{V_s} = \frac{-g_m \left( \frac{r_\pi}{r_\pi + R_S} \right) (r_o \parallel r_{oc} \parallel R_L) (1 - j\omega/\omega_z)}{(1 + j\omega/\omega_{p1})(1 + j\omega/\omega_{p2})}$$

The low-frequency voltage gain (loaded) is  $-g_m \left( \frac{r_\pi}{r_\pi + R_S} \right) (r_o \parallel r_{oc} \parallel R_L)$

The zero is higher than the transition frequency  $\omega_T = g_m / (C_\pi + C_\mu)$

The first (lowest frequency) pole is (approximately)

$$\omega_{p1} = \{ (R_S \parallel r_\pi) [C_\pi + (1 + g_m R_{out}') C_\mu] + R_{out}' C_\mu \}^{-1}$$

where  $R_{out}' = r_o \parallel r_{oc} \parallel R_L$

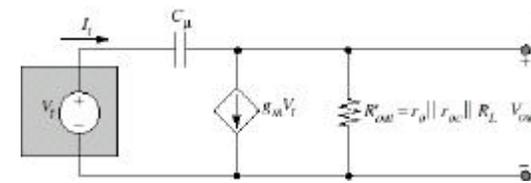
The second pole is (approximately)

$$\omega_{p2} = \frac{(R_S \parallel r_\pi)(R_{out}')}{(R_S \parallel r_\pi) [C_\pi + (1 + g_m R_{out}') C_\mu] + R_{out}' C_\mu}$$

- Brute force analysis is not particularly helpful for gaining insight into the frequency response ...

## The Miller Approximation

- Therefore, we consider the effect of  $C_\mu$  on the input node only



neglect the feedforward current  $I_\mu$  in comparison with  $g_m V_\pi$  ... a good approximation

$$I_t = (V_t - V_o) / Z_\mu$$

$$V_o = -g_m V_t R_L / (R_L + R_{out}') = A_{vC_\mu} V_t$$

where  $A_{vC_\mu}$  is the low frequency voltage gain across  $C_\mu$

$$I_t = V_t (1 - A_{vC_\mu}) / Z_\mu$$

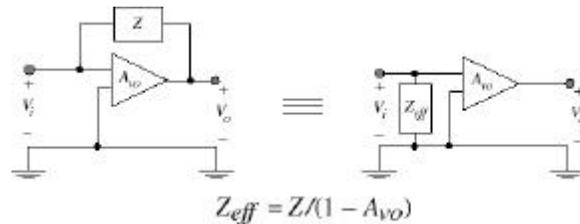
$$Z_{eff} = V_t / I_t = Z_\mu / (1 - A_{vC_\mu})$$

$$Z_{eff} = \frac{1}{j\omega C_\mu} \left( \frac{1}{1 - A_{vC_\mu}} \right) = \frac{1}{j\omega (C_\mu (1 - A_{vC_\mu}))} = \frac{1}{j\omega C_M}$$

$C_M = (1 - A_{vC_\mu}) C_\mu$  is the **Miller capacitor**

## Generalized Miller Approximation

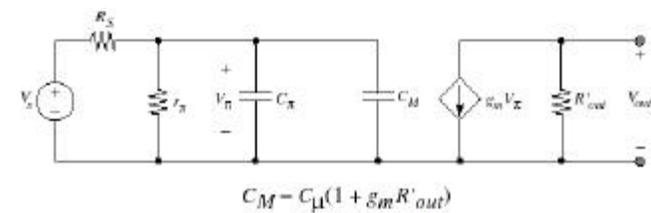
- An impedance  $Z$  connected across an amplifier with voltage gain  $A_{vZ}$  can be replaced by an impedance to ground ... multiplied by  $(1 - A_{vZ})$



- Common-emitter and common-source:  
 $A_{vZ} = \text{large and negative for } C_{\mu} \text{ or } C_{gd} \rightarrow \text{capacitance at the input is **magnified**}$
- Common-collector and common-drain:  
 $A_{vZ} \approx 1 \rightarrow \text{capacitance at the input due to } C_{\pi} \text{ or } C_{gs} \text{ is greatly **reduced**}$

## Voltage Gain vs. Frequency for CE Amplifier Using the Miller Approximation

- The Miller capacitance is lumped together with  $C_{\pi}$ , which results in a single-pole low-pass RC filter at the input



- Transfer function has one pole and no zero after Miller approximation:

$$\omega_{3dB}^{-1} = (r_{\pi} \parallel R_S)(C_{\pi} + C_M)$$

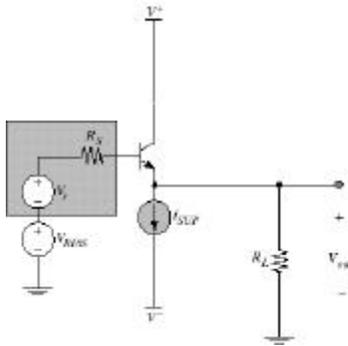
$$\omega_{3dB}^{-1} = (r_{\pi} \parallel R_S)[C_{\pi} + (1 + g_m r_o \parallel r_{oc} \parallel R_L)C_{\mu}]$$

$$\omega_{3dB}^{-1} \approx \omega_1^{-1} \text{ from the exact analysis (final term } R_{out}'C_{\mu} \text{ is missing)}$$

- The break frequency is reduced by the Miller effect ... which results in a large Miller capacitor at the input.

## Common Collector Frequency Response

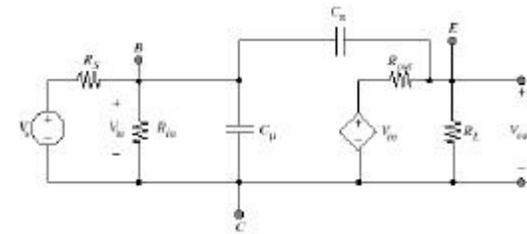
Basic approach: two-port model from Chapter 8, “decorated” with device capacitances



The DC output voltage  $V_{OUT}$  is selected to be 0 V, so that the output voltage is just the small-signal voltage  $v_{out}$  (phasor representation  $V_{out}$ )

## Common-Collector Frequency Response

- Voltage buffer two-port model has input at base and output at emitter:



Note carefully where the capacitances are connected!

“Millerize” the base-emitter capacitor: gain across it is  $A_{v\pi} = \frac{R_L}{R_L + R_{out}}$

$$C_M = C_\pi(1 - A_{v\pi}) = C_\pi \left( 1 - \left( \frac{R_L}{R_L + R_{out}} \right) \right) = C_\pi \left( \frac{R_{out}}{R_L + R_{out}} \right)$$

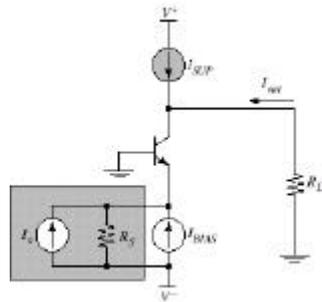
The break frequency of the common-collector is:

$$\omega_{-3dB} = \frac{1}{(R_S || R_{in})(C_\mu + C_M)}$$

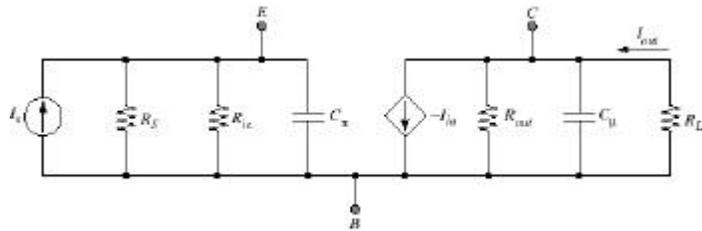
can approach the transition frequency for small source resistances

## Common-Base Frequency Response

Similar approach: start with two-port and add capacitors ....



Two-port model:



## Common-Base Frequency Response

- Poles are separate (no coupling between input and output)

$$\omega_{p, in} = \frac{1}{(R_S || R_{in})C_\pi}$$

$$\omega_{p, out} = \frac{1}{(R_{out} || R_L)C_\mu}$$

- The input pole is beyond the transition frequency, due to the low value of  $R_{in} = 1/g_m$
- The output pole is a function of  $R_L$  since the output resistance is so large. For small load resistances, the output pole can approach the transition frequency

Summary of single-stage amplifiers:

CE/CS: Miller effect decreases the bandwidth severely

CC/CD: wideband

CB/CG: wideband

## Multistage Amplifiers

- Single-stage transistor amplifiers are inadequate for meeting most design requirements for **any** of the four amplifier types (voltage, current, transconductance, and transresistance.)
- Therefore, we use more than one amplifying stage. The challenge is to gain insight into when to use which of the **9** single stages that are available in a modern BiCMOS process:

*Bipolar Junction Transistor:* CE, CB, CC -- in npn and pnp\* versions

*MOSFET:* CS, CG, CD -- in n-channel and p-channel versions

\* in most BiCMOS technologies, only the npn BJT is available

- How to design multi-stage amplifiers that satisfy the required performance goals?

\* **Two fundamental requirements:**

### 1. Impedance matching:

output resistance of stage  $n$ ,  $R_{out, n}$  and input resistance of stage  $n + 1$ ,  $R_{in, (n+1)}$ , must be *in the proper ratio*

$$R_{in, (n+1)} / R_{out, n} \rightarrow \infty \quad \text{or} \quad R_{in, (n+1)} / R_{out, n} \rightarrow 0$$

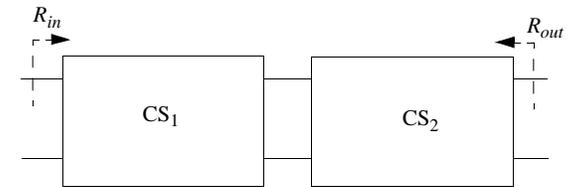
to avoid degrading the overall gain parameter for the amplifier

### 2. DC coupling:

we will directly connect stages: effect on DC signal levels must be considered, too

## Example 1: Cascaded Voltage Amplifier

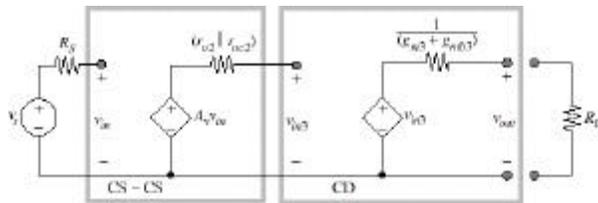
- Want  $R_{in} \rightarrow \infty$ ,  $R_{out} \rightarrow 0$ , with high voltage gain.  
Try CS as first stage, followed by CS to get more gain ... use 2-port models



- solve for overall voltage gain ... higher, but  $R_{out} = R_{out2}$  which is too large

### Three-Stage Voltage Amplifier

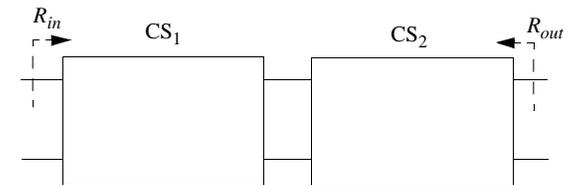
- Fix output resistance problem by adding a common drain stage (voltage buffer)



- Output resistance is not that low ... few  $k\Omega$  for a typical MOSFET and bias --> could pay an area penalty by making  $(W/L)$  very large to fix.

### Cascaded Transconductance Amplifier

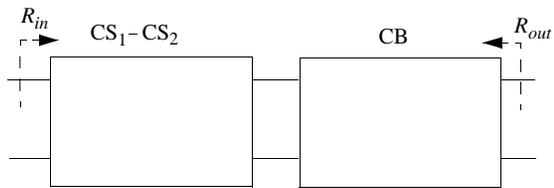
- input resistance should be high; output resistance should also be high  
initial idea: use CS stages (they are "natural" transconductance amps)



- Overall  $G_m = -g_{m1} (r_{o1} \parallel r_{oc1}) g_{m2} = A_{v1} g_{m2} \dots$  can be very large  
BUT, output resistance is only moderately large ... need to increase it

## Improved Transconductance Amplifier

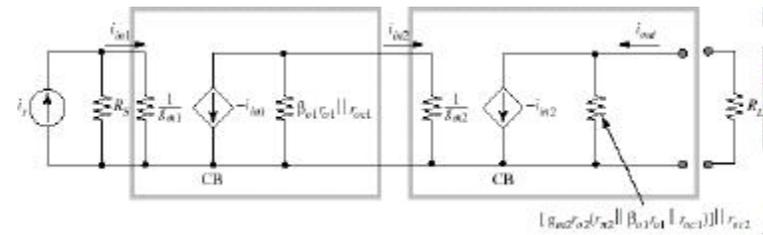
- Output resistance: boost using CB or CG stage



- high-source resistance current sources are needed to avoid having  $r_{oc3}$  limit the resistance

## Two-Stage Current Buffers

- since one CB stage boosted the output resistance substantially, why not add another one ...



- The base-emitter resistance of the 2<sup>nd</sup> stage BJT is  $r_{\pi 2}$  which is much less than the 2<sup>nd</sup> stage source resistance = 1<sup>st</sup> stage output resistance

$$R_{S2} = R_{out1} = \beta_{o1} r_{o1} \parallel r_{oc1}$$

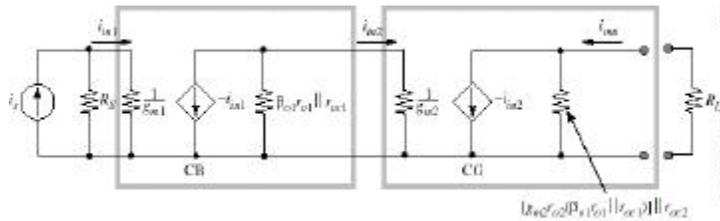
- Therefore, the output resistance expression reduces to

$$R_{out} \approx g_{m2} r_{o2} r_{\pi 2} \parallel r_{oc2} = \beta_{o2} r_{o2} \parallel r_{oc2}$$

... no improvement over a single CB stage

## Improved Two-Stage Current Buffer: CB/CG

- The addition of a common-gate stage results in further increases in the output resistance, making the current buffer closer to an ideal current source at the output port



- The product of transconductance and output resistance  $g_{m2} r_{o2}$  can be on the order of 500 - 900 for a MOSFET -->  $R_{out}$  is increased by over two orders of magnitude ... practical limit ... on the order of 100 M $\Omega$

Of course, the current supply for the CG stage has to have at least the same order of magnitude of output resistance in order for it not to limit  $R_{out}$ .

- General “resistance matching” ... try not to lose much in doing a current divider or a voltage divider. Which of these is appropriate depends on whether the signal is current or voltage at the port.